

# DESIGN OF STEEL PORTAL FRAME BUILDINGS TO EUROCODE 3





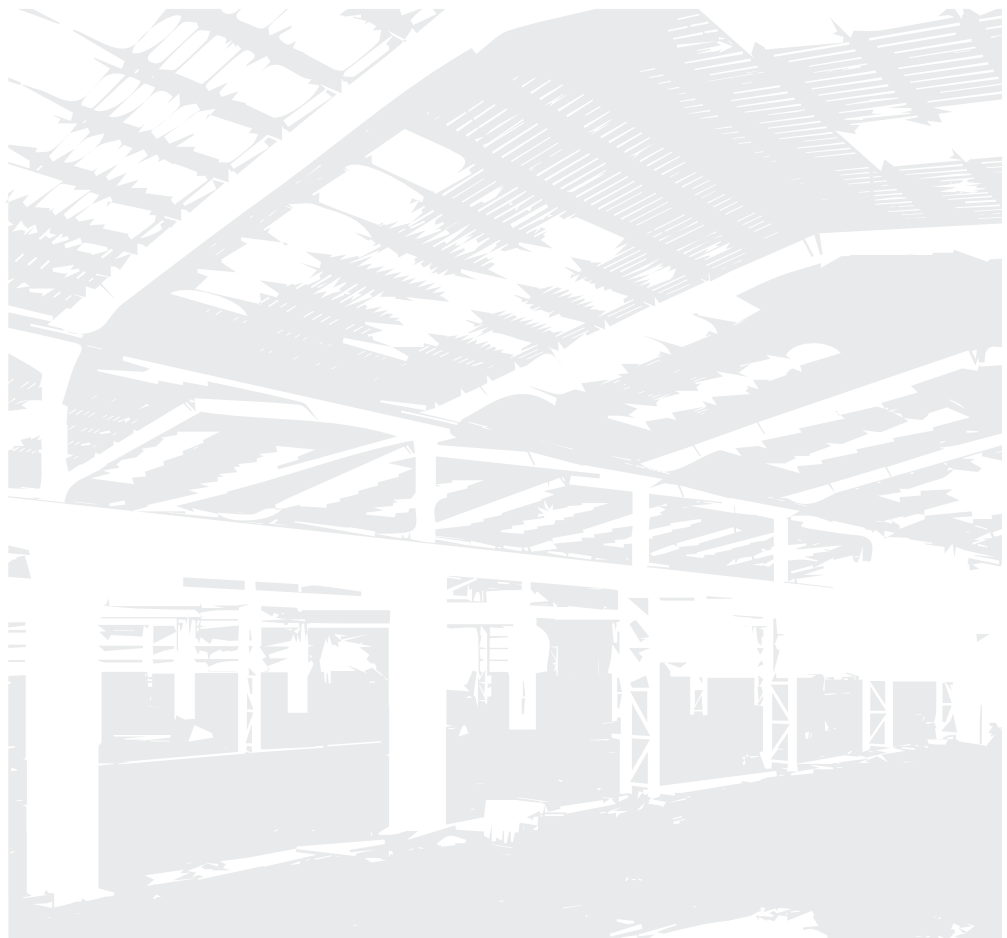
# DESIGN OF STEEL PORTAL FRAME BUILDINGS TO EUROCODE 3





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**J R Henderson** BSc, PhD, CEng MStructE





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# FOREWORD

Steel portal frames are the most common form of single-storey construction in the UK, usually designed using bespoke software. Whereas the design of portal frames was well-covered in BS 5950-1, the guidance in BS EN 1993-1-1 is less detailed and not UK-specific. This publication extends the guidance presented in publications P397 and P400. It deals with portal frames with more than one bay, “hit and miss” frames, plastic analysis and design as well as elastic. It deals with issues which are not covered by BS EN 1993-1-1 on elastic checks on haunches and gives guidance on the effects of initial imperfections based on research carried out over the last two years.

P397 remains useful as it contains worked examples not retained in the present publication. The in-plane buckling checks of column and rafter considered individually, presented in P397 are superseded by in-plane verification of the frame considered as a whole.

It is assumed that the reader is familiar with the anatomy of a portal frame.

A number of industry experts contributed to the development of this publication and their support is acknowledged with gratitude. The expert group comprised:

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# SUMMARY

This publication provides guidance for the elastic and plastic design of portal frames in the UK in accordance with the Eurocodes and their UK National Annexes.

The publication gives an overview of the main portal frame elements, loading and initial design before providing more detail on frame analysis and the design of the major components.

The key technical issues that differ from previous practice are:

- The assessment of frame stability (the significance of second order effects);
- The allowance for second order effects, if these are significant;
- The inclusion of allowances for imperfections in the analysis;

In addition, member verifications for situations where the Eurocode is silent:

- Elastic sections with restraint to one flange;
- Stability of haunches (elastic).





# INTRODUCTION

Approximately 50% of constructional steelwork in the UK is used in the primary framework of single-storey buildings. In this market sector portal frames are the most common structural form in pitched roof buildings. Portal frames are lightweight, efficient and familiar to UK designers in both design and detailing. This form of construction was comprehensively covered in BS 5950-1<sup>[1]</sup>, which devoted a whole section to advice on portal frame design. BS EN 1993-1-1<sup>[2]</sup> does not cover portal frames in such depth, but provides design principles and general application rules.

This publication guides the designer through the detailed steps involved in the design of portal frames to BS EN 1993-1-1. The geometry of portal frames is such that good understanding can be gained by considering the structural behaviour of 2-D frames in plane and out of plane separately. Some portal frame design software packages are written in this way. Guidance is provided on the manual methods used for initial sizing, determination of actions, assessment of frame stability and verification of members in accordance with BS EN 1993-1-1. The intention is to allow the reader to develop a thorough understanding of the structural behaviour and design process to assist in evaluating results from software. The importance of appropriate design details is emphasised, with good practice illustrated.

Manual design may be useful for initial sizing of members but it is readily acknowledged that using software is a more realistic approach for efficient design which provides the means to achieve the greatest structural efficiency. Widely available bespoke software for portal frame design will:

- Undertake elastic-plastic analysis;
- Allow for second order effects;
- Verify members;
- Verify connections.

Generally, a number of different load combinations will have to be considered during the design of a portal frame. Software that verifies the members for all load combinations will shorten the design process considerably.

3-D structural analysis software packages are widely available and portal frame buildings can also be modelled and analysed using this type of software. The effects of variations between adjacent frames will automatically be taken account of with this approach.

## 1.1 Scope

The guidance in this publication covers both elastic and plastic design of portal frames using hot rolled steel I-sections. The publication refers principally to BS EN 1993-1-1 and its UK National Annex<sup>[3]</sup>. Where appropriate, non-contradictory complementary information (NCCI) is referenced.

Hit and miss frames are considered and effects of the different frame stiffnesses examined. A procedure for using 2-D design software to estimate the forces transferred between hit and miss frames due to their different stiffnesses is presented.

Examples of the verification of elastic segments of columns with tension-flange restraint and of elastic segments of haunches are given.

This publication does not address portal frames with ties between eaves. This form of portal frame is relatively rare. The ties modify the distribution of bending moments substantially and increase the axial force in the rafters dramatically. Second order analysis software must be used for the design of portal frames with ties at eaves level because first order analysis does not allow for the significant second order effects in such frames.

There is no comprehensive worked example of frame analysis provided in this guide but that provided in SCI publication P397<sup>[4]</sup> remains a useful reference. The treatment in P397 of in-plane buckling is superseded by the treatment in this publication.

SCI publication P292<sup>[5]</sup> entitled *In-plane stability of portal frames to BS 5950-1:2000* is also an excellent source of basic information on the behaviour of portal frame structures.

## 1.2 Why choose a portal frame?

Steel portal frames are known to provide a highly efficient and cost-effective way to support an envelope, enclosing a useable volume. Steel portal frames are highly suited to carrying relatively modest loads. By their very nature they are relatively flexible; less onerous deflection limits are generally applied to portal frames than for other forms of construction. The careful detailing of cladding, flashings etc. is sufficient to ensure that the flexible behaviour of a steel portal frame is not detrimental to the performance of the envelope.

Although the deflection of steel portal frames can be reduced, for example by the use of ever larger steel sections, the cost-effectiveness of the solution will be adversely affected. If deflections are critical, or the frame is carrying high loads (from suspended machinery, for example), it may be more appropriate to select an alternative structural form, such as a truss.









# SOME FEATURES OF PORTAL FRAME BUILDINGS

## 2.1 General

A portal frame building comprises a series of unbraced transverse frames, braced longitudinally. The primary steelwork consists of columns and rafters, which form the portal frames, and longitudinal bracing, as shown in Figure 2.1. The end frame (gable frame) can be either a portal frame or a braced arrangement of columns and rafters.

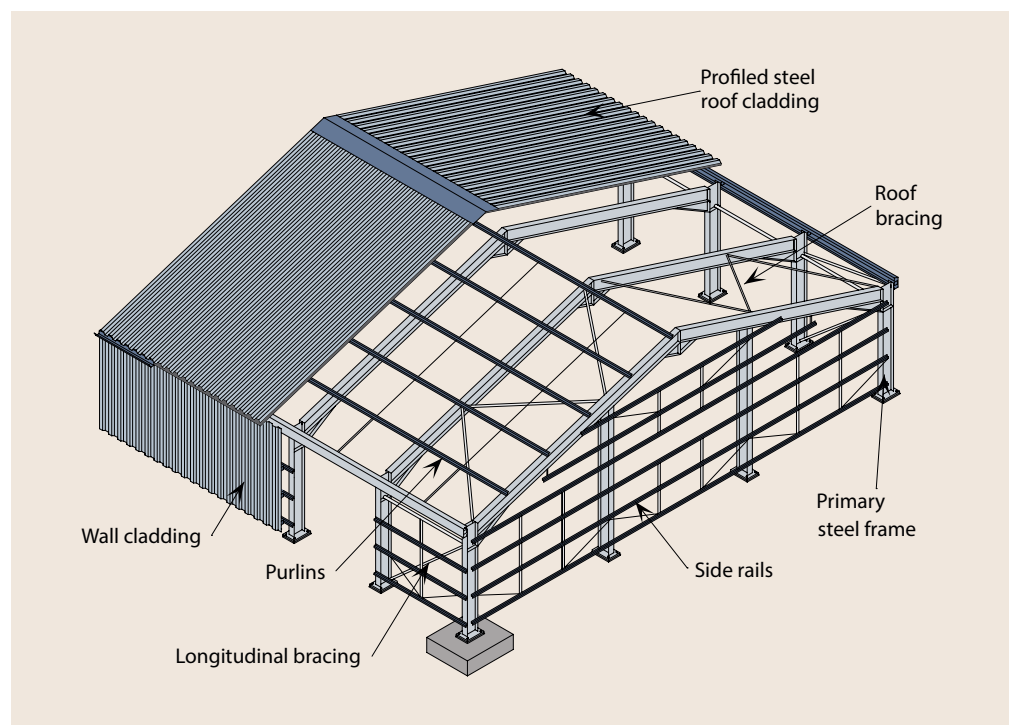


Figure 2.1  
Principal building  
components

The secondary steelwork supporting the cladding consists of side rails for walls and purlins for the roof. The secondary steelwork also plays an important role in restraining the primary steelwork members against buckling out of plane.

When considering structural behaviour, the essential nature of a portal frame is that in the plane of the frame, rigid joints between the primary members at the eaves and apex of the roof form the structural system which resists loads in that plane. This is the attribute that allows a series of frames to enclose a volume which is unencumbered by internal bracing. The portal frame members are orientated with their webs in the plane of the frame to benefit from the major axis strength and stiffness of the members and

form a continuous structure. The structural stability of the frame in this plane therefore has to be considered as a whole.

Perpendicular to the plane of the frame, the longitudinal bracing and secondary steelwork provide points of lateral restraint which define the lengths over which the primary members can buckle. The out-of-plane stability of the members can therefore be considered individually.

The roof and wall cladding separate the enclosed space from the external environment and provide thermal and acoustic insulation. The cladding transfers loads to secondary steelwork and restrains the flange of the purlin or rail to which it is attached.

Further details are provided in SCI publication P397 and on the website [www.steelconstruction.info](http://www.steelconstruction.info).

## 2.2 Hit and miss frames

Portal frame buildings may have multiple spans with lines of internal columns and a valley in the roof above them. In order to increase the utility of the enclosed volume, some of the internal columns may be omitted and a valley beam introduced to provide support to the rafters in these locations. Alternate columns may be omitted giving rise to the term “hit and miss frames” where the hit frames have internal columns. Two or more adjacent columns can be omitted if required. The size of the valley beams increases as necessary to carry the miss frames over the longer span.

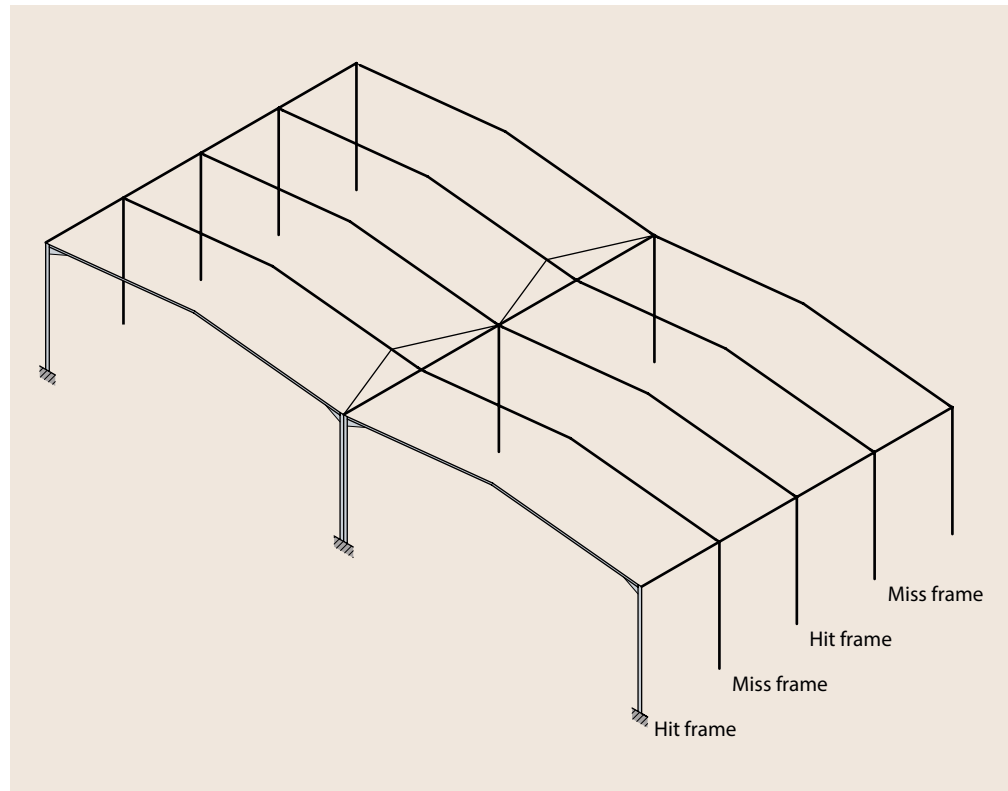


Figure 2.2  
Two-bay “hit and miss” portal frame

Valley beams may be simply supported or continuous through the supporting columns. A simply supported valley beam will be heavier than a continuous one but the connections to the supporting columns are likely to be simpler and cheaper than the alternative. A continuous valley beam may require haunches at the columns to allow for an economical connection.

Careful consideration of the geometry of the connection to the column is necessary to allow for both the haunched rafter and the valley beam, particularly if stiffeners are required to the column web.

Valley beams may be used to form portal frames with the supporting columns to provide longitudinal stability to the building. This may be advantageous instead of using bracing in the plane of the roof, spanning between vertical bracing in the side walls, to restrain the valley lines.

Longitudinal bracing is usually provided in the plane of the roof to reduce the differential lateral displacement between the stiffer hit frames and more flexible miss frames. (If no bracing is provided, load will be transferred to the stiffer frames through the sheeting). The bracing is available to carry all the load but in practice, there may be some sharing of load between bracing and sheeting.

## **2.3 Cranes**

Portal frame buildings often house processes which require electric overhead travelling (EOT) cranes. The size of crane varies with the process and can be of 60 tonne payload or more. The loads applied to the portal frame structure are not only vertical, due to the payload and self-weight of the crane; horizontal loads result from braking and acceleration (surge) and skewing (referred to as crabbing in BS 5950) where equal and opposite horizontal forces transverse to the crane rail are applied at the ends of the crane carriage. Impact loads due to vertical acceleration of payloads when lifting them also occur. The span of the portal frame obviously affects the self-weight of the crane.

EOT cranes travel on rails fixed to runway beams spanning between the portal frame columns. The arrangements for transferring the vertical crane loads to the foundations vary. For some cranes and building geometries, the crane runway beams are supported by brackets fixed to the crane columns. This arrangement results in variable actions applied to the building columns at the bracket position. For other cranes and building geometries, the crane runway beam reactions are taken on columns direct to the foundations. The bending moment from the runway beam support bracket is thus avoided.

An important serviceability issue relates to the outward horizontal deflection of the portal frame columns under the variable vertical actions, resulting in a spread of the frame and therefore of the runway beams. The effects of the spread are seen in the position of the end carriage wheels on the crane rail. The wheels are anchored relative to the rail in one end carriage and allowed to move relative to the rail at the other. This movement must be limited to a value agreed with the crane supplier and is likely

to be in the range 10 to 25 mm. The upper limit is an extreme value and a maximum differential deflection of 15 mm is more likely to be acceptable.

The spread may have a fundamental influence on the choice of the building structural system. For cranes of capacity greater than about 25 tonnes in buildings with spans of over 30 m, a portal frame may not be the most suitable structural form. The size of the frame required to limit the spread to acceptable values may be uneconomic and an alternative structural system may be required, such as a truss supported by lattice columns. It is likely that a portal frame building containing a crane will be designed elastically because plastic design methods will result in a frame which is too flexible.

Possible variations of portal frames required to support crane runway beams include the following arrangements.

- Adopt lattice columns with the inner leg supporting the crane runway beams. (The lattice column bases clearly must be designed as rigid). Elements of equivalent bending and shear stiffness can be determined for the analysis instead of modelling the lattice in full detail.
- Support the crane runway beams vertically on independent pin-ended columns which are restrained laterally by the portal frame columns.
- Adopt a tied portal with an increased roof pitch to mitigate second order effects. The tie results in larger axial forces in the rafters with a corresponding increase in frame flexibility from rafter second order bending.









# DESIGN PROCESS

The process of designing the main frame involves several stages, each of which is described in detail in the following Sections of this guide. For clarity, the outline of the whole design process is introduced in Table 3.1, with reference to the relevant Section of this publication.

DESIGN STEP	SECTION	COMMENTS
Geometry	6.1	Establish the clear span and height based on the client's requirements. The geometry used in the analysis should be a little conservative to allow for subsequent changes in member size.
Actions	4	Establish actions depending on location, site altitude and local topography. Permanent actions may be estimated, based on selected cladding type.
Preliminary design	6.2	Use tables from Appendix A or the preliminary design method described in Section 6.2.
Initial member selection	6.2	Select member sizes based on their cross-section resistance and buckling resistance. It may be assumed that the influence of shear on the bending resistance can be neglected for initial design. Allow about 5% for axial load. <b>Rafter</b> At this stage it may be assumed that sufficient restraints can be introduced to limit member buckling. <b>Column</b> The lateral-torsional buckling resistance of the column is likely to be the critical check, so preliminary checks over the restrained lengths will be necessary. If intermediate restraints cannot be introduced to the column (e.g. because the side rails are not continuous), a larger column section will be required.
Frame stability	7.5	Sensitivity to second order effects must be assessed. It is likely that second order effects must be allowed for, either by amplifying the results of a first order analysis, or by completing a second order analysis. When assessing frame stability, it is recommended that bases are modelled in accordance with the guidance given in Section 7.4.
Member verification	8.1.1 8.1.1, 8.1.2, 8.1.3 8.2	Classification of member cross-sections. Verification of cross-section resistance to bending, shear and compression. Bending interaction with shear is generally not critical. Buckling resistance is checked, establishing the position of the restraints to both flanges and thus the buckling lengths. Out-of-plane buckling resistance is verified against flexural and lateral torsional buckling. Interaction of bending moment and axial force is checked with the use of Expression 6.62 of BS EN 1993-1-1 and interaction factors from Annex B. In-plane buckling resistance verification of members in portal frames is not necessary; See Section 8.
SLS	12	Frame deflections are checked against client requirements.

Table 3.1  
Design sequence



# ACTIONS

This Section covers the actions that should be considered in the design of a steel portal frame, and the combination of those actions at the ultimate limit state and the deflection serviceability limit state.

Rules for actions can be found in BS EN 1991<sup>[6]</sup>, and on the design combinations of actions in BS EN 1990<sup>[7]</sup>. It is important to refer to the National Annex for the relevant Eurocode part and the country the structure is to be constructed in – this Section reflects the recommendations of the UK National Annexes.

## 4.1 Permanent actions

Permanent actions are the self weight of the structure, secondary steelwork and cladding. Where possible, unit weights of materials should be obtained from manufacturers' data. Where information is not available, these may be determined from data in BS EN 1991-1-1<sup>[8]</sup>.

Typical weights of materials used in roofing are given in Table 4.1. For a roof that only carries normal imposed roof loads (i.e. no suspended machinery or similar), the self-weight of the cladding plus secondary steelwork is typically 0.2 to 0.4 kN/m<sup>2</sup> when expressed over the plan area of the roof.

<b>MATERIAL</b>	<b>WEIGHT (kN/m<sup>2</sup>)</b>
Steel roof sheeting (single skin)	0.07 – 0.12
Aluminium roof sheeting (single skin)	0.04
Insulation (boards, per 25 mm thickness)	0.07
Insulation (glass fibre, per 100 mm thickness)	0.01
Liner trays (0.4 mm – 0.7 mm thickness)	0.04 – 0.07
Composite panels (40 mm – 100 mm thickness)	0.1 – 0.15
Steel purlins (distributed over the roof area)	0.03
Steel decking	0.2

Table 4.1  
Typical weights of  
roofing materials

Another component of loading to be considered as a permanent action is the self-weight of any building services. Depending on the use of the building, the weight of the services varies significantly and careful consideration should be given to the values assumed in the design. Some services (e.g. overhead lighting) are evenly spread over the whole building but there may be significant loads in relatively small areas if there is a primary route for the distribution of piped services.

At the preliminary design stage, the service loading is usually assumed to be between 0.15 and 0.4 kN/m<sup>2</sup> on plan over the whole roof area. This distributed load should be appropriate for the design of the frames. Individual purlins may be required to support higher distributed loads. Also, high service loads are likely to result in substantial point loads applied to purlins; the attachment details and member resistance should be verified in accordance with the manufacturer's recommendations. If the loads are too large for the purlins, they should be supported directly from the rafters or by additional members spanning between the portal frames.

It is also important to recognise that the services may be removed during the life of the structure and, where service loads have a beneficial effect in opposing wind uplift, they should be neglected.

At the final design stage, the structure should be checked for the actual service loads if these are available.

## 4.2 Variable actions

### 4.2.1 Imposed roof loads

Imposed loads on roofs that are not accessible, except for normal maintenance and repair, are classed under category H in BS EN 1991-1-1<sup>[8]</sup>. For that category of roof, the UK NA to BS EN 1991-1-1<sup>[9]</sup> gives imposed loads on roofs that depend on the roof slope. A point load,  $Q_k = 0.9$  kN is given, which is used for local verification of roof materials and fixings, and a uniformly distributed load,  $q_k$ , applied vertically and used for the design of the structure. The loading for roofs not accessible except for normal maintenance and repair is given in Table 4.2.

Table 4.2  
Imposed loads  
on roofs, taken  
from the UK NA to  
BS EN 1991-1-1

ROOF SLOPE, $\alpha$	$q_k$ (kN/m <sup>2</sup> )
$\alpha < 30^\circ$	0.6
$30^\circ \leq \alpha < 60^\circ$	$0.6[60 - \alpha]/30$
$\alpha \geq 60^\circ$	0

It should be noted that, following Clause 3.3.2(1) of BS EN 1991-1-1<sup>[8]</sup>, there is no requirement to combine imposed loads on roofs with either snow loads or wind actions.

### 4.2.2 Snow loads

Snow loads in the UK should be determined from BS EN 1991-1-3<sup>[10]</sup> and its National Annex<sup>[11]</sup>.

The characteristic snow load on the ground,  $s_k$ , depends on the site location and altitude. The characteristic snow load on a roof,  $s$ , is taken as  $s_k$  multiplied by the snow load shape coefficient  $\mu$  (which allows for the roof shape), the exposure coefficient,  $C_e$ , and the thermal coefficient,  $C_t$ . BS EN 1991-1-3 Clause 5.2(7) recommends that both  $C_e$  and  $C_t$  be taken as 1.0.

According to Clause NA.2.2 of the UK National Annex to BS EN 1991-1-3, the following design situations should be considered:

- Undrifted snow;
- Drifted snow (removal of snow from one slope);
- Exceptional snow drifts, which should be treated as accidental actions.

Both undrifted and drifted snow should be considered in persistent design situations at ULS, and should be combined with other actions using expressions 6.10 or 6.10a and 6.10b of BS EN 1990 (see Section 4.6 of this publication). The exceptional snow drift case is an accidental action, and should be considered in combination with other actions using Expression 6.11b.

Exceptional snow drifts should be considered:

- Behind parapets;
- In valleys of multi-span frames;
- Behind obstructions on the roof;
- At changes in roof level;
- From snow blown off adjacent buildings.

Because the exceptional snow drifts are considered as accidental actions, it is likely that they have little or no influence on the design of the main frame members. However, exceptional snow drifts are likely to be an important design consideration for the secondary steelwork, which may mean selecting larger sections, or a thicker gauge, or specifying reduced centres.

### **4.2.3 Wind actions**

Wind actions in the UK should be determined from BS EN 1991-1-4<sup>[12]</sup> and its National Annex<sup>[13]</sup>. The UK NA is a substantial document, with many provisions to be observed when determining wind actions in the UK. Guidance on the determination of wind actions is given in SCI publication P394<sup>[14]</sup>.

For single span portal frame buildings in the UK designed to previous National Standards, it was uncommon for load combinations including wind to determine sizes of members. With design to the Eurocodes, wind actions are more significant than previous practice for the following reasons.

- Wind actions can appear as the leading variable action in the ULS combination of actions, with a partial factor of 1.5 applied to characteristic values.
- Although wind actions do not occur simultaneously with imposed roof loads, wind actions are to be combined with snow loads. The combination of actions including both wind and snow may lead to greater bending moments than the combination including only imposed roof loads with permanent actions.

Combinations at ULS and SLS always include wind and must be checked as part of the final design process, although wind load may not be significant in gravity load cases and may have been omitted from preliminary design. Circumstances where wind actions may be significant include:

- where deflections at SLS are critical, i.e.:
  - if the portal frame supports an overhead travelling crane, or
  - if masonry or some other relatively brittle wall construction is used;
- in uplift conditions (a reversed bending moment diagram), as this will determine restraint positions for the rafters (see Section 8.4.2).

Several alternative routes can be chosen to calculate wind actions. More design effort will generally lead to reduced loads. The procedure of calculating wind actions includes five stages:

1. Calculation of the peak velocity pressure;
2. Determination of external pressure coefficients;
3. Determination of internal pressure coefficients;
4. Calculation of the structural factor;
5. Calculation of wind forces.

Wind pressures are calculated as the product of the peak velocity pressure, the structural factor and pressure coefficients. External and internal pressure coefficients are given in the Eurocode – but note that external pressure coefficients for the roof should be taken from the UK National Annex. In the Eurocode, coefficients are given for elements with loaded areas of up to  $1 \text{ m}^2$  and loaded areas of over  $10 \text{ m}^2$ , with logarithmic interpolation for areas between the two. The UK NA simplifies this, allowing the use of the coefficients for  $10 \text{ m}^2$ , known as  $c_{pe}$ , for any loaded area larger than  $1 \text{ m}^2$ .

For the purpose of calculating overall loads on the structure, for example to determine design effects for bracing systems, the Eurocode provides force coefficients, which include friction effects. When designing individual portal frames, loads on elements are required. These loads depend on the internal and external pressures.

### ***Peak velocity pressure***

Calculating the peak velocity pressure can be carried out using one of the four alternative approaches summarised in Table 4.3. Each approach demands different levels of information about the site, and involves different levels of calculation effort.

Although Table 4.3 indicates that building orientation is “not required” for Approach 1, it can be beneficial to account for building orientation, especially if there is an asymmetry, or particular features of the building, (for example, a dominant opening) that mean actions determined with respect to a building face are useful. If the building orientation is not accounted for, the same peak velocity pressure must be used in both orthogonal directions.

It should be noted that software is widely available to calculate wind actions in accordance with Approach 1, relieving the designer of the calculation effort. Usually, software will account for the building orientation, leading to a value of peak velocity pressure normal to each building face.

For regular structures, it is recommended that Approach 3 is adopted if undertaking calculations without software, balancing the need for site-specific information, calculation effort and resulting actions.



	<b>APPROACH 1</b>	<b>APPROACH 2</b>	<b>APPROACH 3</b>	<b>APPROACH 4</b>
Peak velocity pressure	Calculated in 12 directions at 30° intervals	Most onerous factors from 360° around the site	Calculated in 4 quadrants	Calculated in 4 quadrants at ±45° to the normal to the building face
Building orientation	Not required	Not required	Not required	Required
Outcome	Generally the least onerous result	The most onerous result	Generally a less onerous pressure than Approach 2	Generally a less onerous pressure than Approach 2, but specific to each building face
Calculation effort	Significant	Least effort	Modest effort	Modest effort
Application	Both orthogonal directions use the same peak velocity pressure	Both orthogonal directions use the same peak velocity pressure	Both orthogonal directions use the same peak velocity pressure	Actions on each face may use a different peak velocity pressure
Overall Comments	Least onerous peak velocity pressure, but significant calculation effort	Simple, conservative	Recommended approach	Useful for asymmetric structures or where dominant openings are significant

Table 4.3  
Comparison of approaches to determine the peak velocity pressure

### **External pressure coefficients**

External pressure coefficients for walls and roofs are given in the Eurocode and the UK NA. The pressure coefficients on a portal frame building fall into a number of zones, with higher suctions next to corners, such as the vertical corner of a wall, or adjacent to the eaves and ridge of a duopitch roof.

The demarcation between roof zones does not correspond to the zones on the walls, which complicates the assessment of actions on individual frames in typical structures. Some engineering judgement is required to identify the most onerous combinations of actions on the most heavily loaded frame. In many low rise industrial structures, the penultimate frame is likely to be the most critical frame.

### **Internal pressure coefficients**

Internal pressure coefficients are given by Clause 7.2.9 of BS EN 1991-1-4. Where there are no dominant openings, the value of the internal pressure coefficient can be calculated based on the opening ratio in the face under consideration and taken from Figure 7.13 of the Eurocode.

The UK NA gives the permeability of a limited selection of forms of construction. If estimating wall permeability is not possible, or not considered justified for a particular case, the Eurocode recommends that  $c_{pi}$  should be taken as the more onerous of +0.2 and -0.3.

If all four walls of a rectangular building are equally permeable, the relationship between the building geometry and the internal pressure coefficient is shown in Figure 4.1.

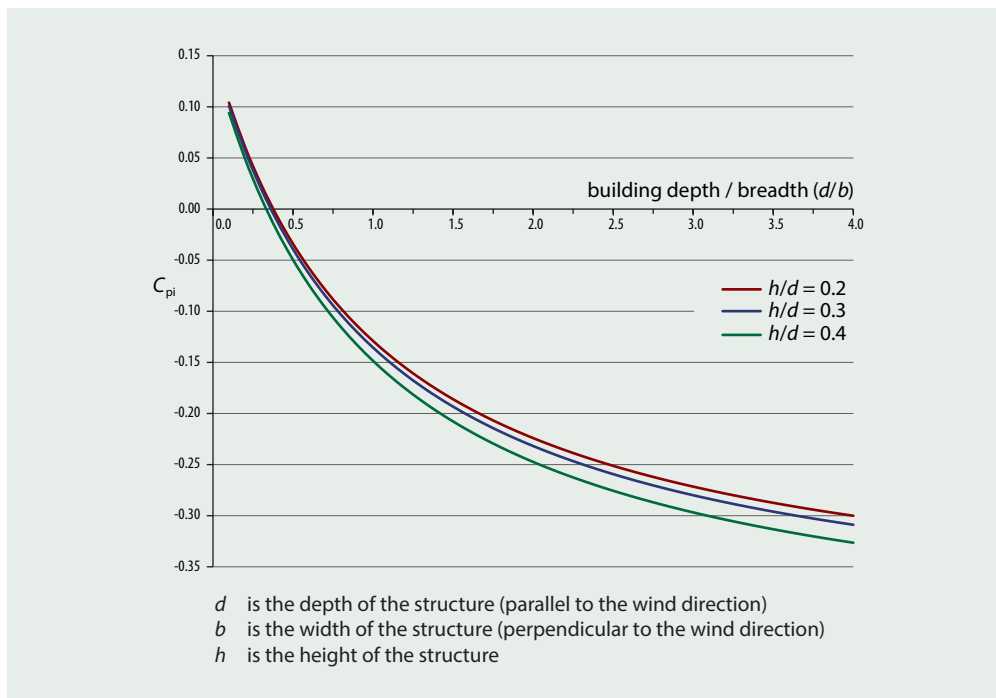


Figure 4.1  
Internal pressure coefficient for rectangular buildings with walls of equal permeability

For orthodox completely clad portal frame buildings, Figure 4.1 shows that the highest internal suction coefficient is approximately -0.3 which led to the recommendation in BRE Digest 436<sup>[15]</sup> that for this type of structure, an internal pressure coefficient of -0.3 is appropriate.

Figure 4.2 can be used to determine internal pressure coefficients for the two orthogonal wind directions. The internal pressure coefficients appropriate for each orthogonal wind direction for a building with one plan dimension twice that of the other are shown in Figure 4.2 taking  $h/d$  as 0.2, which is a typical value.

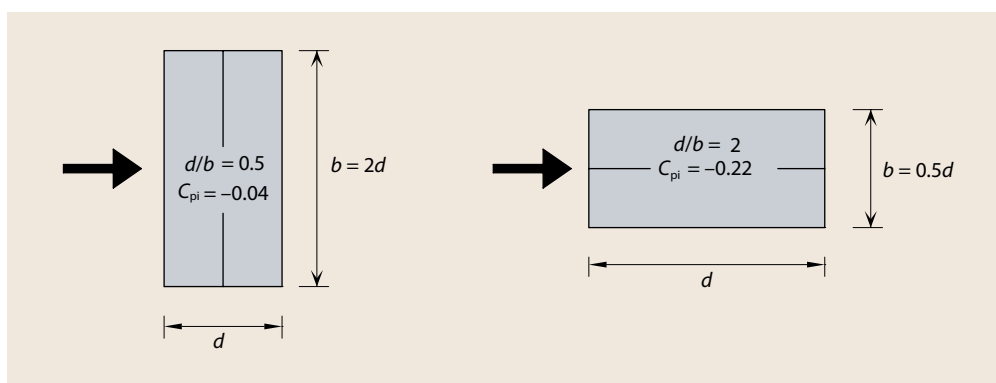


Figure 4.2  
Internal pressure coefficients for a rectangular building with walls of equal permeability

### Dominant openings

Where there is a dominant opening, the internal pressure or suction can be as high as 75% or 90% of the  $C_{pe}$  value at the opening, depending on the size of the opening compared to the openings in the other faces. The designer must decide if the openings might be open

during a severe storm, or if it is reasonable to assume that the openings will be shut. If the openings are assumed to be shut, an accidental design situation must be considered, with the dominant opening being open (Clause 7.2.9(3)). Common practice in the UK is to carry out this second verification with a probability factor  $c_{\text{prob}}$  applied to the basic wind velocity. Practice is to use a  $c_{\text{prob}}$  factor of 0.8, which leads to a reduced peak velocity pressure of 0.64 of the original pressure. The use of a  $c_{\text{prob}}$  factor of 0.8 presumes that procedures will be in place to ensure the openings are closed in a severe storm.

### **Structural factor**

When calculating wind forces, a structural factor  $c_s c_d$  may be applied. For low-rise steel buildings, this may be conservatively taken as 1.0. According to the UK NA, a factor of less than unity may be determined by considering  $c_s$  and  $c_d$  separately. The  $c_s c_d$  factor can only be applied when determining overall force coefficients and external pressure coefficients; it is not to be applied when determining internal pressures.

### **Calculation of wind forces**

For the calculation of overall loads using pressure coefficients, the National Annex allows a factor (from Clause 7.2.2(3) of BS EN 1991-1-4) to be applied to all the horizontal force components for both walls and roof for certain building geometries, due to the lack of correlation between the maximum forces calculated for the windward and leeward faces. The factor varies between 0.85 and 1.0 depending on  $h/d$ . When the wind is considered blowing parallel to the apex, no reduction should be applied to the frame under consideration.

## **4.3 Thermal actions**

A change in the temperature of a steel structure causes a thermal strain in the steel elements. The magnitude of the thermal strain is equal to the coefficient of thermal expansion which is stated in BS EN 1993-1-1 as  $\alpha = 12 \times 10^{-6}$  per K for temperatures less than or equal to  $100^\circ\text{C}$  multiplied by the temperature rise. This corresponds to 1.2 mm expansion per degree temperature rise per 100 m of building. The result of the thermal strain can be free expansion of the element if there is no restraint, or if the expansion is fully restrained, an axial stress is induced.

Portal frame buildings are usually provided with vertical bracing in the side walls to achieve lateral stability in the longitudinal direction. If bracing is provided at each end of the building, axial forces will be developed on the thermal expansion of structural elements which are continuous between the braced bays. The magnitude of the axial force depends on the difference in temperature between that at completion of the structure and the temperature at the time in question and the stiffness of the restraint system.

A minimum temperature range in the UK of  $-5^\circ\text{C}$  to  $+35^\circ\text{C}$  is often used, giving a variation from a mean temperature of  $15^\circ\text{C}$  of  $\pm 20$  K. The maximum unfactored axial compressive stress in a fully restrained element for a temperature rise of 20 K is given by  $20\alpha E = 50.4 \text{ N/mm}^2$ .

If initial investigations using this temperature range suggest a potential problem, a more considered value for the temperature range can be determined, taking into account meteorological data and the operating conditions of the building.

In practice, axial stresses of  $50 \text{ N/mm}^2$  may not be realised because of slip at bolted connections or elastic buckling of secondary elements to relieve the axial load. Continuous longitudinal elements such as crane runway beams, crane rails, valley girders and eaves beams should be considered carefully and designed for axial loads due to thermal actions if necessary. The magnitude of the axial loads depends on the stiffness of the restraint. Substantial elements such as crane runway beams may potentially deliver large forces due to thermal expansion which are several times those due to wind loads.

Alternatively, expansion joints can be introduced to allow thermal expansion to occur. It is suggested in Steelwork Design Guide to BS 5950 Volume 4<sup>[16]</sup> that if expansion joints are provided at 150 m centres, longitudinal members need not be designed to resist stresses due to restraint of expansion. Positioning vertical braced bays mid-way between expansion joints will allow unrestrained expansion away from the braced bay.

In the transverse direction, changes in temperature will result in changes in length of the portal frame rafter. Expansion of the rafter results in bending of the columns and axial compression in the rafter. In one and two-bay frames, elastic analysis shows that the columns are sufficiently flexible for the axial force in the rafter to be low and as a result, the induced bending moment at the eaves is also low. The rafter force reduces with increasing column height. In a one-bay frame of 30 m span with a horizontal rafter and 12 m high columns with representative properties, a 20 K temperature rise results in an axial force of only 0.14 kN with a corresponding eaves bending moment of 1.68 kNm. Reducing the column height with the same sections to an unlikely 4 m increases the axial force tenfold to about 1.4 kN. Almost all the rafter expansion appears as outward displacement of the column top: the reduction in length due to the shortening of the rafter under axial compression is very small. Similar results are found in frames with three and four bays.

In general, for portal frames of normal proportions, thermal effects in the in-plane direction can be neglected.

## 4.4 Crane loads

Cranes impose both vertical and horizontal loads on the structure. Consideration should be given to the following loads:

- Vertical actions, comprising the self-weight of the crane bridge, crab, hook plus the lifted load;
- Horizontal actions due to crane surge and crabbing.

More information on crane loading is given in *Eurocode load combinations for steel structures*<sup>[17]</sup>.

## 4.5 Accidental actions

Rules for accidental actions and accidental design situations are given in BS EN 1991-1-7<sup>[18]</sup>. The three accidental actions that may need to be considered are:

- Fire (see Section 4.5.1);
- Drifted snow (see Section 4.2.2);
- The opening of a dominant opening that was assumed to be shut at ULS (see Section 4.2.3).

Each project should be individually assessed to determine whether any other accidental actions need to be considered.

### 4.5.1 Actions due to fire

Fire is considered as an accidental design situation, resulting in indirect actions (such as forces and moments caused by expansion) and modification of material properties. The UK Building Regulations state that a member supporting only a roof is excluded from requirements for fire resistances, so actions due to fire do not normally need to be evaluated. However, special provisions are generally required when structures are close to a boundary, to prevent fire spread. These requirements are given in Building Regulations, rather than the Eurocodes.

When the building is close to the boundary, there are several requirements aimed at stopping fire spread by keeping the boundary intact:

- The use of fire resistant cladding;
- Application of fire protection to the steel columns up to the underside of the haunch;
- The provision of moment-resisting bases (as it is assumed that in the fire condition rafters go into catenary).

Comprehensive advice is available in SCI publication P313<sup>[19]</sup>.

### 4.5.2 Impact

The structure should be protected from impact loads, so consideration of these loads will not normally form part of the design of the portal frame.

### 4.5.3 Robustness

Robustness is the ability of a structure to withstand events like fire, explosion, impact or the consequence of human error, without being damaged to an extent disproportionate to the original cause.

BS EN 1990 sets the requirement to design and construct robust buildings in order to avoid disproportionate collapse under accidental design situations. BS EN 1991-1-7<sup>[18]</sup> gives details of how this requirement should be met.

The design calculations for robustness are carried out separately from those for normal verifications, and substantial permanent deformation is acceptable for the accidental

design situation. Generally, the resistances of elements and connection components are based on ultimate strength, rather than yield strength.

In the UK, official guidance documents are published to explain how to achieve the requirement for robustness in a structure. These documents apply as follows:

- In England and Wales – Building Regulations 2000: Approved Document A<sup>[20]</sup>.
- In Scotland – The Scottish Building Standards Agency (SBSA) Technical Handbooks<sup>[21]</sup>.
- In Northern Ireland – The Building Regulations (Northern Ireland), Technical Booklet D<sup>[22]</sup>.

The strategy to be adopted depends on the classification of the structure, as defined in the Building Regulations and Annex A of BS EN 1991-1-7. Portal frames are usually structures of Consequence Class 1 (agricultural buildings) or 2a (most non-agricultural portal frame structures). If a portal frame structure is open to significant numbers of the public and greater than 5000 m<sup>2</sup>, it should be considered a Class 3 structure.

Design guidelines and recommended practice on the issues connected with robustness are given in SCI Publication P391<sup>[23]</sup>. The practical application of the rules to portal frames is summarised in Table 4.4. The calculated tying forces in the direction of the portal frames will be easily accommodated by the normal connections in a portal frame. In the longitudinal direction, there is no vertical load on the eaves strut, so the design tying force will be 75 kN – easily accommodated by orthodox members and connections.

CONSEQUENCE CLASS	BUILDING TYPE & OCCUPANCY	REQUIREMENT	COMMENT
1	Agricultural buildings	No specified requirements, but minimum 75 kN tying force is recommended for all connections	Achieved by all orthodox connections
2a	Industrial buildings, public access but < 2000 m <sup>2</sup>	Horizontal ties	Tying force depending on the vertical loading on the member, but not less than 75 kN
2b	Public access, < 5000 m <sup>2</sup>	Horizontal ties and vertical ties	Tying force depending on the vertical loading on the member, but not less than 75 kN (the additional rules for vertical tying are not applicable for single storey portal frames)
3	Public access, ≥ 5000 m <sup>2</sup>	Risk Assessment	As class 2b, but with a risk assessment. No changes to connections, but possible outcomes might include impact protection, or additional redundancy

Table 4.4  
Summary of  
robustness  
requirements

## 4.6 Equivalent horizontal forces

The effect of vertical actions applied to a frame which is not plumb can be modelled by applying horizontal forces to a theoretical model which are proportional to the applied actions and the magnitude of the sway imperfection. These forces are called equivalent

horizontal forces (EHF) and are combined with external actions for convenience. The same effect could be achieved by modelling the frame out of plumb. This is discussed in more detail in Section 7.2.1

## 4.7 Combinations of actions

BS EN 1990 gives rules for establishing combinations of actions. Values of partial factors and combination factors to be used in the UK are given in the UK National Annex. BS EN 1990 gives expressions for the effects of combined actions for both the ultimate limit state (ULS) and the serviceability limit state (SLS). For the SLS, onward reference is made to the material parts of the Eurocodes (for example BS EN 1993-1-1 for steelwork), to identify which expression should be used and the SLS limits that should be observed.

All combinations of actions that can occur together should be considered. If certain actions cannot be applied simultaneously, they should not be combined.

### Ultimate limit state combinations

For persistent (referring to conditions of normal use) or transient (e.g. during execution or repair) design situations, the UK NA allows the designer to use the STR set of expressions to establish the ULS forces and moments for member verification. Expression 6.10 or the less favourable of 6.10a and 6.10b may be used, as shown in Table 4.5. Taking the less favourable of expressions 6.10a and 6.10b will generally result in the most economic solution for gravity loads. Expression 6.10 is advantageous for uplift.

Table 4.5  
Design values of  
actions for persistent  
or transient design  
situations (taken from  
BS EN 1990)

EXPRESSION	PERMANENT ACTIONS		VARIABLE ACTIONS	
	UNFAVOURABLE	FAVOURABLE	LEADING	ACCOMPANYING
6.10	$\gamma_{Gj,sup} G_{kj,sup}$	$\gamma_{Gj,inf} G_{kj,inf}$	$\gamma_{Q,1} Q_{k,1}$	$\gamma_{Q,1} \psi_{0,i} Q_{k,1}$
6.10a	$\gamma_{Gj,sup} G_{kj,sup}$	$\gamma_{Gj,inf} G_{kj,inf}$	$\gamma_{Q,1} \psi_{0,i} Q_{k,1}$	$\gamma_{Q,1} \psi_{0,i} Q_{k,1}$
6.10b	$\xi \gamma_{Gj,sup} G_{kj,sup}$	$\gamma_{Gj,inf} G_{kj,inf}$	$\gamma_{Q,1} Q_{k,1}$	$\gamma_{Q,1} \psi_{0,i} Q_{k,1}$

For structures in the UK, the following values of partial factors and combination factors, taken from the UK NA, should be used:

- $\gamma_{Gj,sup}$  = 1.35 partial factor for unfavourable permanent actions
- $\gamma_{Gj,inf}$  = 1.0 partial factor for favourable permanent actions
- $\gamma_{Qj,sup}$  = 1.5 partial factor for unfavourable variable actions
- $\gamma_{Qj,inf}$  = 0 partial factor for favourable variable actions
- $\psi_0$  = 0.5 combination factor for wind actions
- $\psi_0$  = 0.5 combination factor for snow loads for site altitude below 1000 m above sea level
- $\psi_0$  = 0.7 combination factor for imposed roof loads
- $\xi$  = 0.925

The factors applied to the characteristic values of actions for combinations at the Ultimate Limit State, based on equation 6.10 and using the values given in the UK NA, are shown in Table 4.6.

ACTIONS	PERMANENT	IMPOSED	SNOW	WIND	WIND UPLIFT	EHF
factors for combinations of actions	1.35	1.5				To be included
	1.35		1.5			To be included
	1.35		1.5	0.5 x 1.5 ( $\gamma_Q \times \psi_0$ )		*
	1.35		0.5 x 1.5 ( $\gamma_Q \times \psi_0$ )	1.5		*
	1.0				1.5	*

Table 4.6  
Factors for design combinations at ULS

Note: \* indicates that EHF may not need to be included if  $H_{Ed} \geq 0.15 V_{Ed}$ . Since the EHF are a proportion of the ultimate loads, no additional factor is required.

Note that, in accordance with BS EN 1991-1-1, imposed roof loads are not considered in combination with either wind actions or snow loads. Snow loads and wind actions are considered in combination, with each action in turn as the leading variable action.

Guidance on combinations of actions including cranes is given in *Eurocode load combinations for steel structures*<sup>[17]</sup>.

### Serviceability limit state combinations

BS EN 1990 defines three combinations of actions for the deflection serviceability limit state; the characteristic combination, the frequent combination and the quasi-permanent combination. The UK NA to BS EN 1993-1-1 recommends the use of the characteristic combination of actions when checking SLS, and that permanent actions need not be included. Therefore, in the UK the following expression can be used to determine the effects of SLS combinations of actions:

$$Q_{k,1} \text{ "+" } \sum_{i>1} \psi_{0,i} Q_{k,i}$$

(where "+" means 'combined with')

Detailed consideration of serviceability aspects of frame design is given in Section 12.









# ELASTIC AND PLASTIC ANALYSIS

## 5.1 Elastic and plastic theory

The elastic theory is the most common basis for the analysis of general structures. Structures remain elastic under the application of load such that the load paths in the structure remain the same under all magnitudes of loading and deflections are proportional to the load. The principle of superposition therefore applies.

The plastic theory of behaviour of steel structures has been developed in the United Kingdom starting in 1936<sup>[24]</sup> and many single storey portal frame buildings are designed in this way. The design method exploits the well-known ability of steel to deform plastically under load once its yield strength is reached. This ability allows gross deformations to occur in a structure, in the form of rotations at the position of maximum bending moment. These rotations are known as plastic hinges.

Deformation at a plastic hinge occurs at constant bending moment so that any load which is greater than that which causes the plastic hinge to form is resisted by a different structural system (the original system plus a plastic hinge) and follows a different load path. The principle of superposition therefore does not apply. Significant redistribution of internal forces can occur and the redistribution leads to efficient use of material by utilising the resistance of an element at more than one section along its length.

A simple illustration of the behaviour and potential for savings in material is the example of a uniform fixed-ended beam, subjected to a uniformly distributed load of increasing magnitude. As load on the structure is increased, successive hinges form until the structure becomes a mechanism and deformation occurs under constant load. When designed elastically for a uniformly distributed load  $w$ , the free bending moment is  $wL^2/8$  at mid-span. The fixing moments at each end are  $wL^2/12$  and the mid-span moment is  $wL^2/8 - wL^2/12 = wL^2/24$ . The design bending moment for elastic design is therefore equal to  $wL^2/12$ .

If plastic hinges are assumed to form at the fixed ends and rotation takes place as the uniform load is increased, the beam will become a mechanism when a third plastic hinge forms at mid-span. At this point the magnitude of the bending moment at mid-span equals those at the supports and has the value  $wL^2/16$ . The potential saving is illustrated by the ratio of the design bending moments:  $(1/16)/(1/12) = 3/4$ .

As stated in Horne & Morris<sup>[25]</sup>, “simple plastic design is primarily a strength design and other factors such as fatigue, buckling and deflection limitations can influence the final choice of member size. In extreme cases these other design criteria may dominate the design to the exclusion of strength. ...Plastic design is based on the assumption that premature failure does not occur until sufficient hinges have developed to produce a mechanism and so instability, whether due to local or lateral buckling must be prevented.”

Thus a fundamental requirement is that stability of the elements of a structure is maintained, both locally and globally. This means in practice:

- i. That the slenderness of the parts of a cross-section in compression is sufficiently low to avoid failure by local buckling before a plastic hinge can form;
- ii. That the elements are sufficiently restrained laterally and torsionally to prevent flexural or lateral torsional buckling; and
- iii. That there is effective restraint at the location of a hinge so that plastic rotation can occur without the occurrence of buckling.

Methods of frame analysis at the ultimate limit state can therefore be divided broadly into two types – elastic analysis and plastic analysis, depending on their theoretical basis. The latter general term covers both rigid-plastic and elastic-plastic analysis. Rigid-plastic analysis is a simplification in which the elastic deformation of the structure is ignored.

Elastic analysis will usually give less economical designs for portal structures than plastic analysis but it is not uncommon for an initial analysis to be elastic. Most commercially available software for portal frame analysis will carry out an elastic-plastic analysis. Both types of analysis are described in Sections 5.3 and 5.4 respectively.

## 5.2 First and second order analysis

The internal moments in compression elements in a structure are always increased by the deformations of the structure under applied load and the extent to which this occurs depends on the stiffness of the structure. The increase is due to the effects of axial load on both the overall behaviour of the frame, (known as  $P-\Delta$  effects) and on the behaviour of individual members (known as  $P-\delta$  effects). Both these effects are of second order and are not determined by (first order) linear elastic analysis, where the effects of changes in the geometry of the structure are ignored.

Imperfections in a structure must also be considered. These include geometrical imperfections such as lack of verticality, lack of straightness, lack of flatness, lack of fit eccentricities in joints and residual stresses. The structural analysis should take into account global imperfections for frames and bracing systems and local imperfections for individual members.

Computer analysis software packages are available which take into account the effects of deformation of a structure under load. These often involve an iterative procedure

because the internal actions are increased by the effects of the applied actions on the deformed geometry of the structure. These software packages may deal with the elastic behaviour of the structure only, ignoring yielding, or may also allow for material non-linearity in terms of progressive yielding and the formation of plastic hinges.

Second order elastic and elastic-plastic design software take into account second order effects by various methods. A structure should be modelled appropriately to make sure that the analysis model is relevant to the software package. For example, some software packages only take into account the relative displacements at the ends of elements.

The deformations along the length of a member are only taken into account if the member is modelled by a series of elements (say ten). If this is not done, the second order effect on frame behaviour of the flexibility of the element will not be included in the analysis results: lower displacements and bending moments will be calculated.

Other finite element software packages allow for these effects by modifying bending terms in the stiffness matrix by factors similar to stability functions. If this type of software package is used, the structural members do not need to be sub-divided into a number of elements.

The significance of second order effects must always be considered, and such effects allowed for if necessary. If second order effects are small, a first order analysis is sufficiently accurate. If second order effects are significant, a second order structural analysis package can be used and this will always produce results with increased internal actions. Alternatively, various methods can be used with first-order elastic or plastic analysis, to allow for second order effects.

### **5.2.1 Frame behaviour**

When a portal frame is loaded, it deflects: its shape under load is different from the unloaded shape. The deflection has a number of effects.

- Arch-action means the rafters thrust outwards and the vertical forces at the tops of the columns are eccentric to the bases, which leads to further deflection.
- The apex drops and reduces the arching action.
- The vertical actions cause bending in the rafters; axial compression in initially curved members causes increased curvature. Increased curvature can be considered as a symptom of reduced stiffness.

Taken together, these effects mean that a frame is less stable (nearer collapse) than a first-order analysis suggests. The first step in assessing frame stability is to determine whether the difference between the results of a first and second order analysis are significant.

The deflected shape of a portal frame under combined vertical and horizontal loads is shown diagrammatically in Figure 5.1.

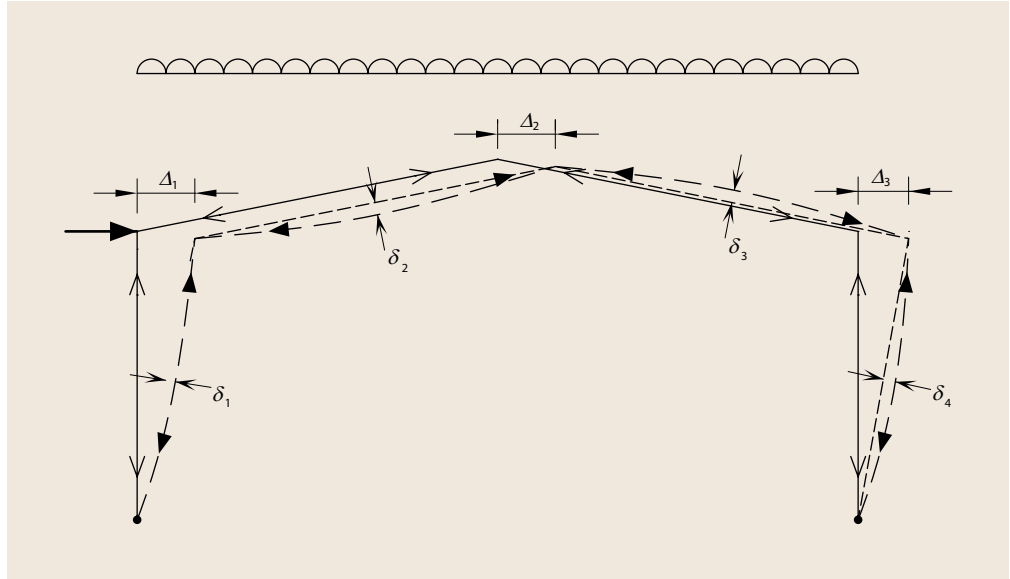


Figure 5.1  
Asymmetric or sway  
mode deflection

As shown in Figure 5.1, when considering the effects of deformed geometry, there are two categories of second order effects:

- Effects of displacements of the intersections of members, ( $P-\Delta$  effects);
- Effects of deflections within the length of members, ( $P-\delta$  effects).

$P-\delta$  effects arise from two different causes:

- Bending due to external actions, which curve the member;
- Curvature due to initial member imperfections.

### 5.3 Elastic frame analysis

A typical bending moment diagram resulting from an elastic analysis of a frame with pinned bases is shown in Figure 5.2. The haunch length is chosen so that under predominantly gravity combinations of actions the hogging bending moment at the end of the haunch is approximately equal to the sagging bending moment adjacent to the apex.

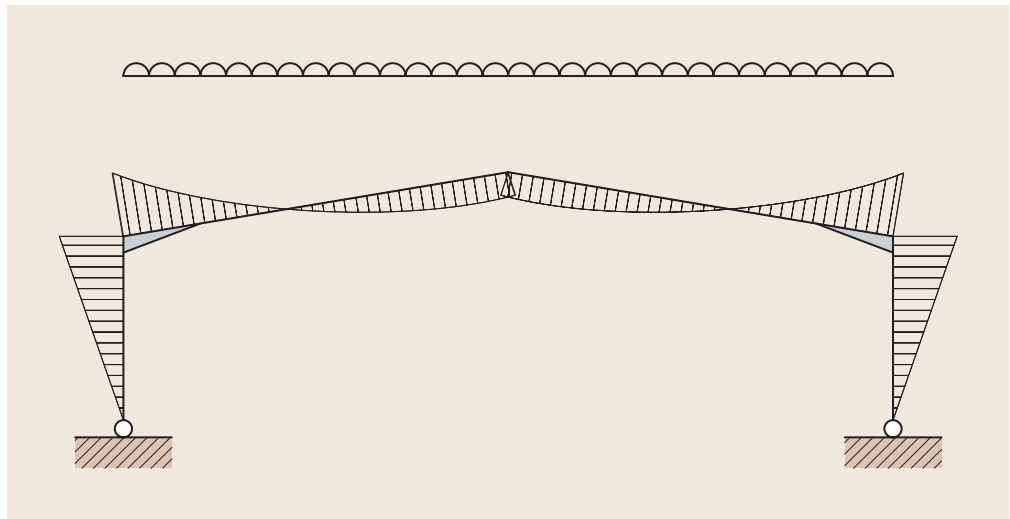


Figure 5.2  
Bending moment  
diagram resulting  
from the elastic  
analysis of a  
symmetrical portal  
frame under  
symmetrical loading

BS EN 1993-1-1 allows the plastic cross-sectional resistance to be used with the results of elastic frame analysis, provided the section is Class 1 or Class 2. In addition, for Class 1 and 2 sections, the Standard allows 15% of the maximum moment to be redistributed as defined in BS EN 1993-1-1 Clause 5.4.1.4(B). In practice, the redistribution from the initial elastic analysis is rarely used in steel design.

## 5.4 Plastic frame analysis

Plastic frame analysis involves the determination of a set of plastic hinges which form a mechanism and the corresponding plastic bending moments, in a frame of given geometry and applied actions. As with elastic analysis, second order effects should be considered and allowed for if necessary. Plastic analysis can either be rigid-plastic, where the elastic behaviour between plastic hinges is neglected, or elastic-plastic. BS EN 1993-1-1 Clause 5.4.3(5) states that rigid-plastic analysis may be used if no effects of the deformed geometry (second order effects) have to be considered. This is because the analysis method does not involve the determination of deflections. The analysis method can be used in conjunction with a separate elastic analysis with which the designer takes second order effects into account.

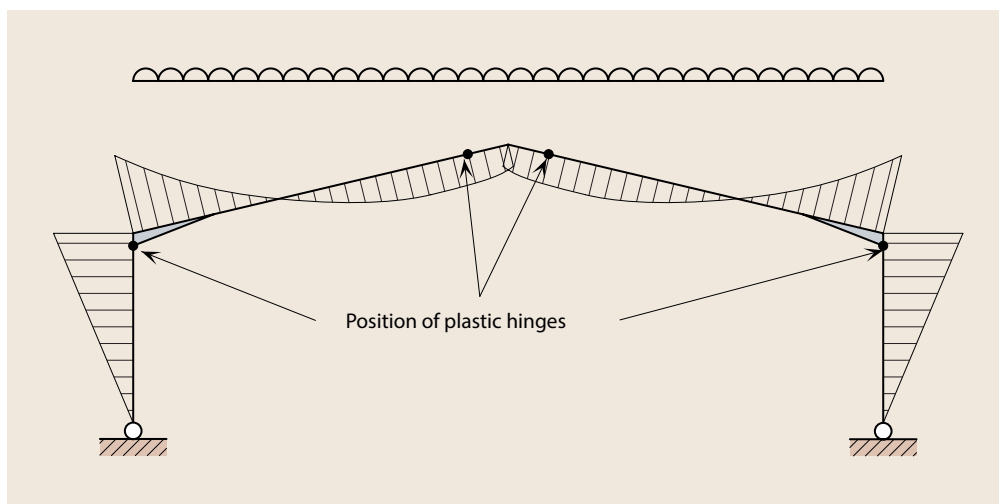
Plastic hinge rotations occur at sections where the bending moment reaches the plastic moment of resistance of the cross-section at load levels below the full ULS loading. Premature failure due to local buckling must be prevented as described in Section 8.2 of this document.

The requirements to prevent local buckling are articulated in BS EN 1993-1-1, Clauses 5.6 and 5.4.3 (2) and (3). A uniform member must have a cross-section at the plastic hinge location “which can form a plastic hinge with the rotation capacity required from plastic analysis without reduction of the resistance”. This cross-section is defined as Class 1 in Clause 5.5.2 (1). The maximum width to thickness ratios for compression parts are given in Table 5.2.

An idealised ‘plastic’ bending moment diagram for a symmetrical portal under symmetrical vertical loads is shown in Figure 5.3. The potential positions of the plastic hinges for the plastic collapse mechanism are shown. The first hinge to form is normally adjacent to the haunch (shown in the column in this case). At higher load levels, depending on the proportions of the portal frame, hinges form just below the apex, at the point of maximum sagging moment.

A portal frame with pinned bases has a single degree of indeterminacy. Therefore, two hinges are required to create a mechanism. The four hinges shown in Figure 5.3 only arise because of symmetry. In practice, due to variations in material strength and section size, only one apex hinge and one eaves hinge will form to create the mechanism. The eaves hinge could also occur at the other end of the haunch in the rafter, instead of the column. As there is uncertainty as to which hinges will form in the real structure, a symmetrical arrangement is assumed, and restraints provided at potential hinge positions on each side of the frame.

Figure 5.3  
Bending moment  
diagram resulting  
from the plastic  
analysis of a  
symmetrical portal  
frame under  
symmetrical loading



Where deflections (at SLS) govern design, there may be no advantage in using plastic analysis for the ULS. If stiffer sections are selected in order to control deflections, it is quite possible that no plastic hinges form and the frame remains elastic at ULS.

BS EN 1993-1-1 contains guidance on the verification of segments containing plastic hinges, stable lengths adjacent to plastic hinges, the necessary restraint at plastic hinges and limitations on member classification when carrying out plastic analysis and design.





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# INITIAL DESIGN

The following overall requirements should be considered at the initial design stage of the structure, depending on the building form and use:

- Space use, for example, specific requirements for handling of materials or components in a production facility;
- Accommodation of loading doors and loading docks;
- Flexibility of space in current and future use;
- Future expansion;
- Speed of construction;
- Environmental performance, including services requirements, air tightness and thermal performance;
- Aesthetics and visual impact;
- Acoustic insulation, particularly in production facilities;
- Access and security;
- Sustainability considerations;
- Design life and maintenance requirements, including end of life issues.

## 6.1 Building layout

The determination of the overall height and width of the frame is critical to give adequate clear internal dimensions and adequate clearance for the internal functions of the building. Precise dimensions can only be determined by carrying out a preliminary design to determine member sizes. Guidance on preliminary sizing of members is given in Appendix A. It is unlikely that modest conservatism in the analysis model will be detrimental, so precise dimensions are generally not critical.

Although frames can be spaced to suit particular building features, regular spacing is generally considered to have advantages:

- The loading on each frame is identical, and only one frame design is required.
- The stiffnesses (and therefore deflections) of the frames are the same.
- The foundation setting out is regular.
- Longitudinal members, such as the purlins, eaves strut, etc. are all regular.

### 6.1.1 Clear span and height

The clear span and height required by the client are key to determining the dimensions to be used in design, and should be established clearly at the outset. The critical client requirement is likely to be the clear distance between the flanges of the opposing columns – the structural span will therefore be larger, by the column section depth. Any requirements for brickwork or blockwork around the columns should be established, as this may also affect the design span.

Where a clear internal height is specified, this will usually be measured from the finished floor level to the underside of the haunch or suspended ceiling.

The calculation of the design height to eaves (the intersection of the rafter and column centre lines) for analysis should allow for:

- The distance from the top of the foundation to the finished floor level;
- The specified clear internal height;
- The requirements for any ceiling below the lowest point of the haunch;
- The requirements for any services below the lowest point of the haunch;
- The depth of the haunch;
- Half the depth of the rafter (calculated vertically).

### 6.1.2 Haunch dimensions

The depth of the haunch is often defined differently, depending on the context:

- For some software, the haunch depth is defined as the vertical distance from the intersection point of the centre-line of the rafter and the column to the bottom of the haunch at the end plate (Figure 6.1).

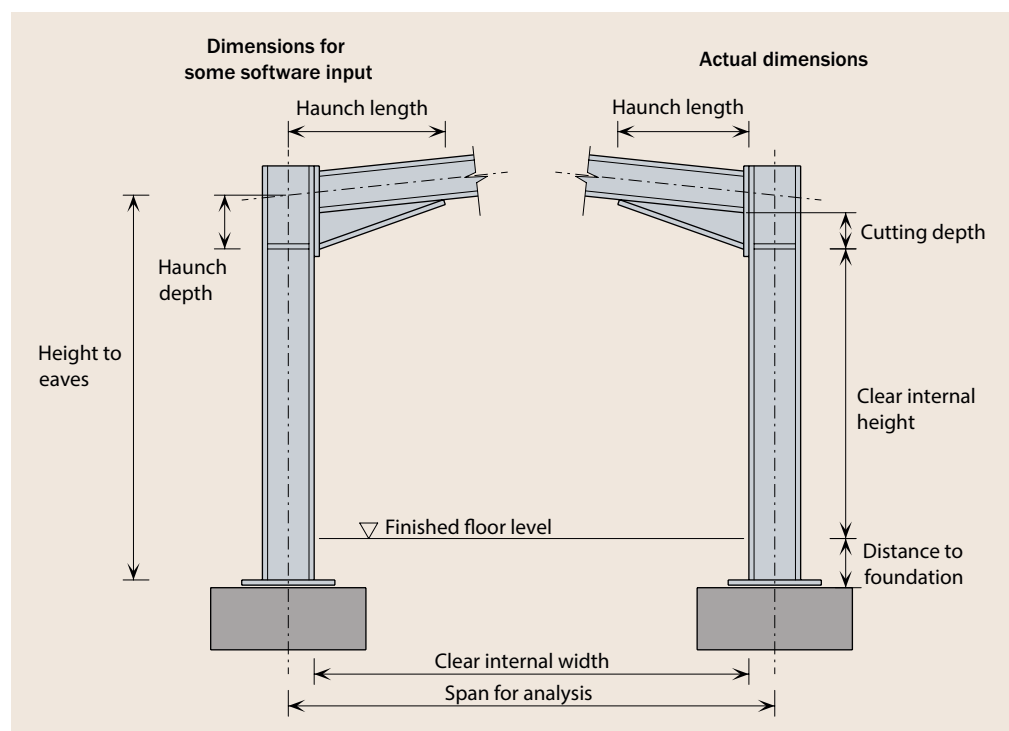


Figure 6.1  
Dimensions used for  
analysis and clear  
internal dimensions

- Steelwork contractors generally specify the cutting depth of the haunch as the depth from the underside of the rafter to the bottom of the haunch (Figure 6.2).
- In some instances, the haunch depth is referred to as the depth from the top of the rafter to the bottom of the haunch.

Similarly, the length of the haunch may be defined either from the centre-line or from the face of the column. The haunch length measured horizontally from the column centre-line to the end of the tapered section is usually chosen to be 10% of the portal span. This length means that in elastic design the hogging bending moment at the “sharp” end of the haunch is approximately the same as the maximum sagging bending moment towards the apex, as shown in Figure 6.2.

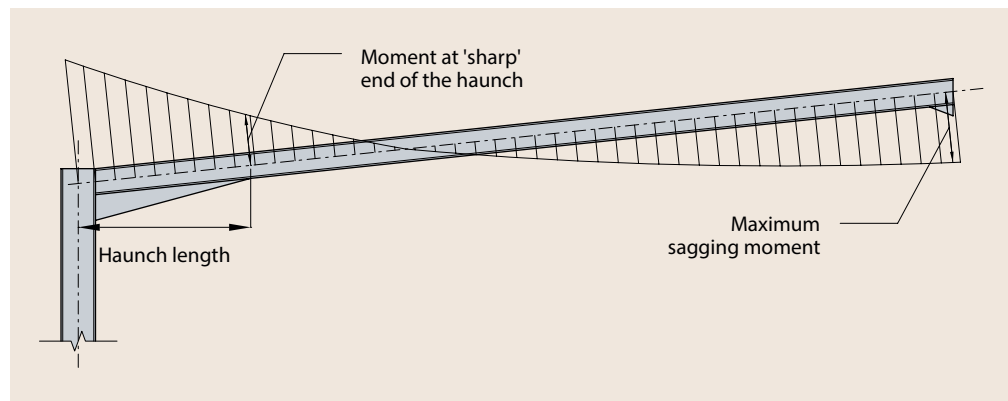


Figure 6.2  
Rafter bending  
moment and  
haunch length

### **Positions of restraints**

During initial design, the rafter members are normally selected according to their cross-sectional resistance to bending and axial force. In later design stages, stability against buckling needs to be verified and restraints positioned judiciously (Figure 6.3).

The initial selection of a column section is likely to be based on its buckling resistance, rather than its cross-sectional resistance. Compared to a rafter, there is usually less freedom to position rails to restrain buckling, as rail positions may be dictated by doors or windows in the elevation.

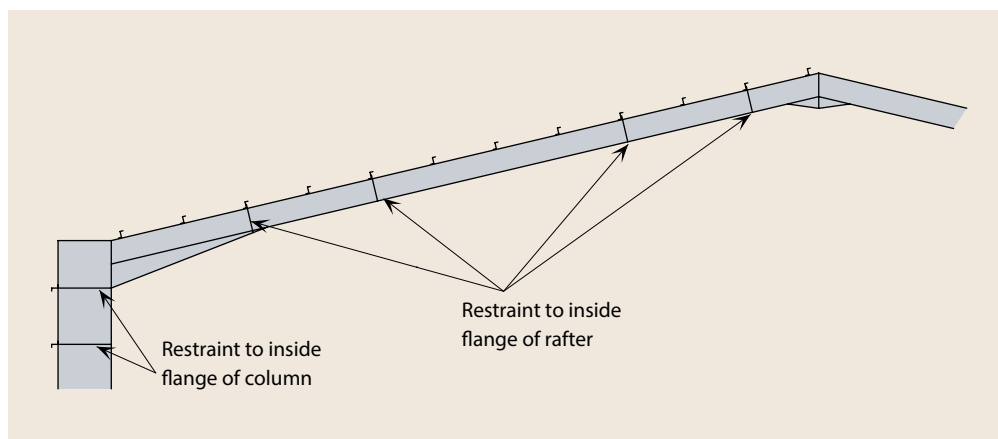
If the provision of sufficient intermediate restraints to the column is not possible, the buckling resistance will determine the initial section size. It is therefore essential to determine at this early stage whether the side rails can be used to provide restraint to the columns. Only continuous side rails are effective in providing restraint. Side rails interrupted by (for example) roller shutter doors cannot be relied on to provide adequate restraint, unless additional bracing is provided.

Where the compression flange of the rafter or column is not directly restrained by purlins or side rails, restraint can be provided at specific locations by column and rafter stays, as shown in Figure 6.3.

Further advice on the positioning of restraints is given in Section 8.2 of this publication.



Figure 6.3  
General arrangement  
of restraints to the  
inside flange



### 6.1.3 Steel grade and sub-grade

S355 material is usually selected, as grade S275 is rarely available in the UK and the higher strength material offers strength:cost benefits compared to S275 steel.

Steel sub-grade would normally be selected in accordance with BS EN 1993-1-10<sup>[26]</sup>, although the use of PD 6695-1-10<sup>[27]</sup> is strongly recommended as a simpler route. Within a portal frame, details will generally fall under the category of “welded generally”. Care should be taken if the steel is exposed to low temperatures (either externally, or in a refrigerated warehouse, for example), when a tougher sub-grade will probably be required.

Selection of a steel sub-grade in accordance with the Eurocodes is covered in SCI Document ED007<sup>[28]</sup>.

## 6.2 Preliminary analysis

Although efficient portal frame analysis and design will use bespoke software, which is likely to be using elastic-plastic analysis, preliminary manual elastic analysis is simple. In most circumstances, a reasonable estimate of the maximum bending moments will be obtained by considering only the vertical loads but it should be borne in mind that wind effects are always additive so a suitable margin should be allowed. Appropriate sections can then be chosen on the basis of this analysis.

For preliminary analysis without selecting sections, it is common to assume that the second moment of area of the column is 1.5 times that of the rafter section.

For the pinned base frame shown in Figure 6.4, the bending moment at the eaves,  $M_E$ , and at the apex,  $M_A$ , can be calculated as follows:

$$M_E = \frac{wL^2(3+5m)}{16N} \quad \text{and} \quad M_A = \frac{wL^2}{8} + m \times M_E$$

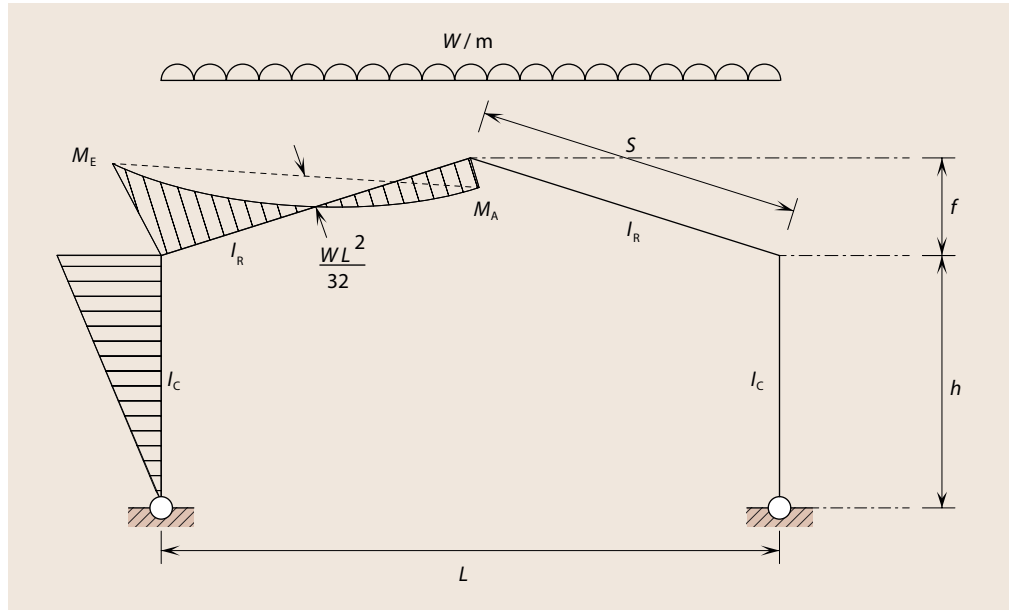


Figure 6.4  
Details of a pinned  
base portal frame

where:

$$\begin{aligned}
 N &= B + mC \\
 C &= 1 + 2m \\
 B &= 2(k + 1) + m \\
 m &= 1 + \phi \\
 \phi &= f/h \\
 k &= \frac{I_R}{I_C} \frac{h}{S}
 \end{aligned}$$

As noted above, it may be assumed for preliminary analysis that  $I_C = 1.5 \times I_R$

### Second order effects

It is likely that many economic frames will be sensitive to second order effects (see Section 5.2), which are likely to increase the design moments by up to 15%. If undertaking a preliminary analysis, bending moments from a first order analysis should be amplified to allow for these second order effects.

### 6.2.2 Selection of members

Because the primary effect in a portal frame is bending, beam sections are invariably selected for rafters and columns. The larger second moment of area of a UB section (compared to a UC section) also helps to control deflections.

For plastic design, in accordance with BS EN 1993-1-1:2005 Clause 5.4.1(3), at a plastic hinge, members should be doubly symmetric or singly symmetric with the plane of symmetry in the plane of rotation of the hinge. Class 1 cross-sections are required at plastic hinge locations in accordance with Clause 5.6(2)a). This is so that sufficient rotations can occur without local buckling to allow the redistribution of bending moments. If the sequence of formation of hinges can be predicted with confidence, the final hinge to form could be in a Class 2 member because no subsequent redistribution is required.

However, this does mean that collapse could be sudden, due to the onset of local buckling (i.e. a brittle failure mode not a ductile one). Restraint should be provided at the hinge position as required by the code.

The rafter should be selected such that its cross-sectional resistance  $M_{c,y,Rd}$  with a modest allowance for axial load (say 10%), exceeds the maximum design moment, which will be at the sharp end of the haunch, or near the apex. Generally, it will be possible to introduce sufficient restraints that an out of plane check between restraints will show the lateral torsional buckling resistance of the rafter is not critical. See Section 8.4.1.

If a more detailed check is warranted at the initial design stage, the lateral torsional buckling resistance may be taken as the value of  $M_{b,Rd}$  over a length equal to the purlin spacing (say 2000 mm maximum). When calculating  $M_{b,Rd}$  it may be conservatively assumed that the bending moment is uniform (i.e.  $C_1 = 1.0$ ) and equal to the maximum value in the segment between the purlins. The lateral torsional buckling resistance may be obtained from SCI publication P363<sup>[29]</sup>. An allowance should also be made for the axial load in the rafter.

The column should be selected such that its cross-sectional resistance,  $M_{c,y,Rd}$ , is at least equal to the moment at the underside of the haunch. In addition, the lateral torsional buckling resistance between restraints must exceed the applied moment including an allowance for the axial compression in the column (say 10%). The lateral torsional buckling resistance is almost certainly the critical check.

If a plastic hinge develops in the column at the underside of the haunch, a second restraint will be required below and relatively near the haunch restraint, otherwise the column will be uneconomic.

If no restraints are assumed (or none can be utilised) between the underside of the haunch and the base, a value of  $C_1 = 1.77$  (for a triangular bending moment diagram) may be assumed when calculating  $M_{b,Rd}$  over the column height. If intermediate restraints are utilised, a value of  $C_1 = 1.1$  for the length between restraints is a reasonable initial assumption. If no intermediate restraints are necessary, the column is likely to remain elastic.







# FRAME ANALYSIS

## 7.1 Introduction

3-D analysis software packages are widely available and their use is becoming the norm. Building Information Management (BIM) is increasingly being adopted as a means of producing integrated designs and comprehensively storing details of building components and their interrelationships. When using 3-D analysis software, all structural elements (both primary and secondary) can be included in the analysis model. One of the advantages is that distribution of area loads to members can be carried out efficiently by the software.

Portal frames are essentially planar structures and their behaviour can be satisfactorily modelled using 2-D analysis. Hand methods were naturally 2-D and elastic and plastic 2-D analysis software packages are also widely available. In this section, the discussion relates to 2-D analysis methods to promote understanding of the structural behaviour and provide methods for checking both 2-D and 3-D analysis results using alternative methods.

## 7.2 Frame imperfections

The Eurocode distinguishes the following types of imperfections to be taken into account:

- Global imperfections for frames and bracing systems;
- Local imperfections for individual members.

Frame imperfections are dealt with by applying equivalent horizontal forces to the structure determined in accordance with BS EN 1993-1-1 Clause 5.3.2. The initial bow in members is allowed for in the member checks and is discussed in Section 7.2.2.

### 7.2.1 Global imperfections and EHF

Global imperfections are assumed to follow the shape of the elastic buckling mode of the structure and for erection out-of-plumb, the relevant buckling mode is in sway. The effect of such a global imperfection is most simply modelled by assuming an initial sway which can be achieved by applying equivalent horizontal forces (EHF) in addition to other actions. This simple approach is recommended in this publication – the alternative is to model the frame out-of-plumb.

The magnitudes of the equivalent horizontal forces (EHF) are based on the initial sway imperfection  $\phi$ , given by Equation 5.5 of BS EN 1993-1-1 as:



$$\phi = \phi_0 \alpha_h \alpha_m$$

where:

$$\phi_0 = 1/200$$

$\alpha_h$  is the reduction factor for height  $h$  applicable to columns:

$$\alpha_h = \frac{2}{\sqrt{h}} \quad \text{but} \quad \frac{2}{3} \leq \alpha_h \leq 1.0$$

$h$  is the height of the structure in metres. For a portal frame,  $h$  should be taken as the height of the columns. For columns 9.0 m high or greater,  $\phi_0 \alpha_h = 1/300$

$\alpha_m$  is the reduction factor for the number of columns in a row

$$= \sqrt{0.5(1 + 1/m)}$$

$m$  is the number of vertical members contributing to the horizontal force on the lateral stability system. For single span portal frames  $m = 2$ .

The equivalent horizontal force at the top of each column can be determined as  $\phi N_{Ed}$ , in which:

$\phi$  is the initial sway imperfection

$N_{Ed}$  is the design value of the compression force in the column; for a portal frame it is equal to the design value of the vertical reaction at the column base. The compression force in the valley column in a hit frame includes the reactions from the valley beams.

The equivalent horizontal forces should be applied in the same direction at the top of each column in the frame. The most onerous direction should be considered, normally in the same direction as the wind actions.

The initial sway imperfections may be disregarded if:

$$H_{Ed} \geq 0.15 V_{Ed}$$

where:

$H_{Ed}$  is the design value of the horizontal loads

$V_{Ed}$  is the design value of the vertical loads.

It is possible that global imperfections need not be included in combinations including wind actions. This can be assessed by comparing the net vertical loads with the net horizontal loads (wind actions generally lead to asymmetric horizontal and vertical reactions at the bases). It is conservative simply to include the EHF in all combinations – their impact is modest in portal frames, because the vertical loads are generally relatively small. For portal frames supporting mezzanine floors or cranes, the impact of the EHF will be more significant.

### 7.2.2 Local imperfections

Imperfections in members (in the form of an initial bow) are allowed for in the formulae given in Section 6.3 of BS EN 1993-1-1, when calculating the buckling resistance of the

member. However, in accordance with Clause 5.3.2 (6) of BS EN 1993-1-1, if frames are sensitive to second order effects (see Section 7.3.2 of this publication), local imperfections must be allowed for in the analysis model for each compressed member if it has a moment-resisting joint at least at one end, and the non-dimensional slenderness  $\bar{\lambda}$  exceeds a limiting value. The limiting slenderness is given by:

$$\bar{\lambda} > 0.5 \sqrt{\frac{Af_y}{N_{Ed}}}$$

where:

- $A$  is the area of the section
- $N_{Ed}$  is the design value of the compression force
- $\bar{\lambda}$  is the in-plane non-dimensional slenderness calculated for the member, considered as hinged at its ends.

More conveniently, the limit may be expressed as  $N_{Ed} > 0.25N_{cr}$

where:

$$N_{cr} = \frac{\pi^2 EI_y}{L^2}$$

- $I_y$  is the second moment of area of the section about the major axis
- $L$  is the length of the member.

New work has demonstrated that for portal frames, the effects of in-plane initial imperfections in members are small enough to be ignored (see Appendix E).

## 7.3 Inclusion of second order effects

### 7.3.1 Analysis and design approaches in BS EN 1993-1-1

Where the influence of the deformations of the structure has to be taken into account, second order effects have to be considered in addition to imperfections, in accordance with BS EN 1993-1-1 Clause 5.2.2. This clause recognises a number of alternative ways of allowing for imperfections and second order effects. One approach is to allow for the imperfection of the frame and second order effects in the frame analysis, and then allow for the effects of imperfections in the members when verifying their buckling resistance. Member imperfections are automatically allowed for (together with residual stresses and other second order effects) if the member design is carried out in accordance with Section 6.3 of BS EN 1993-1-1.

A second approach is to account for all effects within the frame analysis. This would mean carrying out a second order analysis that would account for the imperfections (both in members and the frame itself) and include  $P-\Delta$  and  $P-\delta$  effects. If all these effects are accounted for in the analysis, no member buckling checks are necessary, only verification of cross-sectional resistances is required.

Allowing for second order effects does not mean completing a second order analysis, although this will often be the preferred option if using software for analysis and design. Second order effects can also be allowed for by amplifying the results of a first order analysis, as described in Section 7.7 of this publication.

### 7.3.2 Assessing the significance of second order effects

The second order effects due to the deformed geometry are assessed in BS EN 1993-1-1 by calculating the factor  $\alpha_{cr}$ , defined in Clause 5.2.1(3) as:

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}}$$

where:

$F_{cr}$  is the elastic critical buckling load for the lowest global instability mode, based on initial elastic stiffnesses

$F_{Ed}$  is the design load on the structure.

The value of  $\alpha_{cr}$  may be found using software or, as long as the portal frame meets certain geometric limits and the axial force in the rafter is not “significant”, by using the approximation given by Expression 5.2 from BS EN 1993-1-1. Rules are given in Clause 5.2 to identify when the axial force is significant.

When the frame falls outside the specified limits, as is the case for very many orthodox portal frames, the simplified expression cannot be used. In these circumstances, an alternative expression has been developed<sup>[30]</sup> to calculate an approximate value of  $\alpha_{cr}$ , referred to as  $\alpha_{cr,est}$ . Further details are given in Section 7.6.

According to Clause 5.2.1(3) of BS EN 1993-1-1, the effects of deformed geometry can be neglected (and a first order analysis used without modification) if  $\alpha_{cr}$  is above certain limits.

For elastic analysis, the effects of deformed geometry can be neglected if  $\alpha_{cr} \geq 10$ .

For first order plastic analysis, the UK National Annex Clause NA.2.9 indicates a limit of  $\alpha_{cr} \geq 5$  for portal frames subject to gravity loads only under certain conditions which are:

- The portal frame span  $L$  should not exceed 5 times the mean height of the columns;
- The rise of the rafters  $h_r$  should not exceed 0.25 times  $L$ ; or
- For asymmetric rafters,  $(h_r/s_a)^2 + (h_r/s_b)^2 \leq 0.5$  where  $s_a$  and  $s_b$  are the horizontal distances from the apex to the columns.

These conditions are the same as those set out in BS 5950-1:2000 Clause 5.5.4.2 which discusses the conditions for using the sway-check method for verifying the in-plane stability of portal frames.

If these conditions are not met, second order effects must be accounted for.

The limit of  $\alpha_{cr} \geq 5$  is equivalent to the elastic critical load factor for gravity loads implied by BS 5950 Clause 5.5.4.2.2. In this clause, the horizontal deflection at the top of



column  $I$  of height  $h_i$  is limited to  $h_i/1000$ , under a set of notional horizontal forces applied to the tops of each column equal to 0.5% of the column base vertical reaction. For load combinations which include wind or other horizontal loads, the effects of deformed geometry are allowed for under Clause 5.5.4.2.3 by using the amplifier  $\lambda_{sc}/(\lambda_{sc} - 1)$ . If  $\lambda_{sc}$  is less than 5, the clause states that second order analysis should be used.

## 7.4 Base stiffness

Truly pinned bases are not used in orthodox portal frame construction. If such bases are adopted for architectural reasons, the rotational stiffness is effectively zero. Where they are used, careful consideration needs to be given to the temporary stability of the column during erection. Unless bases are detailed as truly pinned joints, the bases will possess some degree of rotational stiffness. In many cases, the stiffness is small and the bases can be classed as nominally pinned.

Benefit may be taken of the stiffness of nominally pinned bases to reduce frame deflections at SLS and to reduce the effects of deformed geometry (improve frame stability), manifest in a higher value of  $\alpha_{cr}$ . If the nominal base stiffness is ignored by modelling the base as perfectly pinned, the outcome is conservative.

The recommended approach for orthodox portal frame construction with nominally pinned bases is as follows.

- Carry out an analysis at ULS assuming perfect pins at the bases, to avoid the requirement to transfer bending moment through the base into the foundation.
- Assess SLS deflections utilising the benefit of the nominal base stiffness (see Section 7.4.1 below), ignoring moments that arise at the base.
- Assess frame stability utilising the benefit of nominal base stiffness (see Section 7.4.1 below), ignoring moments that arise at the base.

It is UK practice to consider typical portal frame base details with four bolts outside the profile of the column section, and where the column is welded to the baseplate, as nominally pinned. Such a base is illustrated in Figure 7.1.

If any base stiffness is assumed at ULS, the base details and foundation must be designed to have sufficient resistance to sustain the calculated moments and forces. Moment-resisting foundations are usually avoided for portal frames because the high costs usually outweigh any savings in the frame from assuming significant base fixity. However, there are exceptions such as:

- To control the deflection of portal frames supporting EOT cranes;
- Where piled foundations are used due to the ground conditions.

Rigid column bases are extremely uncommon in the UK, and therefore no recommendations for design stiffness for rigid bases are given in this publication. Details can be obtained from non contradictory complementary information (NCCI) SNO45a <sup>[31]</sup>.

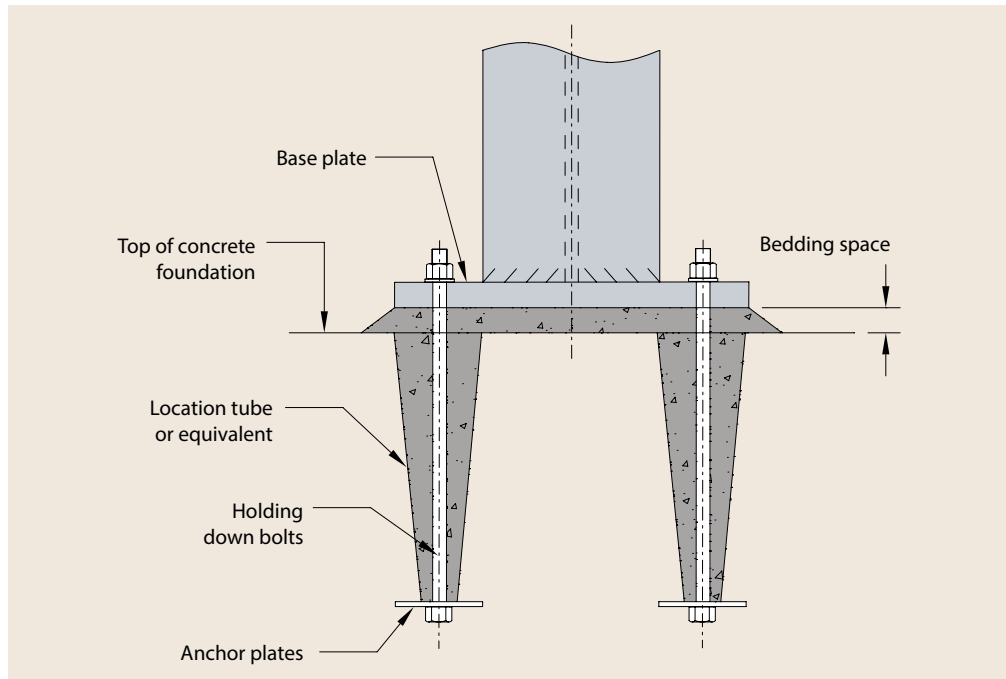


Figure 7.1  
Example of a  
nominally  
pinned base

### 7.4.1 Modelling of nominally pinned bases

For the assessment of frame stability and for the assessment of deflections at SLS, the base may be modelled with a stiffness assumed to be a proportion of the column stiffness, as follows:

- 10% when assessing frame stability (see Section 7.3.2 of this publication);
- 20% when calculating deflections at SLS.

Bespoke software normally has the facility to select values of base stiffness. Alternatively, the base stiffness may be modelled by the use of a spring stiffness or dummy members at the column base.

#### Spring stiffness

The spring stiffness equivalent to 10% of the column stiffness is equal to  $0.4EI_{\text{column}}/L_{\text{column}}$ .

The spring stiffness equivalent to 20% of the column stiffness is clearly twice this value.

#### Modelling with dummy base members

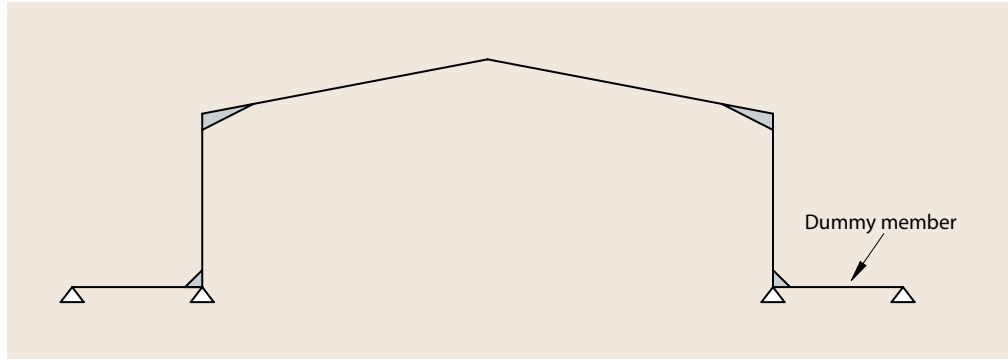
If the software cannot accommodate a rotational spring, the base fixity may be modelled by a dummy member of equivalent stiffness, as shown in Figure 7.2.

The second moment of area ( $I_y$ ) of the dummy member should be taken as:

When assessing frame stability:  $I_y = 0.1 I_{y,\text{column}}$ .

When calculating deflections at SLS:  $I_y = 0.2 I_{y,\text{column}}$ .

Figure 7.2  
Modelling base fixity  
by a dummy member



In both cases, the length of the dummy member is  $L = 0.75 L_{\text{column}}$ , and it is modelled with a pinned support at the extreme end.

When nominal base stiffness is assumed at SLS, and when calculating  $\alpha_{\text{cr}}$ , the resulting moments at the base are ignored.

Reactions from analysis with the use of dummy members should not be used explicitly, as the provision of an additional support will affect the base reactions. The vertical base reaction should be taken as the axial force at the base of the column.

#### 7.4.2 Fire boundary considerations

When the proximity to a site boundary means that the portal base must be designed to resist an overturning moment in the fire condition, it is normal for the base detail to be considerably strengthened, with a thicker baseplate or in some cases, a small haunch. Although it would appear clear that such a base will exhibit at least semi-continuous behaviour, usual practice in the UK is still to consider the base only as nominally pinned for normal temperature design.

### 7.5 Evaluation of in-plane frame stability

The parameter  $\alpha_{\text{cr}}$  provides a measure of the stability of frames. It is the factor by which the design loads would have to be increased to cause elastic instability in a global mode. For a portal frame, the first and second modes of buckling are usually the most important. The first mode is a sway mode and the second mode is a symmetrical mode where both columns in a single-bay portal frame deflect outward and the apex drops. In a portal frame of at least three bays, a “snap-through” mode of buckling may also occur where a pair of rafters invert.

According to BS EN 1993-1-1 Clause 5.2.1(4)B, for the sway mode the value of  $\alpha_{\text{cr}}$  is given by:

$$\alpha_{\text{cr}} = \left( \frac{H_{\text{Ed}}}{V_{\text{Ed}}} \right) \left( \frac{h}{\delta_{\text{H,Ed}}} \right) \quad \text{EN 1993-1-1(5.2)}$$

where, for an individual single span portal frame:

$H_{Ed}$  is the algebraic sum of the base shear on the two columns – due to the horizontal loads and the EHF

$V_{Ed}$  is the total design vertical load on the frame – the algebraic sum of the two base reactions

$\delta_{H,Ed}$  is the maximum horizontal deflection at the top of either column, relative to the base, when the frame is loaded with horizontal loads (e.g. wind) and the EHF

$h$  is the column height.

Because loads on a pitched roof contribute to the lateral deflection at the top of the portal column, it is recommended that  $\alpha_{cr}$  be calculated based on  $H_{Ed}$  and  $\delta_{H,Ed}$  due only to notional horizontal forces (NHF). The NHF should be taken as 1/200 of the design vertical base reaction, and they should be applied to each column, in the same direction, at eaves level. When following this recommendation, note that the NHF used when determining  $\alpha_{cr}$  are distinct from the EHF which are applied to model frame imperfections and may include the effect of  $\alpha_h$  and  $\alpha_m$ .

The expression for  $\alpha_{cr}$  then becomes:

$$\alpha_{cr} = \frac{h}{200\delta_{NHF}}$$

where:

$h$  is the height to eaves

$\delta_{NHF}$  is the lateral deflection at the top of the column due to the NHF

Notes 1B and 2B of Clause 5.2.1 limit the application of the expression to roof slopes no steeper than  $26^\circ$  and where the axial force in the rafter is not significant. Axial force in the rafter may be assumed to be significant if

$$\bar{\lambda} \geq 0.3 \sqrt{\frac{Af_y}{N_{Ed}}}$$

where:

$\bar{\lambda}$  is the non-dimensional slenderness of the rafter pair, for flexural buckling about the major axis

$A$  is the cross-sectional area of the rafter

$f_y$  is the yield strength of the rafter

$N_{Ed}$  is the axial compression in the rafter.

When calculating  $\bar{\lambda}$ , the buckling length is taken as the developed length of the rafter pair from column to column, taken as  $\text{span}/\text{Cos } \theta$ , where  $\theta$  is the roof slope.

A convenient way to express the limitation on the axial force is that the axial force is not significant if:

$$N_{Ed} \leq 0.09N_{cr}$$

where:

$N_{cr}$  is the elastic critical buckling load

$N_{cr,R}$  is the elastic critical load for buckling about the major axis for the complete span of the rafter pair, i.e.  $N_{cr,R} = \frac{\pi^2 EI}{L^2}$

$L$  is the developed length of the rafter pair from column to column, taken as span/Cos  $\theta$ , where  $\theta$  is the roof slope

$I$  is the second moment of area of the rafter about the major axis ( $I_{yy}$ )

$E$  is the modulus of elasticity (210000 N/mm<sup>2</sup>).

If the limits are satisfied, then Expression (5.2), given above, may be used to calculate  $\alpha_{cr}$ . In most practical portal frames, the axial load in the rafter will be significant and Expression (5.2) cannot be used.

The benefit of accounting for base stiffness when performing the stability analysis should not be underestimated, as, for a portal frame with nominally pinned bases, nominal base stiffness can increase the value of  $\alpha_{cr}$  significantly.

## 7.6 Evaluation of in-plane stability when the axial force in the rafter is significant

When the axial force in the rafter is significant, a conservative measure of frame stability, defined as  $\alpha_{cr,est}$  may be calculated as described in reference 30. More accurate (higher) values of  $\alpha_{cr}$  will be obtained from software.

For frames with pitched rafters:

$$\alpha_{cr,est} = \min [\alpha_{cr,s,est}; \alpha_{cr,r,est}]$$

where:

$\alpha_{cr,s,est}$  is the estimate of  $\alpha_{cr}$  for the sway buckling mode

$\alpha_{cr,r,est}$  is the estimate of  $\alpha_{cr}$  for the rafter snap-through buckling mode. This mode only needs to be checked when there are three or more spans, or if the rafter is horizontal, or when the columns are not vertical.

### 7.6.1 Factor $\alpha_{cr,s,est}$

The value of  $\alpha_{cr,s,est}$  is given by:

$$\alpha_{cr,s,est} = 0.8 \left\{ 1 - \left( \frac{N_{Ed}}{N_{cr,R}} \right)_{\max} \right\} \alpha_{cr}$$

where:

$\left( \frac{N_{Ed}}{N_{cr,R}} \right)_{\max}$  is the maximum ratio in any of the rafters

$N_{Ed}$ ,  $N_{cr,R}$ ,  $\alpha_{cr}$  are as previously defined

The lowest value of  $\alpha_{cr}$  for any column is used for the frame as a whole.

The calculation process is:

1. Complete a frame analysis with pinned bases under the design value of combination of actions to determine the vertical base reactions and the axial compression in the rafter,  $N_{Ed}$  (see Figure 7.3).
2. Calculate the values of NHF as 1/200 of the base reactions given by the analysis.
3. Complete a second analysis, with only the NHF on the otherwise unloaded frame, and determine the horizontal deflections  $\delta_{NHF}$  at the tops of the columns (see Figure 7.4). Utilising the beneficial stiffness of nominally pinned bases given in Section 7.4 for this analysis is recommended.

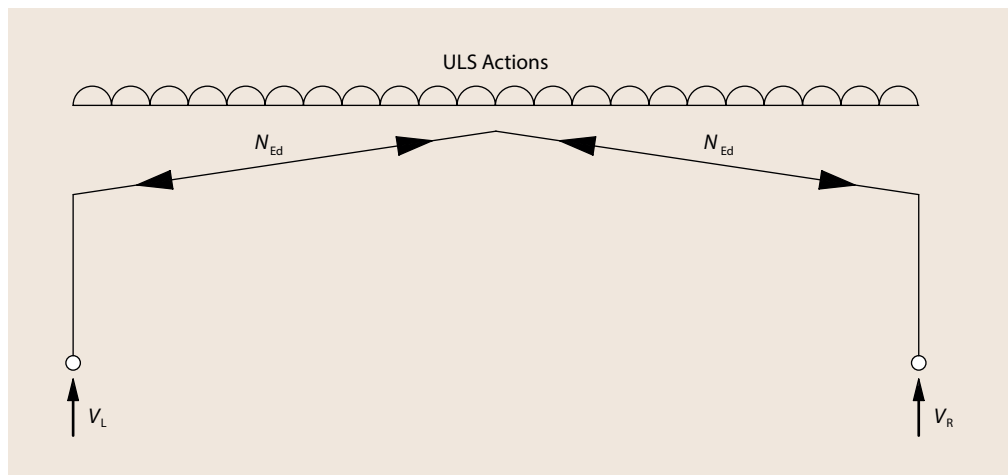


Figure 7.3  
Analysis to establish  
rafter force and  
base reactions

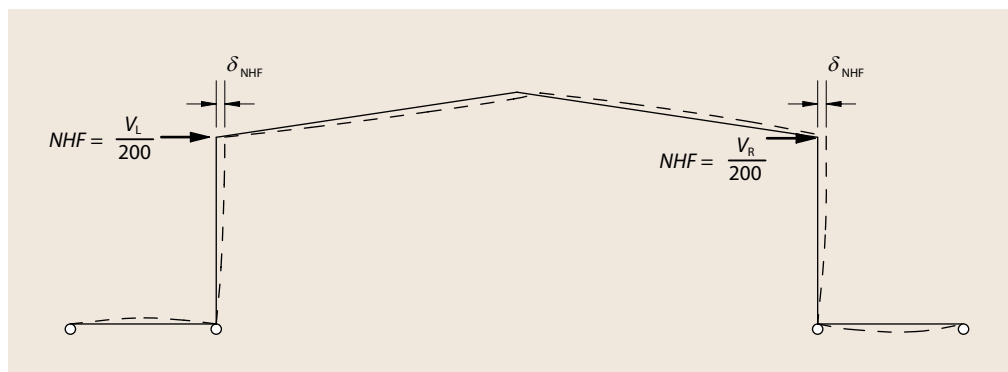


Figure 7.4  
Analysis to establish  
horizontal deflection  
under NHF

### 7.6.2 Factor $\alpha_{cr,est}$

For single span portal frames, this calculation should only be carried out if the rafter is horizontal or when the columns are not vertical.



For frames with rafter slopes not steeper than 1:2 (26°),  $\alpha_{cr,est}$  may be taken as:

$$\alpha_{cr,est} = \left( \frac{D}{L} \right) \left( \frac{55.7(4 + L/h)}{\Omega - 1} \right) \left( \frac{I_c + I_r}{I_r} \right) \left( \frac{275}{f_{yr}} \right) (\tan 2\theta_r)$$

But where  $\Omega \leq 1$ ,  $\alpha_{cr,est} = \infty$

where:

- $D$  is the cross-sectional depth of the rafter
- $L$  is the span of the frame
- $h$  is the mean height of the column from base to eaves or valley
- $I_c$  is the in-plane second moment of area of the column (taken as zero if the column is not rigidly connected to the rafter)
- $I_r$  is the in-plane second moment of area of the rafter
- $f_{yr}$  is the nominal yield strength of the rafters in N/mm<sup>2</sup>
- $\theta_r$  is the roof slope
- $h_r$  is the height of the apex of the roof above the straight line between the tops of the columns
- $\Omega$  is the arching ratio, given by  $\Omega = W_r/W_0$
- $W_r$  is the total design vertical load on the rafters of a frame
- $W_0$  is the value of uniform load for plastic failure of the rafters as a fixed ended beam of span  $L$ .

## 7.7 Modified first order elastic analysis

The ‘amplified moment method’ is the simplest method of allowing for second order  $P-\Delta$  effects in a first order elastic frame analysis; the principle is given in BS EN 1993-1-1, Clause 5.2.2(5B).

A first order linear elastic analysis is carried out and  $\alpha_{cr}$  (or  $\alpha_{cr,est}$ ) determined (see Section 7.3.2). If second order effects are significant, all horizontal actions and other possible sway effects are increased by an amplification factor to allow for the second order effects. The horizontal actions comprise the externally applied actions, such as the wind load, and the equivalent horizontal forces (EHF) used to allow for frame imperfections. Note that the EHF are amplified, not the NHF used to calculate  $\alpha_{cr}$ . It is normally sufficient just to amplify the effects of wind and EHF.

Provided  $\alpha_{cr} \geq 3.0$  the amplification factor is given by:

$$\left( \frac{1}{1 - 1/\alpha_{cr}} \right)$$

If the axial load in the rafter is significant, and  $\alpha_{cr,est}$  has been calculated in accordance with Section 7.6, the amplifier is given by:

$$\left( \frac{1}{1 - 1/\alpha_{cr,est}} \right)$$

If  $\alpha_{cr}$  or  $\alpha_{cr,est}$  is less than 3.0, second order analysis must be used; the simple amplification is not sufficiently accurate.

## 7.8 Modified first order plastic analysis

Second order effects can be allowed for in plastic analysis of portal frame structures without recourse to second order elastic plastic analysis software. Simple design rules based on the Merchant-Rankine reduction method<sup>[30]</sup> can be applied and are described below. These rules are appropriate for two categories of frames:

Category A: regular, symmetric and mono-pitched frames;

Category B: frames that fall outside Category A but excluding tied portals.

Tied portals have larger axial forces in the rafters (balanced by the force in the tie) than are present in untied portal frames and the non-linear behaviour of these frames is complex. The Merchant-Rankine formula is not able to take into account the instability of tied portals with sufficient accuracy. Frames with a value of span/column height ( $L/h$ ) greater than eight are also excluded because these frames are more susceptible to second order effects than other frame geometries. Frames with  $\alpha_{cr} \leq 3.0$  are also excluded. For all these types of frames, second order elastic-plastic analysis software must be used.

### 7.8.1 Merchant-Rankine reduction factor

The Merchant-Rankine reduction factor is given by the following:

$$\left( \frac{\alpha_{p2}}{\alpha_{p1}} \right)_{MR} = \frac{\alpha_{cr} - 1}{\alpha_{cr}}$$

where  $\alpha_{p2}$  is the second order elastic-plastic collapse factor with strain hardening and  $\alpha_{p1}$  is the first order plastic collapse factor (without strain hardening). The first order plastic collapse factor can be determined by first order plastic analysis software or by a hand analysis. The reduction factor is the inverse of the amplification factor in Section 7.7

Frames in Category A:

$$\alpha_{p2} = \alpha_{p1} \left( \frac{\alpha_{cr} - 1}{\alpha_{cr}} \right)$$

Frames in Category B:

$$\alpha_{p2} = \alpha_{p1} \left( \frac{\alpha_{cr} - 1}{1.1\alpha_{cr}} \right)$$

The frame elements should be chosen such that the second order elastic-plastic collapse factor is greater than 1.0.

## 7.8.2 Plastic design procedure

1. The frame geometry and loading are chosen as for elastic design and initial member sizes are selected.
2. An elastic analysis is carried out to determine  $\alpha_{cr}$  using the appropriate foundation stiffness; calculate the Merchant-Rankine reduction factor.
3. Carry out a first order plastic analysis and determine  $\alpha_{p1}$ .
4. Apply the Merchant-Rankine reduction factor and determine  $\alpha_{p2}$ . Check that  $\alpha_{p2}$  is greater than 1.0. If not, select members with a larger  $M_p$  and repeat Steps 2 to 4.

## 7.9 Hit and miss frames

Buildings with two or more spans and valley beams supporting alternate frames have two different frame types which have to be analysed individually when using 2-D plastic analysis software. The frames will have different lateral stiffnesses and different values of  $\alpha_{cr}$ . The lateral displacement under wind load on the side of the building, and spread under vertical load, will be different for each type of frame.

In practice, the adjacent frames in a building move together because they are connected together by primary steelwork such as the valley beam or bracing and by roof sheeting supported on purlins fixed to the rafters. Applied lateral wind load is the same on each frame (assuming uniform frame spacing) but the hit frames are stiffer due to the effect of the internal column with a moment connection at the top. The hit frames attract more lateral load to equalize the lateral displacement and the base shear is higher than on the miss frames. The additional load is transferred through the elements connecting the frames.

If a 3-D analysis is carried out, the forces in the connecting elements will be determined as a matter of course. However, if 2-D analysis is being used, an estimate of the force transferred between adjacent frames can be made using the following procedure which applies to symmetrical two-bay frames. In the case of symmetrical frames, under symmetrical vertical load cases, there is no lateral displacement at the central valley and no load induced.

If no bracing is provided, the valley beam will attract some of the load but it is not clear how much. It would be safe to assume all the load is transferred through the valley beam but this would involve bending of the beam about its minor-axis. If it is desired to design the valley beam as continuous through the hit frames, this minor-axis bending adds complexity to the design of the member and the connection.

The principle of elastic superposition, valid for first order elastic analysis, is not necessarily valid in plastic analysis and not strictly valid in either second order elastic or plastic analyses. The actual value of the additional load should therefore be used in the analysis of a fully loaded frame.

If more than one frame is supported on a valley beam (e.g. hit-miss-miss frames), the valley beam is subject to a uniform moment over the middle third of the beam for equal

frame spacings. If the valley beam is designed plastically, a lot of bracing will be required over its middle third to restrain the beam adequately.

### 7.9.1 Procedure

1. Analyse the miss frame, introducing spring stiffnesses at the valley to represent the valley beam. The vertical stiffness can be determined from an estimated valley beam size. Zero horizontal and rotational stiffnesses can be adopted. Determine the member sizes by plastic design.
2. Apply a unit horizontal load at the valley only and determine the lateral deflection in an elastic analysis.
3. Analyse the hit frame, using the same column and rafter sizes as for the miss frame. Include the valley beam reactions as an applied load at the top of the valley column. Check the member sizes and determine the size of the column.
4. Apply a unit horizontal load at the valley and determine the lateral deflection of this point in an elastic analysis. The deflection is expected to be less than that due to the unit load in the miss frame model because the hit frame is stiffer due to the presence of the central column.
5. Increase the horizontal spring stiffness in the miss frame so as to reduce the horizontal deflection. A horizontal reaction in the spring will be developed.
6. Add this force to the unit load on the hit frame and recalculate the deflection. The new deflection can be determined by increasing the deflection due to the unit load pro-rata. Compare the deflections in the hit and miss frames. Iterate Steps 5 and 6 until the deflections are similar.
7. Determine the spring reaction in the miss frame for the combination load cases involving lateral displacements (i.e. a load case with wind applied to the side of the building). Where wind is the lead variable, the spring force is expected to be twice the value where wind is the secondary variable because the load factors are 1.5 and 0.75 respectively. The unfactored load can be determined from either load case because  $2/1.5 = 1/0.75$ .
8. Add the unfactored lateral load at the valley to the basic wind load case in the hit frame and check the frame with the additional load included. Increase the member sizes as necessary. In following this procedure for an example two-bay frame, the central UC column in the hit frame had to be increased to the next weight in the serial size.
9. Check that the horizontal deflections of the valley of the hit frame and the miss frame are consistent.

### 7.9.2 Procedure to determine bracing loads

It is desirable to introduce a bracing system in the plane of the roof through which to transfer this force directly instead of indirectly through the purlins and roof sheeting, (due to their in-plane stiffness).

The procedure which follows is illustrated in Figure 7.5 and allows for the stiffness of the bracing system.

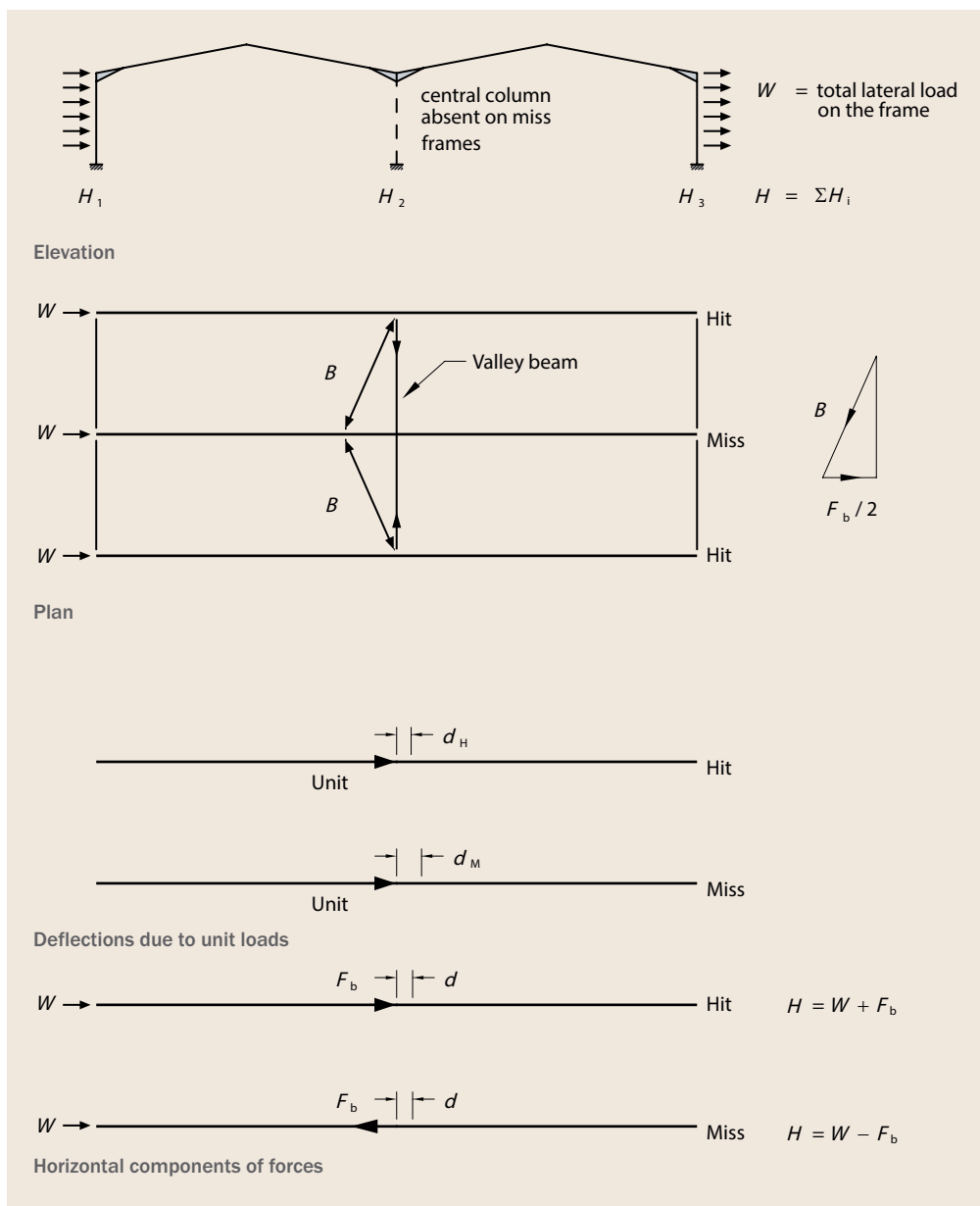


Figure 7.5  
Determination of  
bracing loads

1. Follow Steps 1 to 6 above to determine the spring reaction which gives the same deflections in the hit and miss frames, designated  $\delta_h$  and  $\delta_m$ . This horizontal force is designated  $F_b$  and is an equal and opposite force on the hit and miss frames.
2. The corresponding force in the plane of the rafter is equal to  $F_b / \cos\theta$  where  $\theta$  is the slope of the rafter to the horizontal. Apply equal and opposite forces to the bracing system in the plane of the rafter to determine the deflection of the bracing system  $\delta_b$ . The corresponding horizontal deflection is  $\delta_b / \cos\theta$ .
3. Increase the spring stiffness in the miss frame such that  $(\delta_h - \delta_m) = \delta_b / \cos\theta$ . Iteration may be necessary.
4. Determine the spring reaction in the miss frame for the combination load cases involving lateral displacements (i.e. a load case with wind applied to the side of the building). This reaction is the horizontal component of the force transferred through the bracing.

5. Add the unfactored lateral load at the valley to the wind load case in the hit frame and recheck the frame with the additional load included. Increase the member sizes as necessary.
6. Check that the deflections of the valley of the hit frame and the miss frame are consistent.

## 7.10 Overall stability perpendicular to the portal frames

The overall stability of single span portal frame buildings in the longitudinal direction (at right angles to the span of the portal frames) is usually simply achieved by providing vertical bracing in both side walls. The adequacy of the bracing is checked as for a simple building.

For portal frame sheds of larger dimensions, the overall stability of the building out of plane becomes more sensitive, particularly for buildings with multiple spans. In these buildings, it may be desirable to introduce diagonal bracing on the lines of the internal columns but this would intrude into the usable space in the building. Common arrangements for providing longitudinal stability of buildings with more than one span are:

- Vertical bracing in the perimeter walls and in the lines of internal columns (bracing may also be provided in the roof at the gables but not as part of the longitudinal stability system);
- Vertical bracing in the perimeter walls only and plan bracing in the roof to restrain the tops of the lines of internal columns;
- Vertical bracing in the walls and portalised bays in the longitudinal direction in the lines of internal columns.

All the gravity load in the columns being stabilised must be considered when determining  $\alpha_{cr}$  for the stability system. For the first and third arrangements, the bracing or portal frames on the internal column line must stabilise the gravity load in all the internal columns. For the second arrangement, the perimeter bracing stabilises the whole building so the bracing in each side wall stabilises half the total gravity load on the building.

For the third arrangement, plastic design is not recommended for the stability frames because there may be inadequate ductility at connections and reduced sway stiffness if extensive plasticity occurs, e.g. in “hit-miss-miss” frames.

If simplified methods are used to estimate  $\alpha_{cr}$ , e.g. as in BS EN 1993-1-1 Clause 5.2.1(4)B, care must be taken in calculating the horizontal displacement  $\delta_{H,Ed}$  at the top of the columns to allow for the stiffness of all the contributing structural systems. When plan bracing spanning back to the perimeter walls is used to restrain the interior lines of columns as in 2 above, the deflection of the plan bracing due to the horizontal loads must be taken into account in addition to the deflection of the vertical bracing.

This issue is covered in more detail in Reference 32.









# MEMBER VERIFICATION

Once the analysis has been completed, the frame members must be verified. In general, both the cross-sectional resistance and member buckling resistance must be verified. Member buckling resistance is often referred to as member stability; the terms are equivalent.

As discussed in Sections 5.2, 7.2 and 7.3, in accordance with BS EN 1993-1-1 Clauses 5.2.2(3) and (7) if both global and local imperfections and second order  $P-\Delta$  and  $P-\delta$  effects are allowed for in the analysis, only member resistance (strength) checks are necessary to verify the structure.

Global imperfections are allowed for by applying EHF as described in 7.2.1.  $P-\Delta$  effects are allowed for (if necessary) by performing a second order analysis, either explicitly by using appropriate software or by carrying out a modified first order analysis as described in Sections 7.4 to 7.7. Recent research (Appendix E) has demonstrated that the effects of local imperfections ( $P-\delta$  effects) on the in-plane behaviour of portal frames are small enough to be ignored. The requirements of Clause 5.2.2(3) are therefore satisfied when considering in-plane stability, and only strength checks are necessary. Out of plane, member stability must be checked as well as resistance and  $P-\delta$  effects are accounted for in the member verifications provided in BS EN 1993-1-1.

## 8.1 Cross-sectional resistance

Member bending, axial and shear resistances must be verified. If the shear or axial force is high, the bending resistance is reduced, so resistance to coexisting shear force and bending moment, and coexisting axial force and bending moment needs to be verified. In typical portal frames, neither the shear force nor the axial load is sufficiently high to reduce the bending resistance. When the portal frame rafters form the chord of a bracing system, the axial force in the rafter may be significant, and the combined effects should be verified.

### 8.1.1 Classification of cross-section

In BS EN 1993-1-1, cross-sections are classified according to the width to thickness ratio of the flanges and web, dependent on the magnitude of the bending moment and axial compression on the section. The Class of a section is the least favourable Class of either the flanges or the web.

The Classes indicate the following structural behaviour:

- Class 1 can support a rotating plastic hinge without any loss of resistance from local buckling.
- Class 2 can develop full plastic moment but with limited rotation capacity before local buckling reduces resistance.
- Class 3 can develop yield in extreme fibres but local buckling prevents development of the plastic resistance.
- Class 4 has proportions such that local buckling will occur at stresses below first yield.

### 8.1.2 Column and rafter cross-section verification

The key points for verification are at the positions of maximum bending moment as shown in Figure 8.1:

- In the column at the underside of the haunch;
- In the rafter at the sharp end of the haunch;
- In the rafter at the maximum sagging location adjacent to the apex.

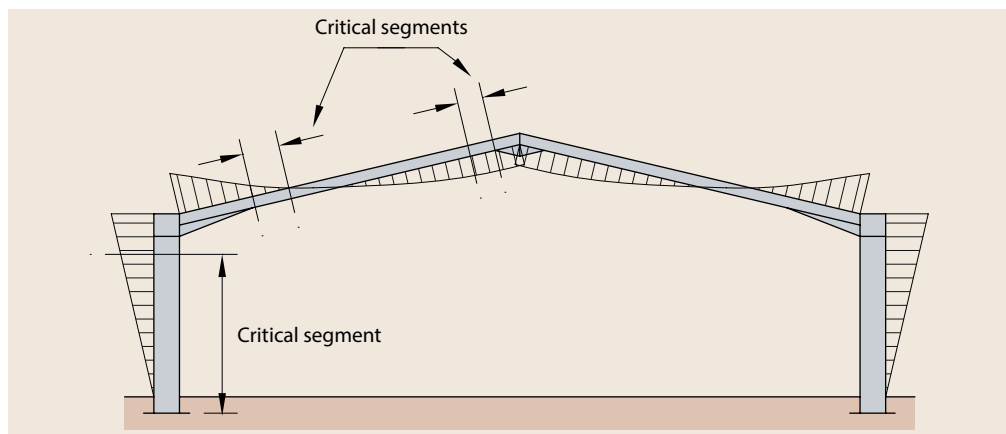


Figure 8.1  
Typical bending  
moment diagram for  
a frame with pinned  
bases subject to  
gravity loading

At the position of the maximum bending moment for each member a number of effects must be considered, as follows:

#### **Bending**

The bending resistance of the cross-section should be verified. This check is usually not critical, as the highest utilisation ratio is found when combined compression and bending are considered.

#### **Shear**

The shear resistance of the cross-section should be verified. Although high shears can serve to reduce moment resistance, shears in portal frame members are generally not high enough to reduce the bending resistance of the cross-section.

#### **Axial resistance**

The tension and compression resistance of the cross-section should be verified. It is very unlikely that this check will be critical.

### **Combined bending and compression**

The presence of compression in the section reduces the moment resistance, though the Eurocode provides limits in Clause 6.2.9.1(4) for Class 1 and Class 2 sections, specifying when the compression is low enough to be ignored. For typical portal frames, it is likely that the axial force will be low enough to have no effect on the bending resistance. For Class 3 cross-sections, the axial force will reduce the bending resistance and Clause 6.2.9.2 needs to be followed.

The most straightforward way of determining the reduced moment resistance is to use the 'Blue Book'<sup>[29]</sup>, which provides values of the design moment resistance reduced for various levels of axial force,  $M_{N,y,Rd}$ .

### **8.1.3 Haunch cross-section verification**

Elements of the haunch (the webs and the flanges) should be classified in accordance with Table 5.2 of BS EN 1993-1-1. A simple approach is recommended, conservatively assuming that the web elements for the rafter and the haunch cutting are in uniform compression, depending on the load combination being considered. If every element is at least Class 2, the cross-sectional resistance may be calculated based on plastic properties.

In the common cases where the web is not at least Class 2, an appropriate approach is to calculate an effective plastic cross section, assuming that the web is only effective for a distance of  $20t_w \varepsilon$  from the flanges, where:

$t_w$  is the thickness of the web

$$\varepsilon = \sqrt{\frac{235}{f_y}}$$

The classification may vary at different points along the haunch, so should be carried out at intermediate positions – to divide the haunch into five segments is appropriate.

### **Bending resistance**

In most cases, the axial force will be very small compared to the cross-sectional resistance and can therefore be ignored when calculating the bending resistance. If plastic or effective plastic cross-sectional properties have been calculated, the bending resistance may be calculated neglecting the (small) part of the cross section allocated to the compression force. This is demonstrated in Appendix D Section 11.1.2 of P397<sup>[33]</sup>.

It is very unlikely in portal frame construction that the applied shear will be sufficient to reduce the bending resistance of the haunch section.

### **Shear resistance**

The shear area of the cross-section,  $A_v$ , can be taken as the total depth of the compound section multiplied by the minimum web thickness.

### **Compression**

The compression resistance may be calculated using the gross area, or, if Class 3 effective properties have been calculated, the effective area. It is most unlikely that compression will be critical.

## **8.2 Member buckling resistance**

Columns and rafters are subject to combined axial force and bending so member verification out of plane involves the consideration of the flexural buckling resistance, the lateral-torsional buckling resistance and the member resistance under combined axial force and bending. The secondary steelwork is used to provide intermediate restraints, to reduce the length of segments and so increase both the flexural and lateral-torsional buckling resistance.

### **8.2.1 Adjacent to plastic hinges**

Torsional restraints are required at plastic hinge locations in accordance with BS EN 1993-1-1 6.3.5.2. The clause allows the restraint to be provided within a distance equal to half the overall depth of the member at the plastic hinge if for practical reasons, such as interference with another connection, it is not possible to provide restraint at the actual hinge position. Segments adjacent to this restraint are treated as segments adjacent to a plastic hinge except as described below for haunched or tapered members which remain wholly elastic.

Lengths of elements which are stable under axial load with a plastic moment at one end can be determined from BS EN 1993-1-1 Annex BB for different member types and bending moment distributions. These lengths determine the positions at which additional torsional restraints are required and vary according to the member shape and moment distribution as described below.

#### **Uniform members**

Where a segment of a member is of uniform section, the segment should be restrained torsionally at a distance away from the torsional restraint at the hinge equal to the stable length,  $L_k$ , of the segment (for a constant moment), or  $L_s$  (for linear and non-linear moment gradients) in accordance with BB.3.1.2.

In addition, at least one intermediate lateral restraint should be provided between the torsional restraints, at a spacing that satisfies the requirements for the stable length,  $L_m$ , in accordance with BB.3.1.1. The spacing between the first and subsequent lateral restraints can be verified by carrying out an elastic check using Clause 6.3.3.

The resistance of segments between restraints to resist out-of-plane buckling effects is then sufficient.

As an alternative to providing a torsional restraint at a distance  $L_k$  or  $L_s$  from the plastic hinge, together with intermediate lateral restraints, a torsional restraint could be



provided at a distance  $L_m$  from the hinge and the remaining member verified using an elastic check in accordance with Clause 6.3.3.

A plastic hinge may form in the column at the underside of the haunch. The table below has been provided for illustrative purposes and gives stable lengths for a selection of UKB sections used as portal frame columns with pinned feet and of height to underside of haunch as given. The lengths have been calculated for axial loads equal to 5% of the squash load and for a maximum bending moment equal to 94% of the plastic moment, assumed to be at the underside of the haunch. The cross-section check gives a unity factor of 0.99. The torsional restraint is provided at a distance of  $L_s$  from the plastic hinge and the segment length can be seen to be greater than  $2L_m$ .

Table 8.1  
Stable lengths  
for a range of  
column sections

COLUMN	COLUMN HEIGHT (m)	SEGMENT LENGTH $L_s$ (mm)	$L_m$ (mm)
686x254x140UKB	13.0	4020	1445
610x229x113UKB	12.0	3640	1325
533x210x82UKB	11.0	3170	1075
457x191x67UKB	10.0	3020	1080

Below the torsional restraint, the column segment is verified as an elastic segment subject to axial load and bending.

A plastic hinge may form in the rafter at the end of the haunch and a torsional restraint should be provided in this position. The torsional restraints adjacent to the plastic hinge are provided in the haunched segment on one side and in the uniform segment of the rafter on the other side. Beyond the torsional restraints, the rafter segments are verified as elastic segments subject to axial load and bending.

### ***Haunched or tapered members***

Where a rotated plastic hinge occurs at the end of a haunch and a restraint is provided within  $h/2$  of the plastic hinge within the tapered segment, if the compression flange of the haunch remains elastic throughout its length, the tapered segment does not need to be treated as a segment adjacent to a plastic hinge. The relevant clause in BS EN 1993-1-1 is 6.3.5.3. An elastic check of the tapered segment can be carried out.

Where the members are haunched or tapered, with constant flanges and the haunch does not remain elastic throughout its length, a segment should be restrained torsionally at a distance away from the torsional restraint at the hinge equal to the stable length  $L_s$  of the segment, for linear and non-linear moment gradients and axial compression, in accordance with BB.3.2.2. The arrangement of restraints is as shown in Figure 8.2.

In addition, at least one intermediate lateral restraint should be provided between the torsional restraints, at a spacing that satisfies the requirements for the stable length  $L_m$  in accordance with BB.3.2.1. The spacing between the first and subsequent lateral restraints can be verified by carrying out an elastic check.

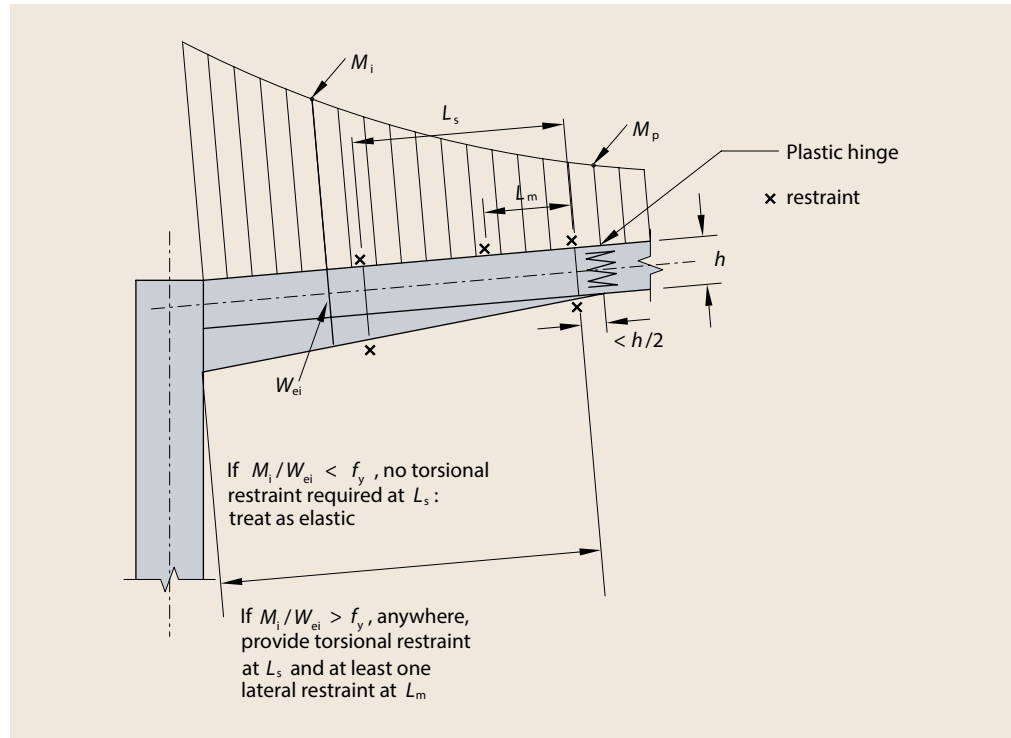


Figure 8.2  
Restraint of  
tapered members

The resistance of segments between restraints to resist out-of-plane buckling effects is then sufficient.

As an alternative to providing a torsional restraint at a distance length  $L_k$  or  $L_s$  from the plastic hinge, together with intermediate lateral restraints at a spacing  $L_m$ , a torsional restraint could be provided at a distance  $L_m$  from the hinge and the remaining member verified using an elastic check.

### 8.2.2 Elastic segments

Elastic verification of segments in plane has been allowed for and out of plane, the relevant expression in BS EN 1993-1-1 is 6.62.

$$\frac{N_{Ed}}{\chi_z \frac{N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\gamma_{M1}} \leq 1 \quad (6.62)$$

where all the symbols are as defined in BS EN 1993-1-1.

For Class 1, 2, 3 and bi-symmetric Class 4 sections,  $\Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$

It is helpful to define  $\chi_z \frac{N_{z,Rk}}{\gamma_{M1}}$  as  $N_{b,z,Rd}$  and  $\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}$  as  $M_{b,Rd}$ .

$M_{z,Ed} = 0$  because the frame is only loaded in its plane.

The expression therefore reduces to:

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

It is essential that Annex B is used to determine the interaction factors  $k_{yy}$  and  $k_{zy}$ . In members of Class 1 or 2, where the ratio of axial load to the axial resistance is 0.4 or less, the value of  $k_{zy}$  is always between 0.95 and 1.0 and the smaller the ratio, the closer  $k_{zy}$  is to 1.0. A value of 1.0 can be assumed for initial design.

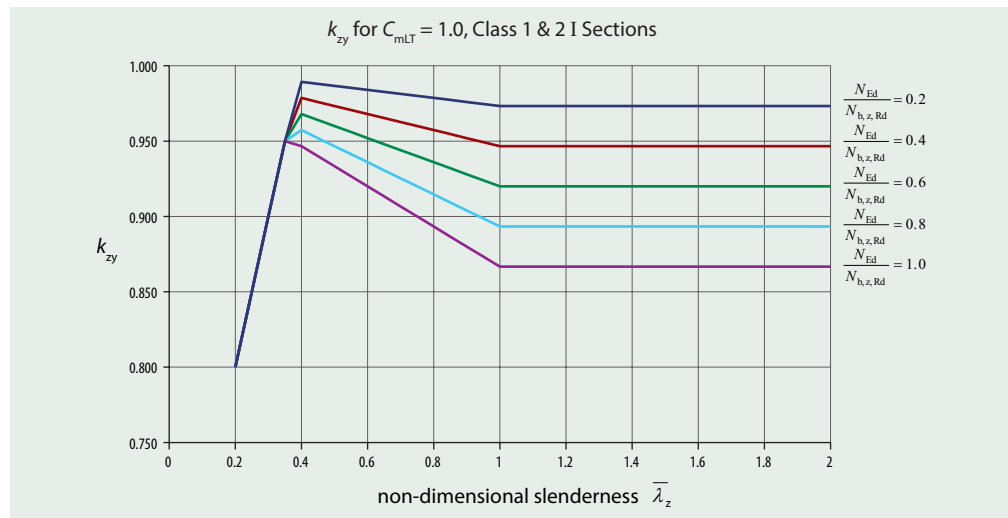


Figure 8.3  
Interaction  
factors  $k_{zy}$  for  
Class 1 and 2  
I sections

Although only out-of-plane verifications are generally required for members in a portal frame, an example of where verification of major axis resistance is required is the central column in a 2-span portal frame structure if the designer has chosen to orientate the column such that its major axis is out of plane. In this case equation 6.61 is relevant, the reduced version of which is as follows:

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

### 8.2.3 Influence of moment gradient

The distribution of the bending moments along an unrestrained length of beam has an important influence on the lateral-torsional buckling resistance. A uniform bending moment is the most onerous loading. When calculating the lateral-torsional buckling resistance, the influence of the moment gradient is accounted for in the  $C_1$  factor when calculating  $M_{cr}$  (see Appendix B.2).

The moment gradient is also important in the interaction expressions 6.61 and 6.62, and is accounted for by various  $C$  factors:  $C_{my}$ ,  $C_{mx}$ , and  $C_{mLT}$ . Although it is conservative to take these  $C$  factors as 1.0, which reflects the onerous case of a uniform bending moment, this is not recommended as it is uneconomic. At many points in a portal frame, a segment between restraints has a significantly varying bending moment, which may be beneficially allowed for in design. It is essential that the  $C$  factors are also taken from Annex B.

### 8.2.4 Restraint and member buckling

Figure 8.4 represents the idealised deflected profile of a portal frame rafter, taken in isolation from the rest of the frame, illustrating the vertical deflection in plane, and the out-of-plane buckled shape of the upper and lower flanges.

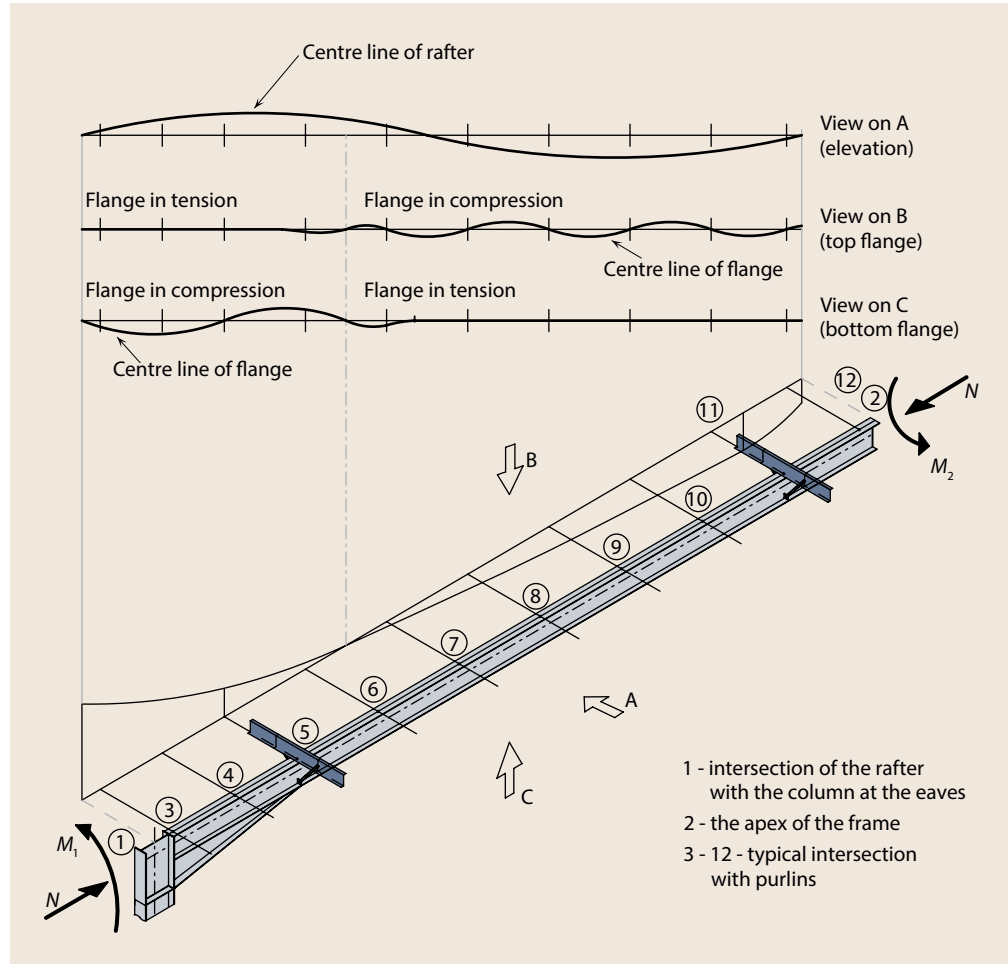


Figure 8.4  
Rafter deflection  
and lateral restraints

Figure 8.4 also illustrates a typical arrangement of purlins providing restraint to the outside flange of the rafter. Stays to the inside flange are provided at positions 5 (adjacent to the sharp end of the haunch) and 11 (near the apex). The bending moment produces compression in the inside flange of the rafter in the eaves region, and compression in the outside flange for the remainder of the rafter. The following points should be noted:

- Purlins provide intermediate lateral restraint to one flange. Depending on the bending moment diagram this may be either the tension or compression flange.
- Restraints to the inside flange can be provided at purlin positions, producing a torsional restraint at that location.

Out of plane, the member is able to buckle between points of restraint. The restraint positions are chosen by the designer of the structure and by judicious positioning of the purlins and stays to the inner flange, the element will sustain the design loads. The secondary steelwork is thus fundamental to the performance of the primary steelwork.

Typical details to restrain the inner flange are shown in Figure 8.5.

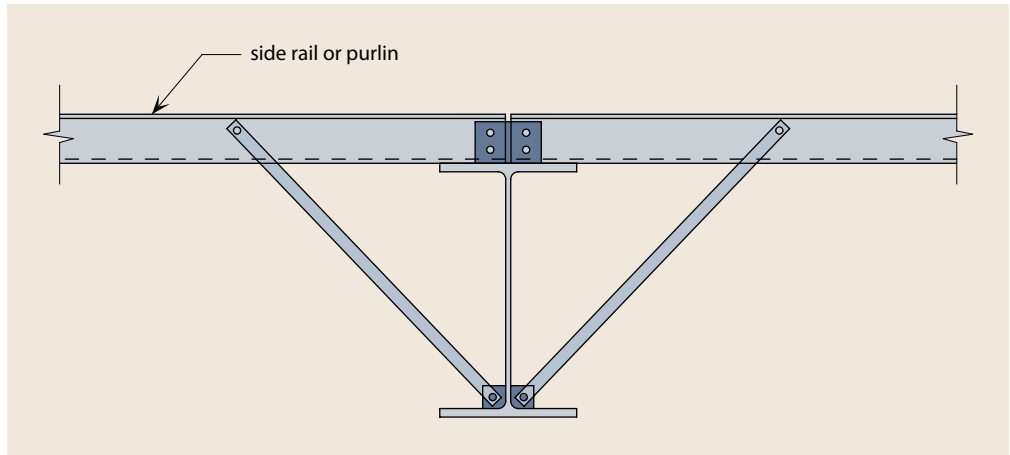


Figure 8.5  
Restraint to  
inner flange

Purlins and side rails must be continuous between one frame and the next in order to offer adequate restraint. A side rail that is not continuous (for example, interrupted by industrial doors) cannot be relied upon to provide adequate restraint as the entire arrangement may rotate, as illustrated in Figure 8.6. If the rail is not continuous, and restraint against torsion is not provided by other elements (e.g. by the door posts), additional bracing must be provided.

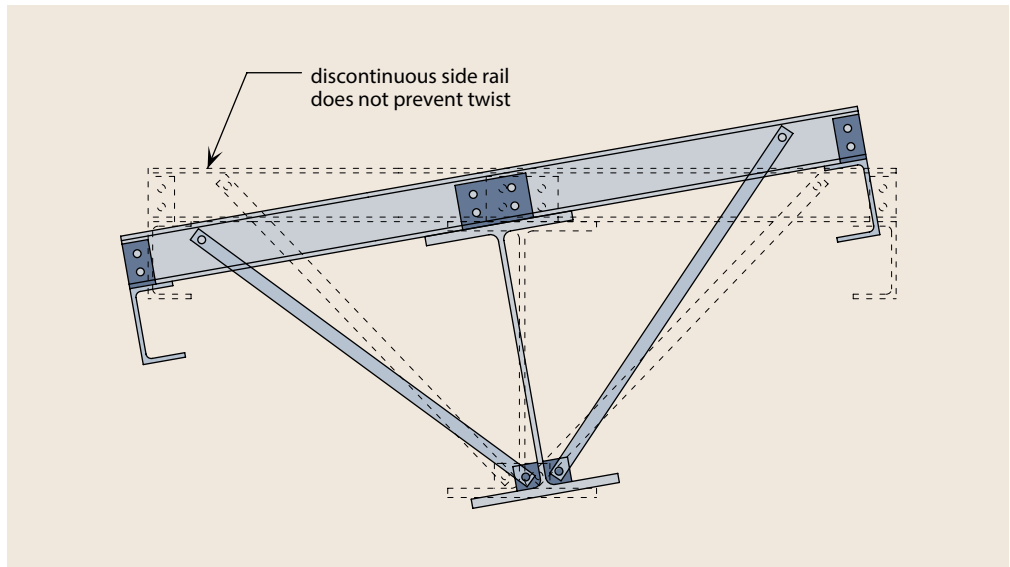


Figure 8.6  
Ineffective restraint  
to inner flange

### 8.2.5 Tension flange restraint

Even if no restraints are provided to the compression flange, restraints to the tension flange can be effective in increasing the buckling resistance of a member. The restraints to the tension flange must be provided at sufficiently close centres such that flexural buckling between these restraints occurs at a higher load than the critical torsional buckling load of the whole segment between torsional restraints. The restraints to the tension flange reduce the lateral buckling of the compression flange, as shown in Figure 8.7. The design approach to utilise the benefits of tension flange restraint is described in more detail in B.3.

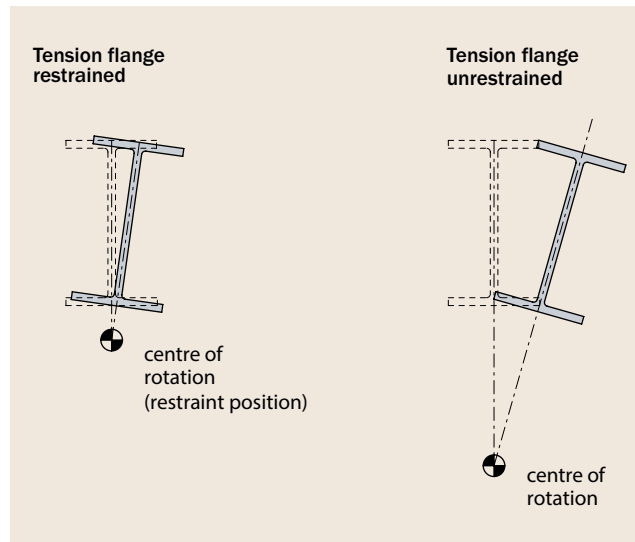


Figure 8.7  
Effect of restraint  
to a tension flange

Tension flange restraint improves flexural buckling resistance about the minor axis much more significantly than lateral-torsional buckling resistance. The increase in flexural buckling resistance is generally of the order of 15%. For lateral-torsional buckling resistance, the increase is only up to approximately 5%. Further guidance can be found in references<sup>[34]</sup> and <sup>[35]</sup>.

## 8.3 Column stability

### 8.3.1 Column stability under gravity loading

It is recommended that a torsional restraint to the column is always provided at the underside of the haunch. It is considered preferable to achieve the restraint using a side rail positioned at this level, with stays to the inside flange, rather than by means of stays from a purlin to the underside of the rafter haunch as these are likely to be longer. It is generally not sufficient to assume that a combination of a side rail and web stiffeners at this position will provide adequate torsional restraint.

As an alternative to a side rail and stays to the inside flange, it may be convenient to provide a hot-rolled member (see Figure 8.8), typically a hollow section, to provide lateral restraint to the inner flange. It is essential to connect this line of bracing on the inner flange to the outer flange (or to the foundation) at some point in the line of the bracing, as the objective is to restrain the inner flange with respect to the outer flange (*i.e.* provide a torsional restraint), not merely to connect all the inner flanges.

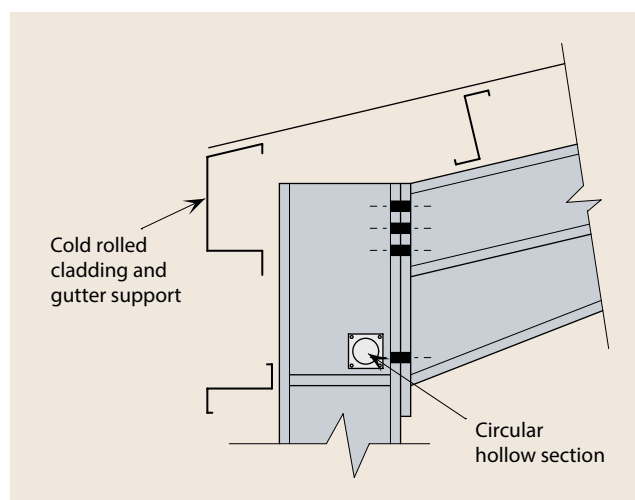


Figure 8.8  
Typical eaves detail  
using a circular  
hollow section as a  
longitudinal bracing  
and restraint member

If the frame is to be designed plastically and the collapse mechanism predicts a plastic hinge forms in the column, the torsional restraint provided immediately below the haunch satisfies the requirement to provide restraint at the plastic hinge. A second torsional restraint is required at a distance  $L_s$  below the restraint at



the plastic hinge for a uniform member (or tapering member) with a linearly varying moment. At least one lateral restraint is required between the two torsional restraints, as described in Section 8.2.1.

There may be more flexibility in the spacing of sheeting rails if a torsional restraint is provided at a distance,  $L_m$ , below the plastic hinge and treat the remaining length of column to the base as elastic. In this case, no intermediate lateral restraint to the tension flange within the length  $L_m$  is required.

Further torsional restraints may be required between the underside of the haunch and the column base because the side rails are attached to the (outer) tension flange; unless restraints are provided, the inner compression flange is unrestrained. As noted in Section 8.2.4, a side rail that is not continuous (for example, interrupted by industrial doors) cannot be relied upon to provide torsional restraint. The column section may need to be increased if intermediate restraints to the compression flange cannot be provided.

Figure 8.9 shows a typical moment distribution in the column under the gravity combination of actions and indicates the positions of restraints on a typical column. An intermediate torsional restraint is shown in the length between the underside of the haunch and the base. Except for the segment adjacent to the plastic hinge, verification using Expression 6.62 of BS EN 1993-1-1 is required between torsional restraints. If there are intermediate restraints to the tension flange between the torsional restraints and these are spaced at sufficiently close centres, they may be used to calculate an increased buckling resistance.

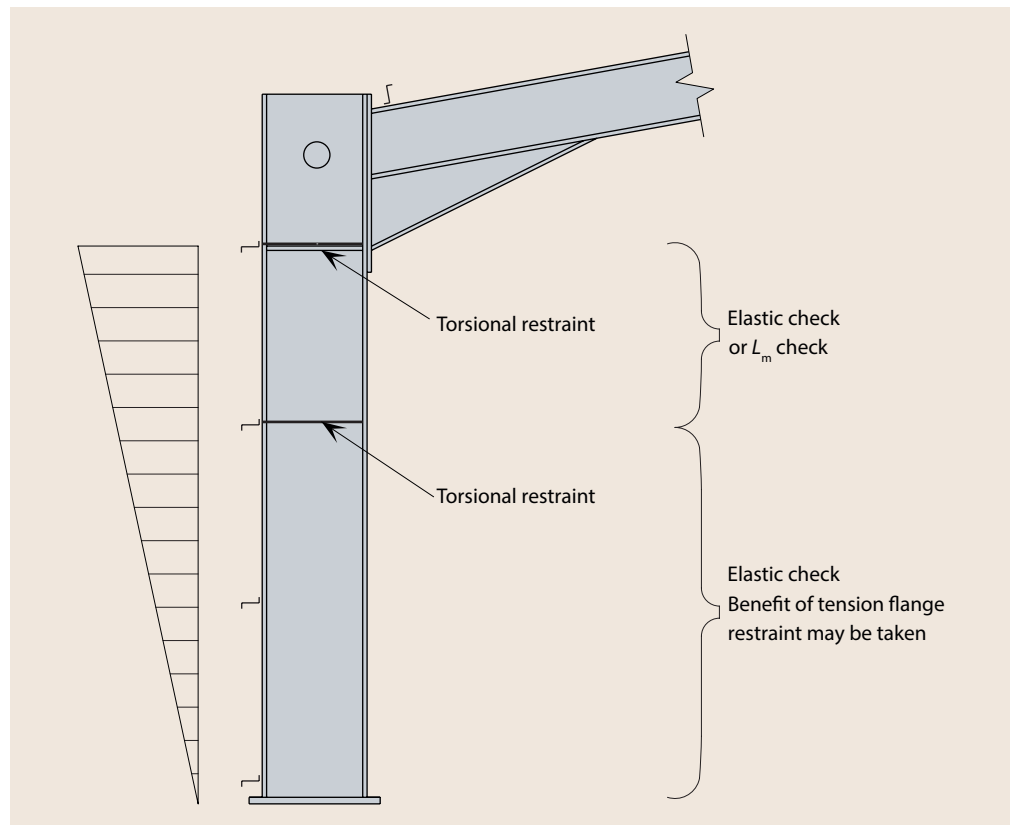


Figure 8.9  
Typical portal frame  
column with plastic  
hinge at underside  
of haunch

When taking advantage of tension flange restraint, the non-dimensional slenderness used to determine the axial resistance should be based on the elastic critical torsional buckling force, because flexural buckling is prevented by the tension flange restraints. The elastic critical moment should also be determined assuming the tension flange is restrained. Appendix C gives the appropriate formulae. Software is also available on the internet.

Even if a frame is analysed and designed elastically, it may be convenient to verify member stability using rules primarily intended for use for segments containing plastic hinges. If a member can be verified using a plastic criterion, then the less onerous elastic situation must also be satisfactory. This approach may be valuable in locations where the bending moment approaches the plastic resistance moment of the section (typically immediately under the haunch). As indicated in Figure 8.9, a typical application of this philosophy is to ensure that a torsional restraint is located within a distance  $L_m$  (see Clause BB.3) from the torsional restraint at the underside of the haunch.

### 8.3.2 Out-of-plane stability under uplift combinations

When the frame is subject to uplift, the column moment will reverse. The bending moments will generally be significantly smaller than those under gravity loading combinations, and the side rails restrain the compression flange.

The out-of-plane stability of the column may be verified between torsional restraints in accordance with Expression 6.62 (see Section 8.2).

## 8.4 Rafter stability

### 8.4.1 Out-of-plane stability under gravity loading

Under gravity loading, rafters are subject to high bending moments, which vary from a hogging moment at the junction with the column to a sagging moment close to the apex, as shown in Figure 8.10. Compression is introduced in the rafters the horizontal

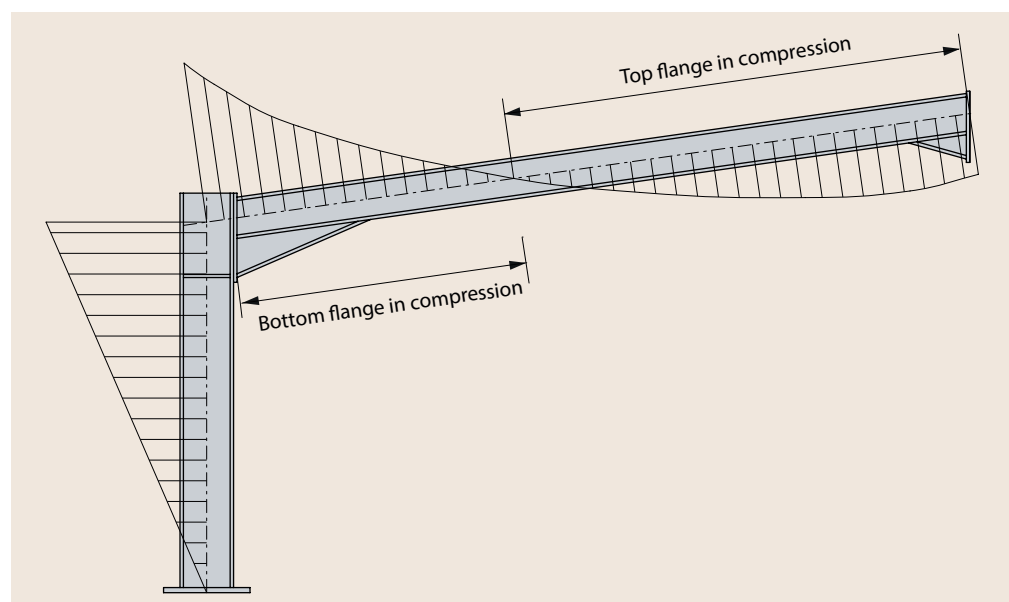


Figure 8.10  
Portal frame  
bending moments,  
gravity actions

component of which is equal to the shear forces in the columns. The rafters are not subject to any minor axis moments.

Figure 8.10 shows a typical moment distribution for the gravity combination of actions, typical purlin positions and typical restraint positions.

Purlins are generally placed at up to 1.8 m spacing but it may be necessary to reduce this spacing in the high moment regions near the eaves. Figure 8.11 identifies three stability zones (A, B, and C), which are discussed separately below.

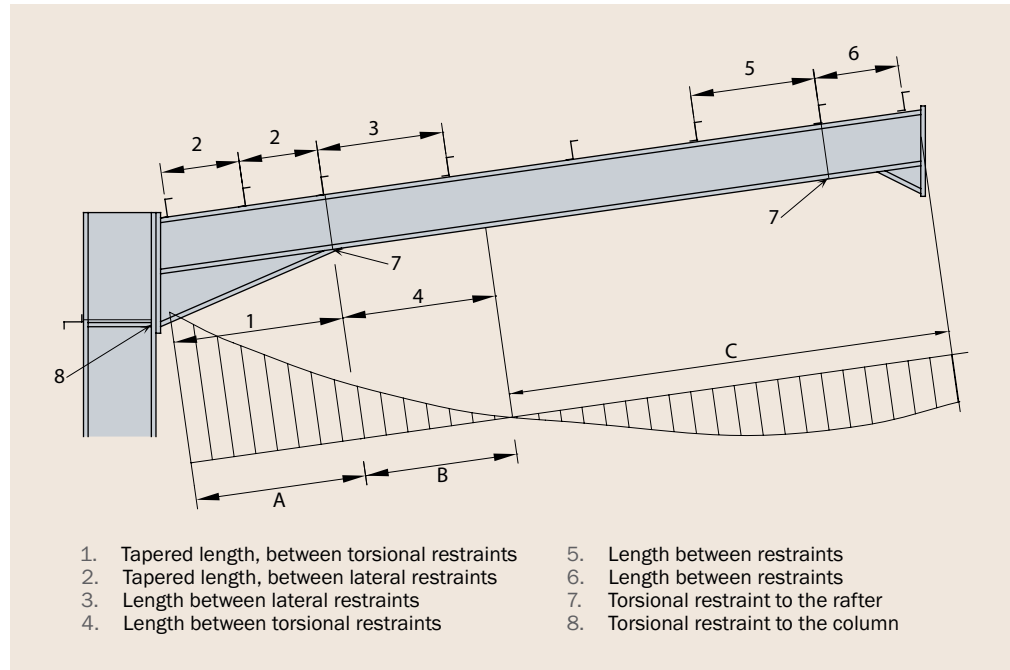


Figure 8.11  
Typical bending moment diagram under the gravity combination of actions

The selection of the appropriate criterion for each zone depends on the shape of the bending moment diagram and the geometry of the section (three flanges or two flanges). The objective is to provide sufficient restraints to ensure the rafter is stable out of plane.

### **Haunch stability in Zone A**

In Zone A, the bottom flange of the haunch is in compression. The stability verification is complicated by the variation in geometry along the haunch. The junction of the inside column flange and the underside of the haunch (point 8 in Figure 8.11) should always be restrained. The 'sharp' end of the haunch (point 7 in Figure 8.11) usually has restraint to the bottom flange, from a purlin and stays, located at this position, forming a torsional restraint at this point.

A plastic hinge may form at the sharp end of the haunch and if so a torsional restraint must be provided at this point, in accordance with clause 6.3.5.2, or within a distance of  $h/2$  along the member, where  $h$  is the overall depth at the plastic hinge. Torsional restraints are required a distance equal to the stable length away from the plastic hinge (unless the haunch remains elastic). This length is calculated using the

appropriate stable length formula depending on whether the segment is of uniform depth or varying depth as described in 8.2.1.

BS EN 1993-1-1 is silent on a method for verifying the elastic resistance of a tapered segment under axial load and varying moment where tension flange restraint is provided. Under these circumstances verification can be made using the following procedure (as set out in Annex G of BS 5950-1:2000).

Satisfy the following criterion at all points in the haunched or tapered member or segment within which the non-restrained flange is in compression:

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{Edi}}{M_{b,Rdi}} \leq 1.0$$

where

- $N_{Ed}$  is the axial compression in the segment;
- $N_{b,Rd}$  is the design buckling resistance of the segment based on the properties of the minimum depth cross-section;
- $M_{Edi}$  is the bending moment about the major axis at the point  $i$  being considered;
- $M_{b,Rdi}$  is the design buckling resistance moment determined using the appropriate modulus  $W_y$  and other properties at the section being considered.

BS EN 1993-1-1 is similarly silent on verifying the elastic resistance of a tapered segment under axial load and varying moment between lateral restraints. Verification can be made using the following procedure similar to that set out in BS 5950-1:2000, Annex B.2.5.

Satisfy the following criterion at a number of cross-sections along the member:

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{Edi}}{M_{b,Rdi}} \leq 1.0$$

where  $M_{b,Rdi}$  is a function of  $W_{pl}$  (or  $W_{el}$ ) at the section being considered and  $M_{cr}$  and  $\bar{\lambda}_{LT}$  are determined at the point of maximum bending moment in the segment. In determining  $M_{cr}$ , the  $C_1$  factor for bending moment gradients is assumed to be equal to 1.0. If a torsional restraint is provided at the sharp end of the haunch, the segment under consideration always has a cross-section with one axis of symmetry.

In practice, if the value of  $L_s$  has been determined and more than one intermediate restraint provided to suit the purlin spacing, it is likely that the resistance of the segment between the lateral restraints is adequate.

If the portal frame is being designed elastically, there is no requirement to provide a torsional restraint at the sharp end of the haunch and the purlin spacing may be such that the sharp end of the haunch is positioned between purlins. This portion of the rafter is described as a haunched segment, as illustrated in Figure BB.6I in BS EN 1993-1-1, and the taper factor for this segment is given by Equation BB.17.

In the uplift case, the top flange is in compression and restrained by the purlins. In carrying out an elastic check for this element, the same approach as that described above for a tapered segment between lateral restraints can be carried out. The elastic critical buckling load  $N_{b,Rd}$  is determined using the properties of the rafter because this will give the lowest value.

### ***Rafter stability in Zone B***

Zone B generally extends from the 'sharp' end of the haunch to the point of contraflexure, or to the first purlin beyond the point of contraflexure (see Figure 8.12)

The bottom flange is partially or wholly in compression over this length.

In this zone, torsional and lateral restraint will be provided at the 'sharp' end of the haunch. At the upper end, restraint will be provided by a purlin beyond the point of contraflexure.

In the UK, practice is to consider the point of contraflexure as a restraint, provided the following conditions are satisfied:

- The rafter is a rolled section
- At least two bolts are provided in the purlin-to-rafter connections
- The depth of the purlin is not less than 0.25 times the depth of the rafter.

If a plastic hinge forms at the sharp end of the haunch, the segment should be justified as a uniform segment adjacent to a plastic hinge as described in Section 8.2.1.

### ***Rafter stability in Zone C***

In Zone C, the purlins provide lateral restraint to the top (compression) flange. It is assumed that the diaphragm action of the roof sheeting is sufficient to carry the restraint forces to the bracing system.

Out-of-plane stability of the rafter between purlins is verified in accordance with Expression 6.62 (see Section 8.2 of this publication). Normally, if the purlins are regularly spaced, it is sufficient to verify the rafter for the maximum bending moment with maximum compression force.

It is normal practice to provide a torsional restraint at the penultimate purlin to the apex, as this will be necessary when considering the uplift combinations of actions – the bottom flange will be in compression. This restraint is indicated at point 7 in Figure 8.11.

In plastic design where a plastic hinge forms near the apex of the portal frame, this is usually at the penultimate purlin. A torsional restraint is required at this point and where the frame is symmetrical, a torsional restraint will be required on both sides of the apex. A second torsional restraint should be provided a distance equal to the stable length down the rafter from the plastic hinge position.

### 8.4.2 Rafter and haunch stability under uplift conditions

Under uplift, most of the bottom flange of the rafter is in compression. A typical bending moment diagram is shown in Figure 8.12, which indicates two zones, (E and F) for which stability is verified.

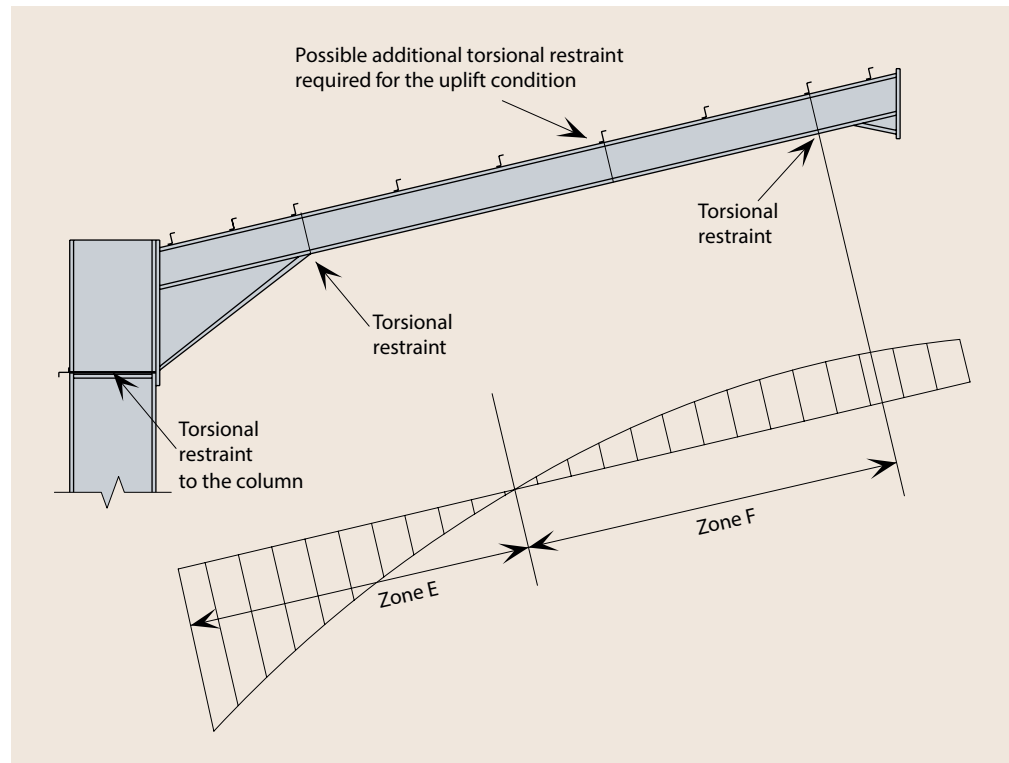


Figure 8.12  
Typical purlin  
and rafter stay  
arrangement for  
wind uplift

This type of bending moment diagram will generally occur under internal pressure and wind uplift. Normally, the design bending moments are smaller than under gravity loading.

#### **Haunch stability in Zone E**

In Zone E (see Figure 8.12), the top flange of the haunch will be in compression and will be restrained by the purlins.

If the purlins are spaced at a distance  $L_m$  apart then this will provide sufficient restraint to the compression flange to prevent buckling. A cross section check is subsequently all that is required, otherwise check the tapered section as described in 8.2.1 or 8.4.1 as appropriate.

#### **Stability in Zone F**

In Zone F, the purlins will not restrain the bottom flange, which is in compression.

The rafter must be verified between torsional restraints. A torsional restraint will generally be provided adjacent to the apex, as shown in Figure 8.12. The rafter may be stable between this point and the virtual restraint at the point of contraflexure, as the moments are generally modest in the uplift combination. If the rafter is not stable over this length, additional torsional restraints should be introduced, and each length between restraints verified.



This verification should be carried out using Expression 6.62.

The beneficial effects of the restraints to the tension flange (the top flange, in this combination) may be accounted for using a modification factor  $C_m$ , taken from Clause BB.3.3.1(1)B for linear moment gradients and from Clause BB.3.3.2(1)B for non-linear moment gradients.

### 8.4.3 Step by step procedures

Step by step procedures for member verification under gravity loading are presented below. The requirements of the cladding will already have suggested the number and location of purlins and sheeting rails. The requirements for restraints and adjustments to the position of the secondary elements are determined, in addition to confirming that the primary elements are adequate.

#### Columns

1. Classify the section.
2. Provide torsional restraints at plastic hinge positions (assumed to be at the underside of the haunch).
3. Determine the position of the first torsional restraint: calculate the stable length for a uniform moment,  $L_k$ .
4. Check whether a reduction in moment resistance for axial force is necessary.
5. Calculate the modification factor  $C_m$ : determine the ratio of the flexural buckling (Euler) load and the elastic critical torsional buckling load for a guessed length  $L_s$  and calculate  $C_m$  and  $L_s$  (start with  $L_s = 1.05 L_k$ ). Iterate to confirm  $L_s$ . Provide torsional restraint.
6. Calculate the position of the first intermediate restraint  $L_m$ .
7. Carry out an elastic check of the segments between the first intermediate restraint, any other intermediate restraint and the torsional restraint using Equation 6.62.  $M_{cr}$  for this segment is that for lateral torsional buckling between the restraints.
8. Verify the elastic segment below the first torsional restraint. Note whether sheeting rails are capable of providing lateral supports to the tension flange: if so use the elastic torsional buckling load to determine the non-dimensional slenderness and calculate the axial resistance. Consider the segment of the column from base plate to the torsional restraint; calculate  $M_{cr}$  for elastic torsional buckling; determine the non-dimensional slenderness and  $M_{b,Rd}$ . Compare with the applied moment. If  $M_{Ed} < M_{b,Rd}$  and a check using Equation 6.62 passes, the section is adequate.
9. If the check at Step 8 fails, consider a shorter segment and carry out a check as in Step 8 until a pass is achieved. Provide a torsional restraint.
10. Check the lower portion explicitly as in Step 8. If a pass is achieved the element is verified. If it fails, provide further torsional restraints between the base plate and the restraint provided at Step 9 and continue until all segments in the column are verified.

Note: It is essential that the sheeting rails are continuous and able to provide effective lateral restraint to the columns as discussed in Section 8.2.4. If the sheeting rails are interrupted, tension flange restraint is not available to the columns. This means that their resistance must be based on  $N_{cr}$  determined for flexural (Euler) buckling over the length from the base to the lowest effective restraint, and  $M_{cr}$  determined for lateral-torsional buckling over the same length. The lack of tension flange restraint changes the relevant buckling mode from a torsional to a flexural one.

### Rafter

1. Classify the section.
2. Provide torsional restraints at plastic hinge positions. These may be at the sharp end of the haunch and/or near the apex (e.g. at the first purlin down the slope from the apex).
3. Determine the position of the first torsional restraint from the sharp end of the haunch:
  - a. Calculate the stable length for a uniform moment  $L_k$ .
  - b. Check whether a reduction in moment resistance for axial force is necessary.
  - c. Calculate the modification factor for non-linear moment gradient  $C_n$ . Determine  $R$  at quarter points. If  $(M_{y,Ed} + aN_{Ed}) < 0$  at a given point, put  $R = 0$ . This is because the bottom flange is in tension at this point. Calculate  $L_s$ . Iterate to confirm  $L_s$ . Provide torsional restraint.
4. Determine the position of the first intermediate restraint  $L_m$ .
  - a. Carry out an elastic check of the segment between the intermediate restraint, any other intermediate restraint and the torsional restraint using Equation 6.62.  $M_{cr}$  for this segment is that for lateral torsional buckling between the restraints.
  - b. Verify the elastic segment between purlins, where the unrestrained flange is in tension using Equation 6.62 with  $N_{cr}$  and  $M_{cr}$  determined for Euler buckling and lateral torsional buckling respectively.

### Haunch elastic check

1. Classify the section
2. Provide torsional restraints at the plastic hinge position at the sharp end of the haunch.
3. Determine the position of the first intermediate restraint ( $L_m$ ). Iteration is necessary because  $L_m$  depends on  $C_1$  which in turn depends on the ratio of the bending moments at the ends of the segment defined by  $L_m$ . Provide a torsional restraint.
4. Verify the elastic segment between the torsional restraint and the column: calculate  $M_{cr}$  for a tapered section based on the properties for the minimum section. Adjust the value of  $M_{cr}$  by  $C_n$  for moment gradient and  $c$  for the haunch taper.
5. Calculate  $N_{cr}$  for the tapered section based on the properties for the minimum section.
6. Consider (say) five sections along the taper and determine  $M_{b,Rdi}$  at each section  $i$ . Calculate  $N_{Ed} / N_{b,Rd} + M_{y,Edi} / M_{b,Rdi} \leq 1$ . If the checks pass, the haunch is verified. If a check fails, the haunch must be strengthened and rechecked.









# BRACING

Vertical bracing in the side walls is required to resist longitudinal actions on the building and provide longitudinal stability. Roof bracing is required to support the tops of the gable columns, to resist any restraint forces or longitudinal forces carried by the purlins, and to carry these forces to the vertical bracing.

## 9.1 Vertical bracing

The primary functions of vertical bracing in the side walls of the frame are:

- To transmit horizontal forces to the foundations. The horizontal forces include forces due to wind and cranes.
- To provide stability during erection.

### 9.1.1 Bracing location

The bracing may be located:

- At one or both ends of the building.
- Within the length of the building.
- In each portion between expansion joints (where these are present).

Where the side wall bracing is not in the same bay as the plan bracing in the roof, an eaves strut is essential to transmit the forces from the roof bracing into the wall bracing and to ensure the tops of the columns are adequately restrained in position. An eaves strut is also required:

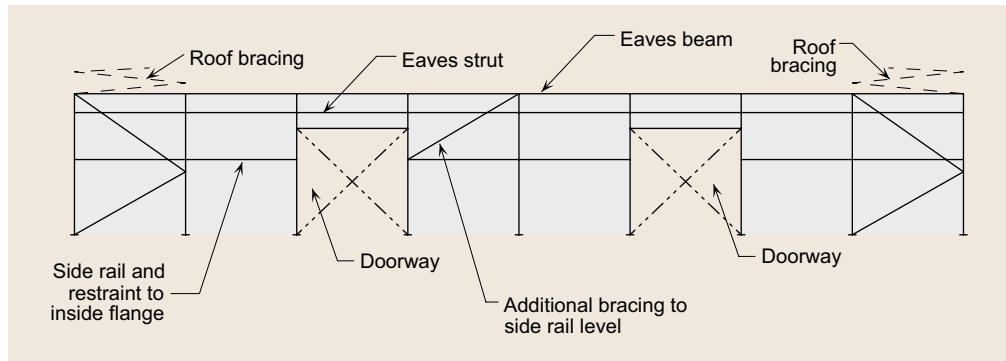
- To assist during the construction of the structure.
- To stabilise the tops of the columns if a fire boundary condition exists.

### 9.1.2 Bracing to restrain columns

If side rails and column stays provide lateral or torsional restraint to the column, it is important to identify the route of the restraint force to the vertical bracing system. If the continuity of side rails is interrupted by openings in the side of the building, additional intermediate bracing may be required such as shown diagrammatically in Figure 9.1. This bracing should be provided as close to the plane of the side rail as possible.

It is not normally necessary for side rails that provide restraint at column stay positions to be aligned with a node of the vertical bracing system. It can be assumed that

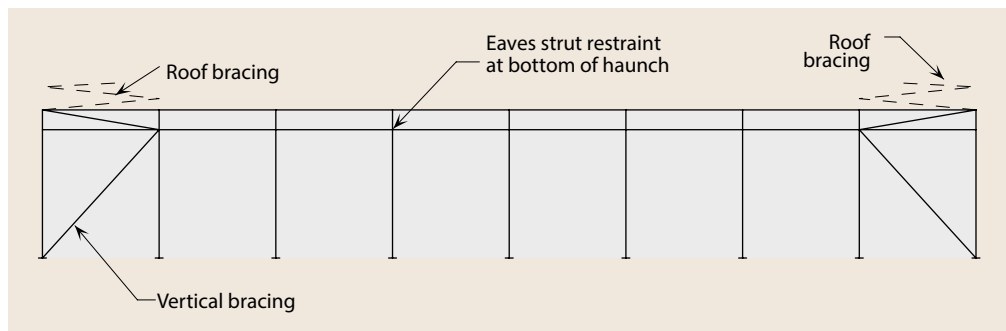
Figure 9.1  
Typical bracing  
pattern in the side  
of a building  
with openings



diaphragm action in the vertical sheeting and the transverse stiffness of the column can transmit the load into the vertical bracing system.

If, instead of a side rail and restraints to the inner flange (see Figure 9.2) a separate member is used to provide restraint to the inside flange, it is essential that it is tied properly into the bracing system. The objective is to provide a torsional restraint with respect to the outer flange, not merely to connect all the inner flanges. This can result in the configuration shown in Figure 9.2. Where there is an opening in the side of the building that interrupts the restraining member, additional intermediate bracing will be required in a similar way to that described above.

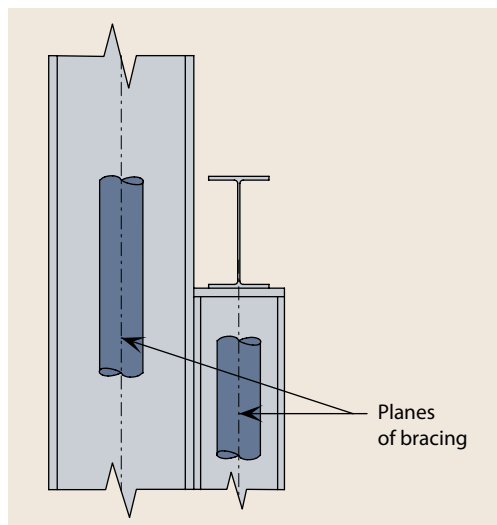
Figure 9.2  
Typical bracing  
pattern in a building  
using a member to  
restrain the underside  
of the haunch



### 9.1.3 Bracing to restrain longitudinal loads from cranes

If a crane is directly supported by the frame, the longitudinal surge force will be eccentric to the column and will tend to cause the column to twist, unless additional restraint is

Figure 9.3  
Detail showing  
additional bracing  
in the plane of the  
crane girder



provided. A horizontal truss at the level of the crane girder top flange or, for lighter cranes, a horizontal member on the inside face of the column flange tied into the vertical bracing, may be adequate to provide the necessary restraint.

For large horizontal forces, additional bracing should be provided in the plane of the crane girder as indicated in Figure 9.3. Bracing requirements taken from Fisher<sup>[36]</sup> are summarised in Table 9.1.



DESIGN VALUE OF LONGITUDINAL FORCE	BRACING REQUIREMENT
Small (<15 kN)	Use wind bracing
Medium (15 – 30 kN)	Use horizontal bracing to transfer force from the crane to plane of bracing
Large (> 30 kN)	Provide additional bracing in the plane of the longitudinal crane forces

Table 9.1  
Bracing requirements  
for crane girders

## 9.2 Roof bracing

Roof bracing is located in the plane of the roof. The primary functions of the roof bracing are:

- To transmit wind forces from the gable posts to the vertical bracing in the walls.
- To transmit any frictional forces from wind on the roof to the vertical bracing.
- To provide a stiff anchorage for the purlins that are used to restrain the rafters.
- To provide stability during erection.
- To restrain the tops of internal columns by bracing back to perimeter wall bracing.

The roof bracing should be arranged to provide support at the top of the gable posts.

In modern construction, circular hollow section bracing members are generally used in the roof, as shown in Figure 9.4, and are designed to resist both tension and compression. Many arrangements are possible, depending on the spacing of the frames and the positions of the gable posts.

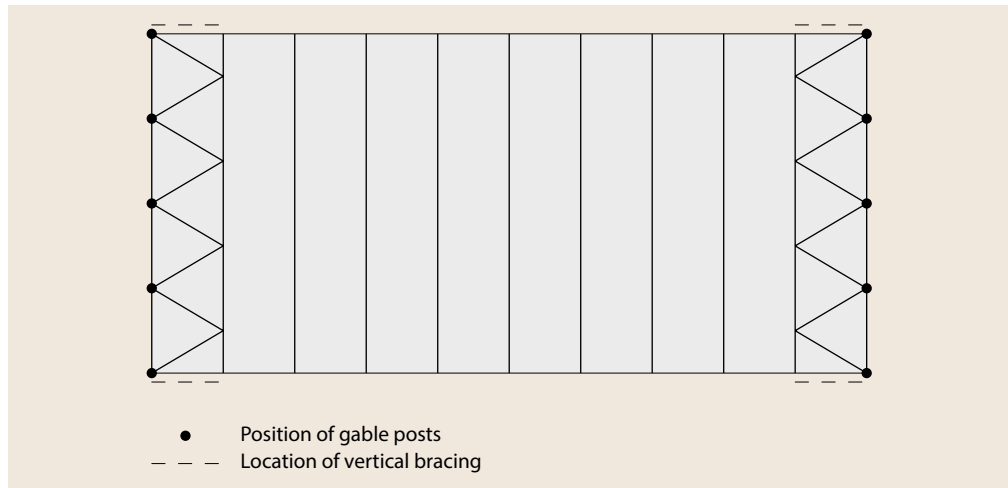


Figure 9.4  
Plan view showing  
both end bays braced

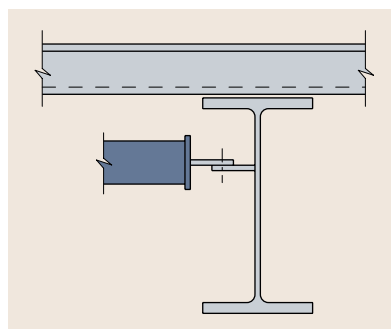


Figure 9.5  
Typical connection  
detail for hollow  
section roof bracing

The bracing is usually attached to gusset plates welded to the web of the rafter, as shown in Figure 9.5. The attachment points are usually positioned close to the top flange, allowing for the size of the bracing member and the connection.

### 9.3 Restraint to inner flanges

Restraint to the inner flanges of rafters or columns is often most conveniently formed by diagonal stays from the purlins or sheeting rails to small plates welded to the inner flange and web. Pressed steel angles are commonly used. Where restraint is only possible from one side, the restraint must be able to carry compression; typically angle sections of minimum size  $40 \times 40$  mm are used. The stay and its connections should be designed to resist a force equal to 2.5% of the maximum force in the column or rafter compression flange between adjacent restraints.

The effectiveness of such restraint depends on the stiffness of the system, especially the stiffness of the purlins. The effect of purlin flexibility on the bracing is shown in Figure 9.6. As a rule of thumb, it will be adequate to provide a purlin or side rail of at least 25% of the depth of the member being restrained. Where the proportions of the members, purlins and spacings differ from proven previous practice, the effectiveness should be verified. In the absence guidance in the Eurocodes, the stiffness may be verified as suggested by Horne and Ajmani<sup>[37]</sup>. Details are also given in SCI publication P164<sup>[38]</sup>.

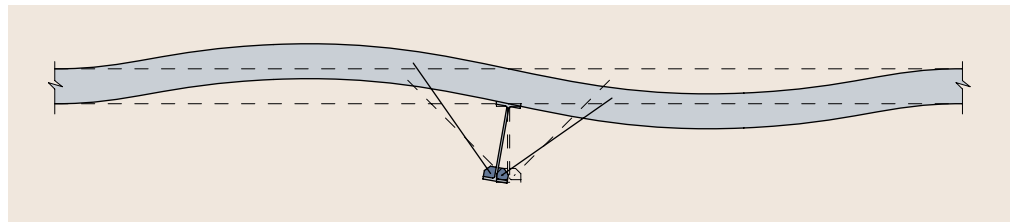


Figure 9.6  
Effect of purlin  
flexibility on bracing







# GABLES

## 10.1 Types of gable frame

Gable frames are typically one of two basic forms:

- A portal frame identical to the remainder of the structure. The gable columns are located in the plane of the end frame but are not designed to support the rafters. This form of gable is used for simplicity, or because there is the possibility of extending the structure in the future.
- A gable frame comprising gable posts with rafters spanning between the posts. Gable frames of this form require bracing in the plane of the gable, as shown in Figure 10.1. The advantage of this form of gable is that the rafters and external columns are smaller than those in a portal frame.

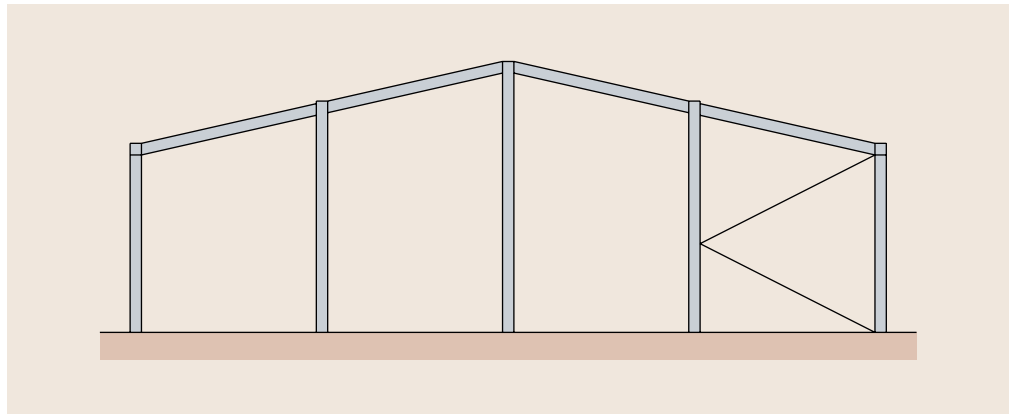


Figure 10.1  
Gable frame from  
columns, beams  
and bracing

## 10.2 Gable columns

In both types of gable frame, gable columns are designed to span between the base and the rafter. At rafter level, the horizontal reaction from the gable column is transferred into the roof bracing, to the eaves, and then to the ground via the bracing in the elevations.

The gable columns are designed for both wind pressure and suction in combination with any axial load. The maximum suction may occur when the gable is on the downwind elevation, as shown in Figure 10.2(a), or, more likely (due to the higher suctions), when the gable is parallel to the wind direction, as shown in Figure 10.2 (b).



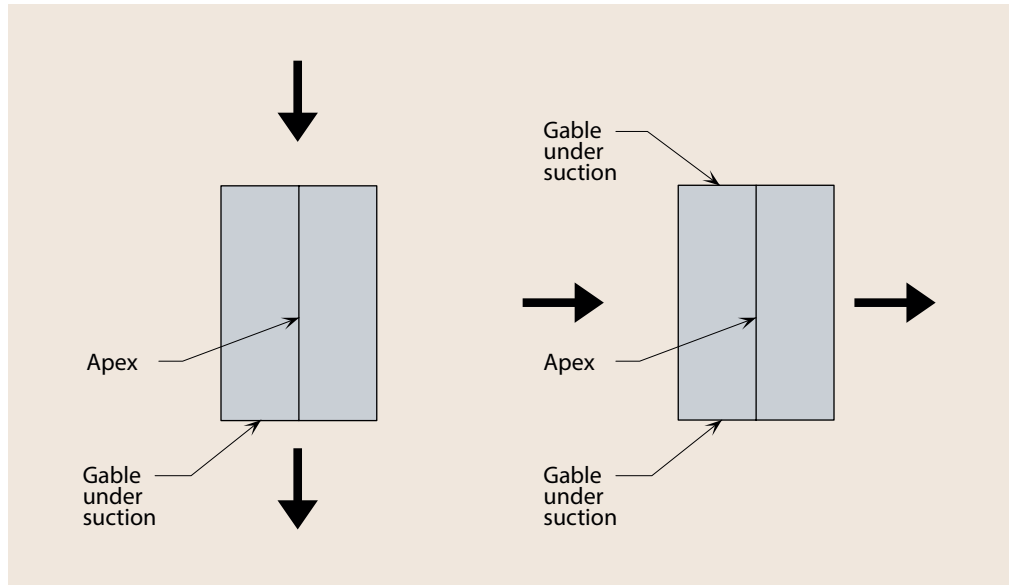


Figure 10.2  
Wind loads on gables

The internal pressure or suction contributes to the net loads on the gable. When the net loads are equivalent to an external pressure, the outside flanges of the gable columns are in compression, but are restrained out of plane by the side rails. When the net loads are equivalent to an external suction, the inside flanges of the gable columns are in compression. This design case may be the more onerous of the two conditions. It may be possible to reduce the length of the unrestrained inside flange of the gable columns by introducing column stays from the side rails, as illustrated in Figure 10.2.









# CONNECTIONS

The major connections in a portal frame are those at the eaves and the apex, which are both required to be moment-resisting. The eaves connection in particular is generally subject to a very large design bending moment. Both the eaves and apex connections are likely to experience loading reversal in certain design situations and this can be an important design consideration for the connection. For economy, connections should be arranged to minimise any requirement for additional reinforcement (commonly called stiffeners). This is generally achieved by:

- Making the haunch deeper (increasing the lever arm to the bolt rows).
- Extending the eaves connection above the top flange of the rafter (providing an additional bolt row).
- Adding bolt rows within the depth of the connection.
- Selecting a stronger column section.

Guidance on the design of moment-resisting connections is given in SCI Publication P398<sup>[39]</sup>.

## 11.1 Eaves connections

A typical eaves connection is shown in Figure 11.1. In addition to increasing the bending resistance of the rafter, the haunch increases the lever arms of the bolts in the tension zone. Generally the bolts in the tension zone (the upper bolts under gravity loading) are

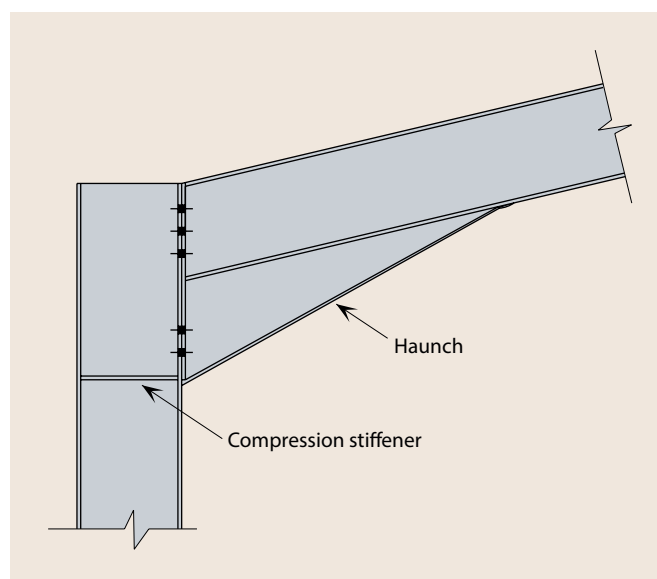


Figure 11.1  
Typical eaves  
connection

nominally allocated only to carry tension, whilst the lower bolts (adjacent to the compression stiffener) are nominally allocated only to carry the vertical shear, which is generally modest. The compression force is transferred at the level of the bottom flange.

Because the portal frame members are chosen for bending resistance,

deep members with relatively thin webs are common in portal frames. A compression stiffener in the column is usually required. The web panel of the column may also need to be reinforced, either with a diagonal stiffener, or an additional web plate (referred to as a supplementary web plate).

The end plate on the rafter is unlikely to require stiffening as it can simply be made thicker, but it is common to find that the column flange requires strengthening locally in the tension zone. Stiffeners are expensive, so good connection design would minimise the need for stiffeners by judicious choice of connection geometry.

If the reversed moment under uplift is significant, it may be necessary to provide a stiffener to the column web at the top of the column, aligned with the top flange of the rafter, to resist the compressive force. A stiffener at the top of the column is often referred to as a cap plate. Under gravity loading, a cap plate increases the resistance of the column web in tension and the column flange in bending.

Property class 8.8 bolts are invariably used in the UK for moment-resisting connections; M24 are common in larger connections and M20 in more modest connections. Plate components are commonly S275 or S355. End plates are generally at least as thick as the bolt diameter.

## 11.2 Apex connections

A typical apex connection is shown in Figure 11.2. Under gravity loading, the bottom of the connection is in tension. The haunch below the rafter serves to increase the lever arms to the tension bolts, thus increasing the moment resistance. The haunch is usually small and short, and is not considered in the global analysis of the frame. In lightly loaded frames, a simple extended end plate may suffice. Plate grades and bolt details are the same as for eaves connections.

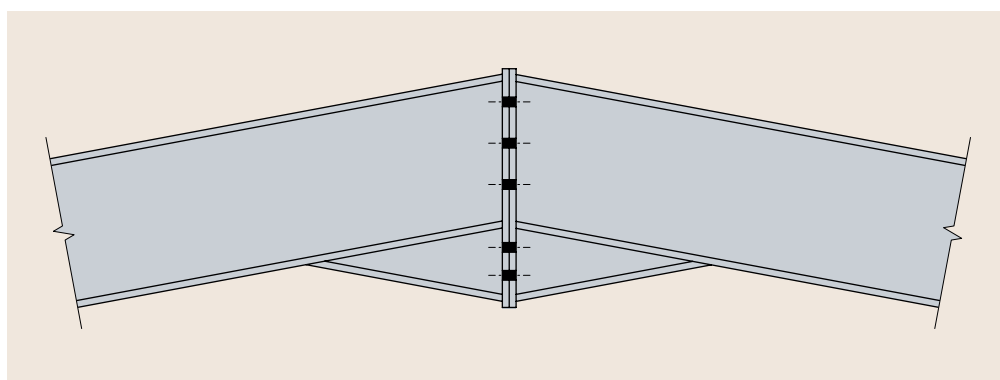


Figure 11.2  
Typical apex  
connection

## 11.3 Bases, base plates and foundations

In the majority of cases, a nominally pinned base is provided, because of the difficulty and expense of providing a rigid base. A rigid base would involve a more expensive steelwork detail and, more significantly, the foundation would also have to resist the moment, which increases foundation costs significantly compared to a nominally

pinned base. However, the bases should be designed for a small moment from temporary wind loads on a free-standing column during erection to eliminate the need for temporary bracing.

Where crane girders are supported by the column, moment resisting bases may be required, to reduce deflections to acceptable limits.

### 11.3.1 Nominally pinned bases

A typical nominally pinned base detail is shown in Figure 11.3.

For larger columns, the bolts may be located entirely within the column profile, as shown. For smaller columns (less than approximately 350 mm), the base plate would be made larger so that the bolts can be moved outside the flanges. Even with larger columns, it is common practice to detail the base with bolts outside the section, as this provides some stability during erection. Because steelwork is erected on levelling packs, typically 100 mm square placed directly under the column, if holding down bolts are detailed at very close centres, there may be very little concrete for the packs to bear on.

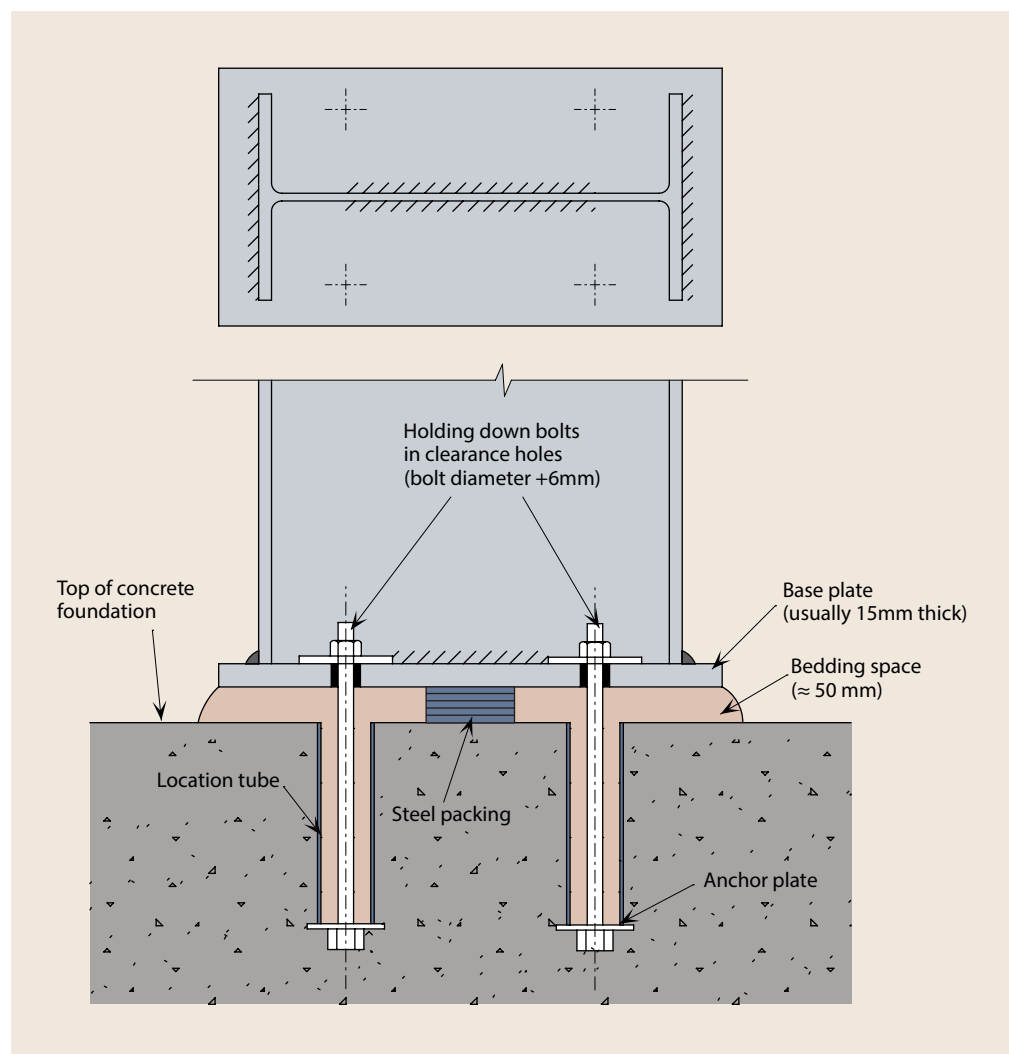


Figure 11.3  
Typical nominally  
pinned base for a  
large column

### 11.3.2 Rigid bases

A rigid, moment-resisting base is typically achieved by providing a bigger lever arm for the bolts and by increasing the plate thickness. Additional gusset plates may be required for bases subject to large bending moments.

The bending moment is transmitted to the concrete base which is likely to be larger and require significant reinforcement which has to be coordinated with the holding down bolts. This may prove difficult if the reinforcement is congested.

For the valley columns of multi-span hit and miss frames, the higher vertical load may result in a larger base which can be designed as rigid without significant cost penalty when also taking account of the benefits of reduced deflections in the frame.

### 11.3.3 Resistance to horizontal forces

Horizontal forces on foundations occur in both gravity and combination load cases. These forces are resisted in different ways, depending on the type of foundations, which in turn depend on the ground conditions. The following are theoretically possible:

- Sliding resistance on the base of the foundation as shown in Figure 11.4(a);
- A tie cast into the floor slab, connected to the base of the column, as shown in Figure 11.4(b);
- A tie across the full width of the frame connecting both columns beneath or within the floor slab, as shown in Figure 11.4(c) and (d);
- Passive earth pressure on the side of the foundation.

In theory, horizontal forces can be resisted by earth pressure on the side of foundations but in practice, because the lateral earth pressure cannot be relied on, it is ignored. The passive resistance of the surrounding ground is uncertain because it may have been disturbed by drainage and service trenches alongside the frame.

The sliding resistance depends on the type of soil under the column base.

The outward force on a portal frame column due to gravity loads can be resisted by a bar connected to the column and cast into the floor slab, and wrapped at the end to allow differential vertical movement. This can be a relatively cheap solution. This detail may lead to some local cracking of the floor slab and, where a high specification floor slab is used, the warranty on the slab may be invalidated. The length of the bar should be determined by the ultimate pull-out resistance required to resist the horizontal force.

A tie across the full width of the frame connected to the column at each side is the most certain way of resisting horizontal forces due to gravity loads. It is more expensive in terms of materials and labour and can be damaged by site activities. A full width tie may impede the erection of the structure, which generally will be undertaken from within the footprint of the building.

Coordination between the designers of the frame, foundations and floor slab is clearly essential to producing a satisfactory solution.



The transfer of shear between the steelwork and foundation must be considered carefully. Common and successful practice is to consider that modest shear forces can be transferred in friction. Transfer by shear in the holding down bolts is less certain, as shear is unlikely to be shared equally between bolts.

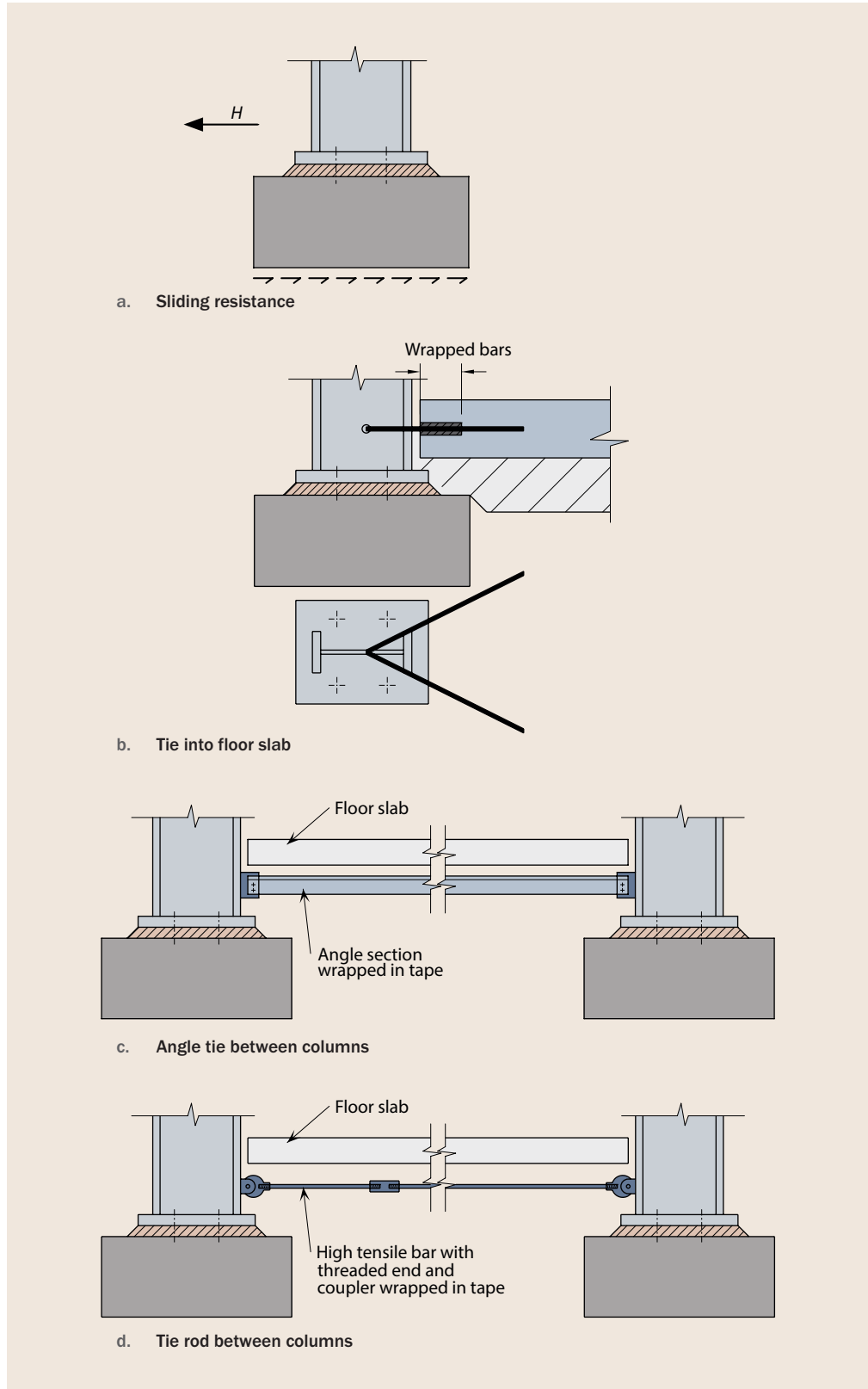


Figure 11.4  
Alternative  
approaches to resist  
horizontal forces at  
column bases

### 11.3.4 Base plates and holding down bolts

The steelwork contractor will usually be responsible for detailing the base plate and holding down bolts; commonly, another designer is responsible for the foundations. It should be made clear in the contract documentation where the responsibility lies for the interface between the steelwork and the foundation, as special reinforcement spacing or details may be required. Best practice is to ensure that the holding down details are integrated with the foundation details.

Holding down bolts are usually property class 8.8. The diameter of the bolts will generally be determined by consideration of the uplift and shear forces applied to them, but will not normally be less than 20 mm. There is often generous over-provision, to allow for the incalculable effects of incorrect location of bolts and combined shear force and bending on the bolt if the grout does not completely fill the void under the baseplate, and to provide robustness to accidental damage.

The length of the bolt should be determined by considering the properties of the concrete, the spacing of the bolts, and the design tensile force. A simple method of determining the embedment length is to assume that the bolt force is resisted by a conical surface of concrete. Where greater uplift resistance is required, angles or plates may be used to anchor the bolts together as an alternative to individual anchor plates.

Advice on the design of holding down systems is given in SCI Publication P398<sup>[39]</sup>.

### 11.3.5 Base design at the fire limit state

If the foundation is designed to resist a moment due to rafter collapse in the accidental fire situation, both the base plate and the foundation itself should be designed to resist that moment. As noted in Section 7.4, it is usual to consider the base as nominally pinned for the frame analysis.

To resist the base moment at the fire limit state, the following options may be considered:

- Increasing the thickness of the base plate.
- Adding more holding down bolts on the tension side of the column.
- Adding a haunch on the compression side.









# SERVICEABILITY ASPECTS OF FRAME DESIGN

## 12.1 Deflections

Elastic analysis is used to determine the deflections of the frame at the serviceability limit state. Only deflections due to variable actions are considered.

In some cases, the frame is pre-set such that the deflections under the permanent actions leave the frame with (for example) vertical columns. The degree of pre-set is partly a matter of calculation and partly a matter of experience – the steelwork contractor should be consulted if pre-setting the frame is being considered.

The maximum acceptable deflections in portal frames will depend on many factors, such as appearance, the building use and cladding type, and should be agreed with the client.

### 12.1.1 Deflection sensitive details

Advice is given in the following paragraphs on the influence of construction details on appropriate deflection limits.

#### ***Sheeting***

Limits on differential deflection between adjacent portal frames are necessary to prevent the fixings between the sheets and the frame from becoming overstressed, resulting in tearing of the sheeting, and leakage.

Portal frames clad in steel sheeting deflect significantly less than the deflection calculated for the bare frame. This is due to the sheeting acting as a stressed skin diaphragm, which provides a considerable stiffening effect to the structure. The actual deflection depends on the building proportions and cladding type, but reductions in horizontal deflections of over 50% (from those calculated for the bare frame) are typical of real structures.

#### ***Gables***

A sheeted and/or braced gable end is very stiff in its own plane and the deflections can be ignored. The calculated differential deflections between the end frame and the adjacent frame (at the ridge and at the eaves) can be very high. This differential deflection will always be modified by the presence of the roof sheeting and roof bracing, particularly if the roof bracing is located in the end bays.

### **Masonry**

When brick or blockwork side walls are constructed such that they receive support from the steel frame, they should be detailed to allow them to deflect with the frame by using a compressible damp proof course at the base of the wall. Suitable restraint should be provided at the top of the brickwork panel and at intermediate points, if necessary. If brickwork is continued around the steel columns, forming stiff piers, it is unreasonable to expect the panels to deflect with the frame. In this case, more onerous deflection limits should be applied to the frame.

### **Base fixity**

In order to provide stability during erection, it has become common to use four holding down bolts, even with nominally pinned bases. In this situation, it is reasonable to use a base stiffness of 20% of the column stiffness when calculating SLS deflections, as noted in Section 7.4.1.

### **Cranes**

Where crane girders are supported directly from portal frames, the need to control deflections at the crane level is likely to result in stiffer sections for the frames. The limit on spread should be determined in agreement with the client and the crane manufacturer.

### **Ponding**

To ensure proper discharge of rainwater from a nominally flat roof, or from a very low-pitched roof (slope less than 1:20), deflections under permanent and variable actions should be checked to ensure that water does not pond.

## **12.1.2 Existing guidance on deflection limits**

The recommendations presented in Table 12.1 are taken from Reference 16. That publication noted (in 1991) that “Early feedback on this table has suggested that some of the values may be more stringent than is necessary. Pending outcome of a wider consultation on this subject the indicative numerical values given in this table should be regarded as provisional.” Despite this note, the deflection limits have been accepted by many designers as the *de facto* standard and are unchanged in Table 12.1.

When considering horizontal deflections, the more onerous of the requirements for the side cladding and the roof cladding should be adopted. For the vertical deflection at the ridge, both the criteria should be observed.

### **Differential deflection**

The criteria for differential deflection between frames will be most critical for the frame nearest the gable end or next to any internal or division walls that are connected to the steel frame.



It is recognised that the in-plane stiffness of the roofing will reduce the differential deflection between adjacent frames to varying degrees, depending on the form of the roofing and geometrical factors such as the slope of the roof and the spacing of the frames. This is particularly important for the penultimate frame adjacent to a stiffer end gable.

Table 12.1 gives recommended limits for both the absolute and the differential deflections of portal frames. The absolute deflection of portal frame buildings depends on the plan proportions of the building as well as on the type of roof system. The absolute deflection limits in Table 12.1 should therefore be compared with the calculated deflection of a bare steel frame, taking account of base fixity, unless the designer is able to justify the stiffening effect of the roof and cladding system. Note that standing seam roof systems possess little stiffness, whereas double skin roof systems are quite stiff.

The differential deflection limits in Figure 12.1 may be compared with the calculated deflection of a frame that has restraint from the roof system.

**a. Horizontal deflection at eaves:**

TYPE OF CLADDING	ABSOLUTE DEFLECTION	DIFFERENTIAL DEFLECTION RELATIVE TO ADJACENT FRAME
<i>Side cladding:</i>		
Profiled metal sheeting	$\leq h/100$	--
Fibre reinforced sheeting	$\leq h/150$	--
Brickwork	$\leq h/300$	$\leq (h^2 + b^2)^{0.5}/660$
Hollow concrete blockwork	$\leq h/200$	$\leq (h^2 + b^2)^{0.5}/500$
Precast concrete units	$\leq h/200$	$\leq (h^2 + b^2)^{0.5}/330$
<i>Roof cladding:</i>		
Profiled metal sheeting	--	$\leq b/200$
Fibre reinforced sheeting	--	$\leq b/250$

**b. Vertical deflection at ridge (for rafter slopes  $\geq 3^\circ$ ):**

TYPE OF ROOF CLADDING	DIFFERENTIAL DEFLECTION RELATIVE TO ADJACENT FRAME
Profiled metal sheeting	$\leq b/100$ and $\leq (b^2 + s^2)^{0.5}/125$
Fibre reinforced sheeting	$\leq b/100$ and $\leq (b^2 + s^2)^{0.5}/165$

Table 12.1  
Recommended  
deflection limits

Notes: The calculated deflections are those due to:

- wind actions
- imposed roof loads
- snow loads
- 80% of (wind actions and snow loads).

The above values are recommendations from reference 16. Feedback on publishing the original recommendations indicated that some of these values are more stringent than necessary.

The values of  $h$ ,  $b$ , and  $s$  are defined in Figure 12.1.

The height  $h$  should always be taken as the height to eaves, not the height of the masonry panel.

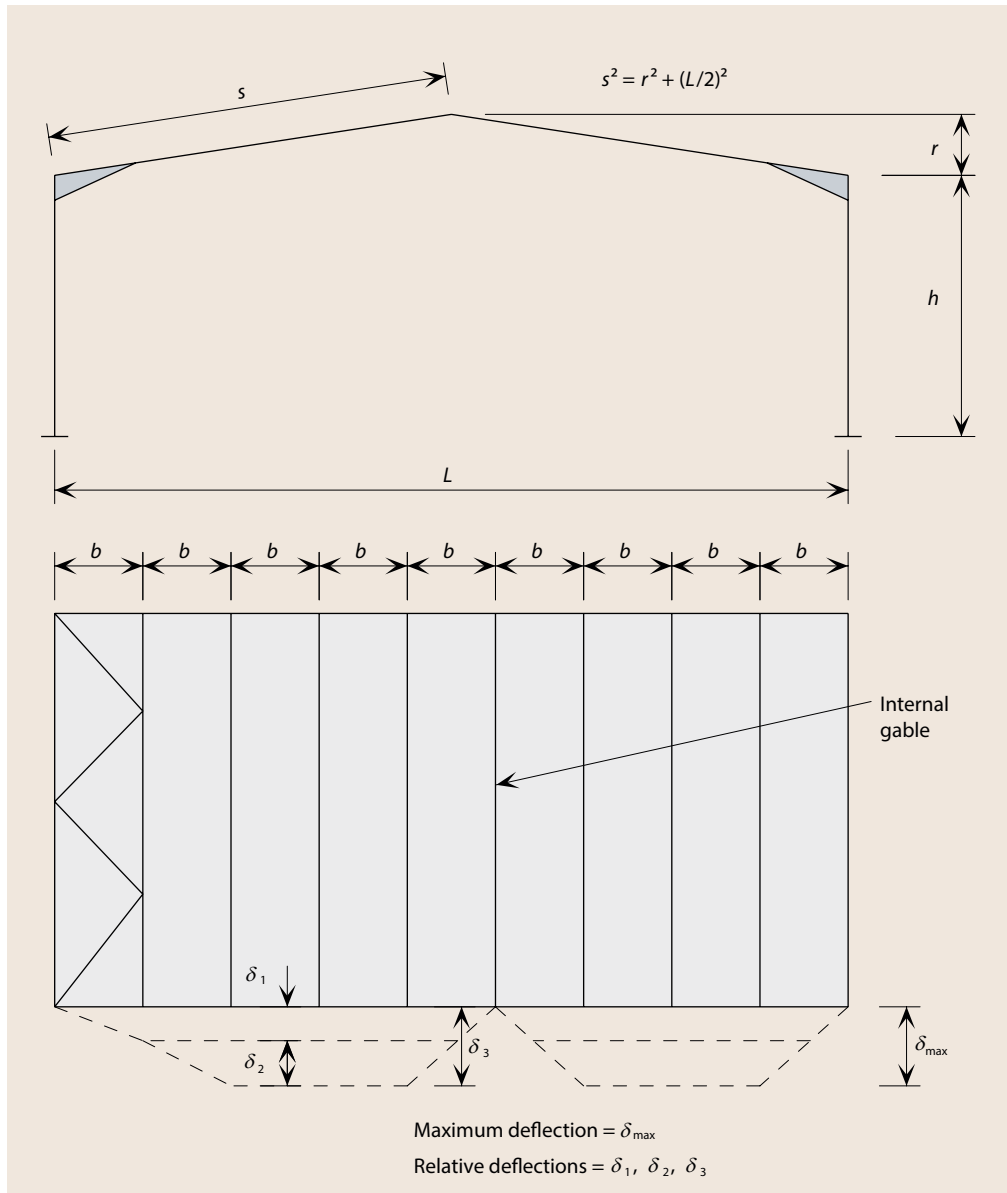


Figure 12.1  
Dimensions to be used in determining deflection limits in Table 12.1

## 12.2 Thermal expansion

In the UK, temperature movements are generally small and no additional calculations are required where the spacing of expansion joints is within the limits in Table 12.2.

COMPONENT	SITUATION	SPACING (m)
Steel framed industrial buildings	Generally	150
	With high internal temperatures	125
Roof sheeting	Down the slope	$20^2$
	Along the slope	no limit
Brick or block walls <sup>1</sup>	Clay bricks	12
	Calcium silicate bricks	8
	Concrete blocks	6

Table 12.2  
Maximum spacing of expansion joints in buildings

- <sup>1</sup> This is a guide only and refers to the expansion joints in the brickwork to structure connection; reference should be made to UK NA to BS EN 1996-2<sup>[40]</sup>.
- <sup>2</sup> Longer lengths are possible where provision for expansion is made.

### **12.2.1 Expansion joints**

The provision of satisfactory expansion joints is neither easy nor cheap, and it is usually better to detail the structure such that joints can be avoided, for example by considering the alternative of resisting expansion by the use of braced bays. Where expansion joints are provided, care should be taken to ensure that they are properly detailed to ensure that they cannot cause leaks in the cladding due to differential movement.



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	vi	Liebherr container cranes factory, Ireland		89	Manufacturing technology centre, Coventry
	3	B&Q distribution centre, Swindon		97	Manufacturing technology centre, Coventry
	9	Sainsbury's distribution centre, Basingstoke		133	Mechanical treatment facility, Basildon
	25	Siemens facility, Lincoln			
	33	Manufacturing technology centre, Coventry			
	41	Liebherr container cranes factory, Ireland			
	59	B&Q distribution centre, Swindon			
	79	Sainsbury's distribution centre, Basingstoke			



# APPENDIX A - PRELIMINARY SIZING

## A.1 Preliminary member sizes

### A.2 Introduction

This Appendix contains tables of member sizes for columns and rafters of single span portal frames at the preliminary design stage. Further detailed calculations will be required at the final design stage. The tabulated sizes take no account of:

- Stability at the ultimate limit state.
- Deflections at the serviceability limit state.

Further checks will therefore be required, which may necessitate increasing the size of the members in some cases.

### A.3 Column and rafter sizes

Table A.1 presents data that will enable a rapid determination of member size to be made for estimating purposes. The span range is 15 to 40 m and the steel grade is S355. The information is based on earlier tables for S275, designed to BS 5950, given in SCI Publication P252<sup>[44]</sup>. The S355 sections have been selected to provide a similar bending resistance to the S275 sections presented in Reference 41. The assumptions made in creating this table are as follows:

- The roof pitch is 6°.
- The steel grade is S355.
- The rafter load is the design value of the permanent actions (including self-weight) plus variable action (the imposed roof load). Wind loading has not been included – the presumption is that “gravity” loading will dominate the choice of member sizes.
- The haunch length is 10% of the span of the frame.
- A column is treated as restrained when torsional restraints are provided along its length (these columns are therefore lighter than the equivalent unrestrained columns).
- A column is treated as unrestrained if no torsional restraint can be provided along its length.

The member sizes given in the tables are suitable for preliminary design only. Where an asterisk (\*) is shown in the table, a suitable section size has not been calculated, as the wind effects and second order effects are likely to be significant.

Table A.1 Preliminary sizes of columns and rafters for symmetrical single-span portal frame with 6° roof pitch (S355 steel)

DESIGN LOAD ON RAFTER (kN/m)	EAVES HEIGHT (m)	SPAN OF FRAME (m)						
		15	20	25	30	35	40	
Rafter	8	254x102x22 UKB	254x146x31 UKB	356x127x39 UKB	356x171x45 UKB	356x171x57 UKB	356x171x67 UKB	356x171x67 UKB
	8	254x102x22 UKB	254x146x31 UKB	356x127x39 UKB	356x171x51 UKB	356x171x67 UKB	356x171x82 UKB	457x191x67 UKB
	8	254x102x22 UKB	254x146x31 UKB	356x127x39 UKB	356x171x51 UKB	356x171x67 UKB	356x171x82 UKB	457x191x67 UKB
	8	*	254x146x31 UKB	356x127x39 UKB	356x171x51 UKB	356x171x67 UKB	356x171x82 UKB	457x191x67 UKB
Restrained column	8	305x165x40 UKB	305x165x46 UKB	406x178x67 UKB	457x191x82 UKB	533x210x92 UKB	610x229x113 UKB	610x229x113 UKB
	8	305x165x40 UKB	305x165x46 UKB	406x178x67 UKB	457x191x82 UKB	533x210x101 UKB	610x229x113 UKB	610x229x113 UKB
	8	305x165x40 UKB	305x165x54 UKB	406x178x67 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x125 UKB
	8	*	305x165x54 UKB	406x178x67 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x125 UKB
Unrestrained column	8	305x165x46 UKB	457x191x67 UKB	533x210x82 UKB	533x210x92 UKB	610x229x113 UKB	610x229x125 UKB	610x229x125 UKB
	8	406x178x60 UKB	457x191x82 UKB	610x229x101 UKB	610x229x113 UKB	686x254x125 UKB	762x267x147 UKB	762x267x147 UKB
	8	406x178x67 UKB	533x210x92 UKB	610x229x113 UKB	610x229x125 UKB	762x267x147 UKB	838x292x194 UKB	838x292x194 UKB
	8	*	610x229x101 UKB	610x229x125 UKB	762x267x147 UKB	838x292x194 UKB	914x305x224 UKB	914x305x224 UKB
Rafter	10	254x102x25 UKB	254x146x31 UKB	356x171x45 UKB	356x171x57 UKB	356x171x67 UKB	356x171x82 UKB	457x191x82 UKB
	10	254x102x25 UKB	254x146x31 UKB	356x171x45 UKB	356x171x57 UKB	356x171x67 UKB	356x171x82 UKB	457x191x82 UKB
	10	254x102x25 UKB	356x127x39 UKB	356x171x45 UKB	356x171x57 UKB	356x171x67 UKB	356x171x82 UKB	457x191x82 UKB
	10	*	356x127x39 UKB	356x171x45 UKB	356x171x57 UKB	356x171x67 UKB	356x171x82 UKB	457x191x82 UKB
Restrained column	10	305x165x40 UKB	406x178x60 UKB	406x178x74 UKB	533x210x92 UKB	610x229x113 UKB	610x229x125 UKB	610x229x125 UKB
	10	305x165x40 UKB	406x178x60 UKB	457x191x82 UKB	533x210x92 UKB	610x229x113 UKB	610x229x125 UKB	610x229x125 UKB
	10	305x165x40 UKB	406x178x60 UKB	457x191x82 UKB	533x210x101 UKB	610x229x113 UKB	610x229x140 UKB	610x229x140 UKB
	10	*	406x178x60 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x140 UKB	610x229x140 UKB
Unrestrained column	10	406x178x54 UKB	457x191x74 UKB	533x210x92 UKB	610x229x101 UKB	610x229x125 UKB	610x229x125 UKB	610x229x125 UKB
	10	457x191x67 UKB	533x210x92 UKB	610x229x113 UKB	686x254x125 UKB	686x254x140 UKB	610x305x149 UKB	610x305x149 UKB
	10	457x191x74 UKB	610x229x101 UKB	610x229x125 UKB	762x267x147 UKB	838x292x194 UKB	838x292x194 UKB	838x292x194 UKB
	10	*	610x229x113 UKB	686x254x140 UKB	838x292x194 UKB	914x305x224 UKB	914x305x224 UKB	914x305x224 UKB
Rafter	12	254x102x28 UKB	356x127x39 UKB	356x171x51 UKB	356x171x67 UKB	457x191x82 UKB	533x210x92 UKB	533x210x92 UKB
	12	254x102x28 UKB	356x127x39 UKB	356x171x51 UKB	356x171x67 UKB	457x191x82 UKB	533x210x92 UKB	533x210x92 UKB
	12	254x102x28 UKB	356x127x39 UKB	356x171x51 UKB	356x171x67 UKB	457x191x82 UKB	533x210x92 UKB	533x210x92 UKB
	12	*	356x127x39 UKB	356x171x51 UKB	356x171x67 UKB	457x191x82 UKB	533x210x92 UKB	533x210x92 UKB
Restrained column	12	305x165x40 UKB	406x178x67 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x140 UKB	610x229x140 UKB
	12	305x165x40 UKB	406x178x67 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	686x254x140 UKB	686x254x140 UKB
	12	305x165x46 UKB	406x178x67 UKB	533x210x92 UKB	610x229x113 UKB	610x229x125 UKB	686x254x140 UKB	686x254x140 UKB
	12	*	406x178x67 UKB	533x210x92 UKB	610x229x113 UKB	610x229x125 UKB	686x254x140 UKB	686x254x140 UKB
Unrestrained column	12	406x178x60 UKB	533x210x82 UKB	610x229x101 UKB	610x229x113 UKB	610x229x125 UKB	610x229x125 UKB	610x229x125 UKB
	12	457x191x74 UKB	610x229x101 UKB	610x229x113 UKB	686x254x140 UKB	610x305x149 UKB	610x305x149 UKB	610x305x149 UKB
	12	457x191x82 UKB	610x229x113 UKB	686x254x140 UKB	838x292x194 UKB	838x292x194 UKB	838x292x194 UKB	838x292x194 UKB
	12	*	610x229x125 UKB	610x229x149 UKB	838x292x194 UKB	914x305x224 UKB	914x305x224 UKB	914x305x224 UKB

Table A.1 Preliminary sizes of columns and rafters for symmetrical single-span portal frame with 6° roof pitch (S355 steel) (continued)

	DESIGN LOAD ON RAFTER (kN/m)	EAVES HEIGHT (m)	SPAN OF FRAME (m)						
			15	20	25	30	35	40	
Rafter	14	6	254x146x31 UKB	356x171x45 UKB	356x171x57 UKB	457x191x67 UKB	457x191x82 UKB	457x191x92 UKB	533x210x92 UKB
	14	8	254x146x31 UKB	356x171x45 UKB	356x171x57 UKB	457x191x67 UKB	457x191x82 UKB	457x191x92 UKB	533x210x92 UKB
	14	10	254x146x31 UKB	356x171x45 UKB	356x171x57 UKB	457x191x67 UKB	457x191x82 UKB	457x191x92 UKB	533x210x92 UKB
	14	12	*	356x171x45 UKB	356x171x57 UKB	457x191x67 UKB	457x191x82 UKB	457x191x92 UKB	610x229x101 UKB
Restrained column	14	6	305x165x46 UKB	406x178x74 UKB	533x210x92 UKB	610x229x113 UKB	610x229x140 UKB	610x229x140 UKB	686x254x170 UKB
	14	8	305x165x54 UKB	406x178x74 UKB	533x210x92 UKB	610x229x113 UKB	610x229x140 UKB	610x229x140 UKB	686x254x170 UKB
	14	10	305x165x54 UKB	406x178x74 UKB	533x210x101 UKB	610x229x125 UKB	610x229x140 UKB	610x229x140 UKB	686x254x170 UKB
	14	12	*	406x178x74 UKB	533x210x101 UKB	610x229x125 UKB	610x229x140 UKB	610x229x140 UKB	686x254x170 UKB
Unrestrained column	14	6	457x191x67 UKB	533x210x82 UKB	610x229x101 UKB	610x229x125 UKB	610x229x140 UKB	610x229x140 UKB	610x305x149 UKB
	14	8	457x191x82 UKB	610x229x101 UKB	686x254x125 UKB	762x267x147 UKB	762x267x147 UKB	762x267x147 UKB	838x292x176 UKB
	14	10	533x210x92 UKB	610x229x125 UKB	762x267x147 UKB	610x305x149 UKB	610x305x149 UKB	610x305x149 UKB	914x305x224 UKB
	14	12	*	686x254x140 UKB	610x305x149 UKB	610x305x149 UKB	838x292x194 UKB	838x292x194 UKB	914x305x289 UKB
Rafter	16	6	254x146x31 UKB	356x171x45 UKB	356x171x67 UKB	457x191x82 UKB	457x191x82 UKB	457x191x82 UKB	610x229x101 UKB
	16	8	254x146x31 UKB	356x171x45 UKB	356x171x67 UKB	457x191x82 UKB	457x191x82 UKB	457x191x82 UKB	610x229x101 UKB
	16	10	254x146x31 UKB	356x171x45 UKB	356x171x67 UKB	457x191x82 UKB	457x191x82 UKB	457x191x82 UKB	610x229x101 UKB
	16	12	*	356x171x45 UKB	356x171x67 UKB	457x191x67 UKB	457x191x82 UKB	457x191x82 UKB	610x229x113 UKB
Restrained column	16	6	305x165x54 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x125 UKB	610x229x125 UKB	686x254x170 UKB
	16	8	305x165x54 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x125 UKB	610x229x125 UKB	762x267x173 UKB
	16	10	406x178x60 UKB	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x125 UKB	610x229x125 UKB	762x267x173 UKB
	16	12	*	457x191x82 UKB	533x210x101 UKB	610x229x125 UKB	610x229x125 UKB	610x229x125 UKB	838x292x176 UKB
Unrestrained column	16	6	457x191x67 UKB	533x210x92 UKB	610x229x113 UKB	610x229x125 UKB	610x229x140 UKB	610x229x140 UKB	610x305x149 UKB
	16	8	457x191x82 UKB	610x229x113 UKB	686x254x140 UKB	610x305x149 UKB	610x305x149 UKB	610x305x149 UKB	838x292x194 UKB
	16	10	533x210x101 UKB	610x229x125 UKB	610x305x149 UKB	610x305x149 UKB	610x305x149 UKB	610x305x149 UKB	914x305x253 UKB
	16	12	*	686x254x140 UKB	610x305x149 UKB	610x305x149 UKB	838x292x176 UKB	838x292x176 UKB	914x305x289 UKB





# APPENDIX B - DETERMINATION OF THE ELASTIC CRITICAL FORCE AND MOMENT

Values of elastic critical force  $N_{cr}$  and elastic critical moment  $M_{cr}$  for an unrestrained length of member are given in Sections B.1 and B.2 respectively.

The value of  $N_{cr}$  is required when calculating  $\alpha_{cr}$  (see Section 7.5) and when calculating the flexural buckling resistance of members in accordance with Clause 6.3.1.2 of BS EN 1993-1-1. Note that an alternative approach to calculate the flexural buckling resistance, not requiring  $N_{cr}$ , is given in Clause 6.3.1.3; the two approaches give identical resistances.

The value of  $M_{cr}$  is required when calculating the lateral-torsional buckling resistance of members in accordance with Clause 6.3.2.3 of BS EN 1993-1-1. For rolled sections, the use of Clause 6.3.2.3 in preference to 6.3.2.2 (the general case) is recommended, as a higher but valid lateral-torsional buckling resistance results from the calculation.

## **B.1 $N_{cr}$ for uniform members**

$$N_{cr} = \frac{\pi^2 EI}{L^2}$$

where

- $E$  is the modulus of elasticity (210000 N/mm<sup>2</sup>)
- $I$  is the second moment of area about the axis of buckling being considered
- $L$  is the buckling length.

## **B.2 $M_{cr}$ for uniform members**

### **B.2.1 General expression**

This expression only applies to uniform straight members and when the cross-section is bi-symmetric. Assuming the ends are not restrained against warping and assuming that the load is not destabilising, then:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}}$$

where:

- $E$  is the modulus of elasticity (210000 N/mm<sup>2</sup>)
- $G$  is the shear modulus (81000 N/mm<sup>2</sup>)
- $I_z$  is the second moment of area about the minor axis
- $I_T$  is the torsional constant of the member
- $I_w$  is the warping constant of the member
- $L$  is the beam length between points of lateral restraint
- $C_1$  is a factor that accounts for the shape of the bending moment diagram.

### B.2.2 $C_1$ factor

According to the UK National Annex to BS EN 1993-1-1,

$$C_1 = \frac{M_{cr} \text{ for the actual bending moment diagram}}{M_{cr} \text{ for a uniform bending moment diagram}}$$

The factor  $C_1$  may be determined from Table B.1 for a member with end moments or with intermediate transverse loading.

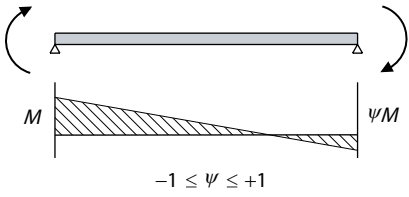


END MOMENT LOADING	$\psi$	$C_1$
 <p style="text-align: center;"><math>-1 \leq \psi \leq +1</math></p>	+1.00	1.00
	+0.75	1.17
	+0.50	1.36
	+0.25	1.56
	0.00	1.77
	-0.25	2.00
	-0.50	2.24
	-0.75	2.49
	-1.00	2.76
INTERMEDIATE TRANSVERSE LOADING		
		1.13
		2.60

Table B.1  
 $C_1$  factor

If a non-linear bending moment diagram is concave, compared to a straight line between end moments, as shown in Figure B.1a, the bending moment shape may be conservatively taken as linear and the value of  $C_1$  can be determined depending on  $\psi$ . For a non-linear diagram shape, as shown in Figure B.1b, which is convex compared to a straight line between the end moments, it is not conservative to assume a linear moment. It will always be conservative to assume a uniform moment ( $C_1 = 1.0$ ). Alternatively,  $C_1$  (and indeed  $M_{cr}$ ) can be determined by software.

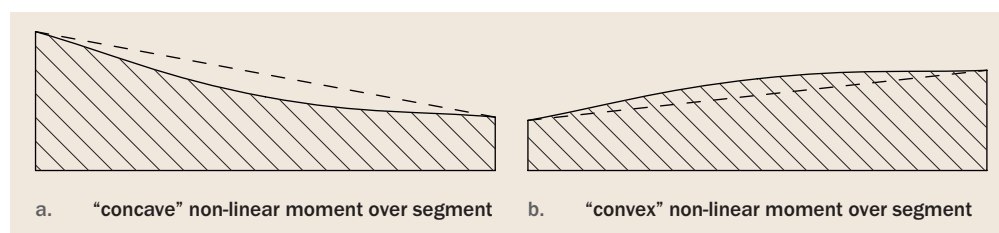


Figure B.1  
Typical bending moment diagrams found in portal frames

### B.3 Monosymmetric sections

Near the column of a portal frame the rafter is usually haunched with a T section (either a cutting from an I section or a welded element) to form a compound section. The cross-section of the element at the haunch has one axis of symmetry so the geometric centroid of the cross-section and the shear centre are not coincident. The elastic critical moment for this type of element depends on the position of the shear centre and the monosymmetry index  $\beta_1$ .

The elastic critical moment for a monosymmetric section where the load is not destabilizing and the ends are not restrained against warping is given by:

$$M_{cr} = -\frac{\pi^2 EI_z}{L^2} \frac{\beta_1}{2} \pm \frac{\pi^2 EI_z}{L^2} \sqrt{\frac{\beta_1^2}{4} + \frac{I_w}{I_z} + \frac{L^2 GI_T}{\pi^2 EI_z}}$$

The second term can either be positive or negative, depending on which flange is in compression, giving two values of  $M_{cr}$ .

The monosymmetry index  $\beta_1$  is given by:

$$\beta_1 = \frac{1}{I_y} \left( \int_A z^3 dA + \int_A y^2 z dA \right) - 2z_0$$

where  $z_0$  is the distance of the shear centre from the geometric centroid of the section.

The integrals can be expressed in terms of the second moments and areas of the flanges and webs of the compound section and the ordinates of their centroids. The effects of root radii are ignored.

$$\beta_1 = \frac{1}{I_y} \left[ \bar{z}_{ft} (I_{zft} + A_{ft} \bar{z}_{ft}^2 + 3I_{yft}) + \bar{z}_{fb} (I_{zfb} + A_{fb} \bar{z}_{fb}^2 + 3I_{yfb}) \right. \\ \left. + \bar{z}_{fc} (I_{zfc} + A_{fc} \bar{z}_{fc}^2 + 3I_{yfc}) + \bar{z}_{wb} (I_{zwb} + A_{wb} \bar{z}_{wb}^2 + 3I_{ywb}) + \bar{z}_{wc} (I_{zwc} + A_{wc} \bar{z}_{wc}^2 + 3I_{ywc}) \right] - 2z_0$$

where the  $I_{zf}$  and  $I_{zw}$  terms are the second moments of area of the flanges and webs about the minor axis and the  $\bar{z}_f$  and  $\bar{z}_w$  terms are the ordinates of the centroids of the flanges and webs relative to the geometric centroid of the compound section.

The  $I_{yf}$  and  $I_{yw}$  terms are the second moments of area of the flanges and webs about the major axis. The suffices ft, fb and fc refer to the top, bottom and haunch flanges respectively. The suffices wb and wc refer to the webs of the beam and haunch.

The shear centre can be determined using the following formula (ignoring the effects of root radii) where  $z_0$  is the ordinate relative to the geometric centroid.

$$z_0 = \frac{1}{I_z} \times [I_{zft} \bar{z}_{ft} + I_{zfb} \bar{z}_{fb} + I_{zfc} \bar{z}_{fc} + I_{zwb} \bar{z}_{wb} + I_{zwc} \bar{z}_{wc}]$$

The position of the shear centre is towards the haunch flange where the haunch is shallow and for the z centroidal axis positive downward is greater than zero.



# APPENDIX C - DESIGN OF MEMBERS WITH DISCRETE RESTRAINTS TO THE TENSION FLANGE

It is possible to take account of restraints to the tension flange. This may lead to a greater design buckling resistance of the member.

## C.1 Behaviour

Research was carried out by Horne, Ajmani and others in the 1960s and 1970s on the behaviour of columns with restraints to the tension flange provided by sheeting rails. This work showed that an element with torsional restraints at the ends and intermediate restraints to the tension flange which are sufficiently close together, will buckle out of plane in a torsional mode. The axis of rotation for the torsional buckling is parallel to the tension flange of the element, through the centroid of the sheeting rails. The intermediate restraints prevent flexural buckling out of plane. The degree of torsional restraint provided to the tension flange has a significant effect on the torsional buckling resistance of the element. However, the effectiveness of the torsional restraint provided by the bolted cleats normally used to connect sheeting rails is uncertain and this effect is not included in the expression for torsional flexural buckling.

The stability of the element out of plane is verified by checking the interaction of axial and bending behaviour where the buckling mode is appropriate to the system of restraints. The relevant critical values of axial resistance and bending resistance are used. The spacing of the intermediate restraints must be such that failure due to the interaction of flexural buckling and lateral torsional buckling will not occur between them. Checks should be carried out to demonstrate this.

Additional limits on the spacing of restraints are imposed where the element carries a high bending moment in the form of a plastic hinge.

## C.2 Spacing of lateral restraints

In segments containing a plastic hinge, the spacing between the hinge and the first intermediate lateral restraint to the tension flange must be less than the value of  $L_m$  as given in Clause BB.3.1.1 of BS EN 1993-1-1, for uniform members. This formula for the limiting length assumes the maximum bending moment in the segment is equal to its plastic resistance. The bending moment in the segment at the position of the first intermediate lateral restraint is likely to be significantly less than the plastic resistance and the spacing of the next lateral restraint can be determined by carrying out an elastic check.



Although the expression for  $L_m$  is relevant to segments between torsional restraints containing a plastic hinge, it can conservatively be used for the less onerous case of a segment without a plastic hinge. Restraints at a closer spacing than  $L_m$  are not necessary.

### Limiting length, $L_m$

$$L_m = \frac{38i_z}{\sqrt{\frac{1}{57.4} \left( \frac{N_{Ed}}{A} \right) + \frac{1}{756C_1^2} \left( \frac{W_{pl,y}}{AI_T} \right) \left( \frac{f_y}{235} \right)^2}}$$

where

- $i_z$  is the radius of gyration about the minor axis  $\left( \sqrt{I_z/A} \right)$
- $N_{Ed}$  is the design value of the compression force in the member
- $A$  is the cross-sectional area of the member
- $W_{pl,y}$  is the plastic section modulus of the member
- $I_T$  is the torsional constant of the member
- $f_y$  is the yield strength
- $C_1$  is a factor that accounts for the shape of the bending moment diagram over the length of the segment and is given in Section B.2.2 of this document.

Where the segment is haunched or tapered, the formulae in Clause BB.3.2.1 should be used.

## C.3 Spacing of torsional restraints

Torsional restraints to the member should be provided in accordance with BS EN 1993-1-1 BB.3.1.2 or BB.3.2.2. For a uniform member with a constant moment, the stable length is as follows.

### Stable length between torsional restraints $L_k$

$$L_k = \frac{\left( 5.4 + \frac{600f_y}{E} \right) \left( \frac{h}{t_f} \right) i_z}{\sqrt{5.4 \left( \frac{f_y}{E} \right) \left( \frac{h}{t_f} \right) - 1}}$$

For uniform elements with linear and non-linear moment gradients and haunched and tapered elements, stable lengths are modified by factors for linear and non-linear moment gradient as indicated in the clauses identified. These factors are given below in Clause C.5.1 of this document.

A further modification for axial load is given in the formula for the stable length of a member subject to a linear moment gradient. This formula should appear as follows in BS EN 1993-1-1 to agree with BS 5950.

$$L_s = \sqrt{C_m} L_k \left( \frac{M_{pl,y,Rk}}{M_{N,y,Rk} + aN_{Ed}} \right)^{0.5} \quad (\text{BB.7})$$

If the axial force  $N_{Ed}$  is tensile it should be taken as zero.

## C.4 $N_{crT}$ for members with discrete restraints to the tension flange

The elastic critical torsional buckling force for a length of uniform section with two axes of symmetry, between torsional restraints and with intermediate restraints to the tension flange is given in Clause BB.3.3.1 as:

$$N_{crT} = \frac{1}{i_s^2} \left( \frac{\pi^2 EI_z a^2}{L_t^2} + \frac{\pi^2 EI_w}{L_t^2} + GI_T \right)$$

where:

$$i_s^2 = i_y^2 + i_z^2 + a^2$$

$i_y$  is the radius of gyration about the major axis ( $\sqrt{I_y/A}$ )

$L_t$  is the length of the segment along the member between torsional restraints to both flanges

$a$  is the distance between the centroid of the member and the centroid of the restraining members, such as purlins restraining rafters

$I_T$  is the torsional constant of the member

$I_w$  is the warping constant of the member

$I_z$  is the second moment of area about the weak axis

$E$  is the modulus of elasticity (210000 N/mm<sup>2</sup>)

$G$  is the shear modulus (81000 N/mm<sup>2</sup>).

For tapered or haunched members,  $N_{crT}$  is calculated using the section properties of the shallow end.

## C.5 $M_{cr}$ for members with discrete restraints to the tension flange

### C.5.1 General expression

For the general case of a beam of varying depth but symmetrical about the minor axis, subject to a non-uniform moment:

$$M_{cr} = (1/c^2) C_m M_{cr0} \quad \text{for beams with a linear bending moment diagram}$$

or

$$M_{cr} = (1/c^2) C_n M_{cr0} \quad \text{for beams with a non-linear bending moment diagram}$$

where:

$M_{cr0}$  is the elastic critical buckling moment for a beam with intermediate restraints to the tension flange, subject to uniform moment, calculated in accordance with Section C.5.2

$c$  accounts for the taper ( $c = 1$  for a uniform member).

The value of  $c$  is given by BS EN 1993-1-1 Clause BB.3.3.3, based on the depth at the shallower end of the member and limited to members where  $1 \leq h_{\max}/h_{\min} \leq 3$ . For non-uniform members with constant flanges, for which  $h \geq 1.2b$  and  $h/t_f \geq 20$ , the following equations may be used:

for tapered members or segments, see Figure C.1(a)

$$c = 1 + \frac{3}{\left(\frac{h}{t_f} - 9\right)} \left(\frac{h_{\max}}{h_{\min}} - 1\right)^{2/3} \quad (\text{BB.16})$$

for haunched members or segments, see Figure C.1(b) and Figure C.1(c)

$$c = 1 + \frac{3}{\left(\frac{h}{t_f} - 9\right)} \left(\frac{h_h}{h_s}\right)^{2/3} \sqrt{\frac{L_h}{L_y}} \quad (\text{BB.17})$$

where:

$h_h$  is the additional depth of the haunch or taper, as shown in Figure C.1

$h_{\max}$  is the maximum depth of cross-section within the length  $L_y$

$h_{\min}$  is the minimum depth of cross-section within the length  $L_y$

$h_s$  is the vertical depth of the un-haunched section

$L_h$  is the length of haunch within the length  $L_y$

$L_y$  is the length between points at which the compression flange is laterally restrained

$(h/t_f)$  is to be derived from the shallowest section.

The relevant dimensions when calculating the taper factor  $c$  are illustrated in Figure C.1.

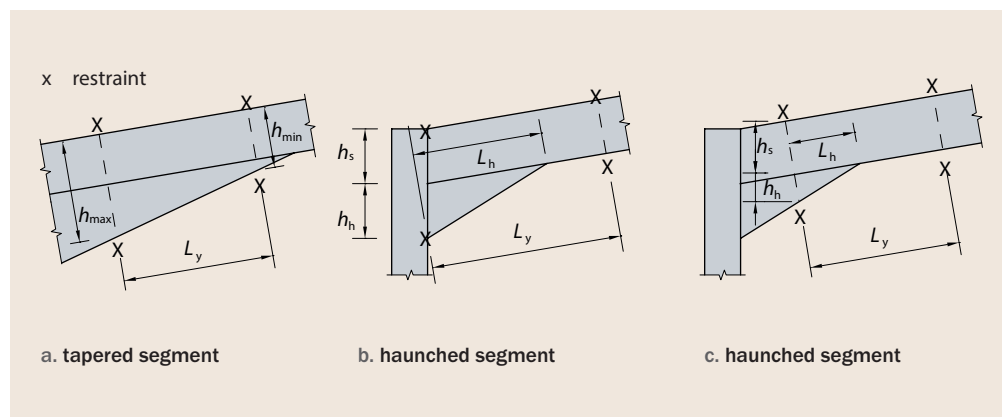


Figure C.1  
Dimensions defining  
taper factor

The factor  $C_m$  accounts for linear moment gradients. The value is given by Expression (BB.13) of BS EN 1993-1-1 as:

$$C_m = \frac{1}{B_0 + B_1\beta_t + B_2\beta_t^2} \quad (\text{BB.13})$$

where:

$$B_0 = \frac{1+10\eta}{1+20\eta}$$

$$B_1 = \frac{5\sqrt{\eta}}{\pi+10\sqrt{\eta}}$$

$$B_2 = \frac{0.5}{1+\pi\sqrt{\eta}} - \frac{0.5}{1+20\eta}$$

$$\eta = \frac{N_{crE}}{N_{crT}}$$

$$N_{crE} = \frac{\pi EI_z}{L_t^2}$$

$L_t$  is the distance between the torsional restraints

$N_{crT}$  is the elastic critical torsional force for members with restraints to the tension flange, as defined in Section C.4 of this document

$\beta_t$  is the ratio of the algebraically smaller end moment to the larger end moment.

The factor  $C_n$  accounts for non-linear moment gradients. The value is given by Expression (BB.14) of BS EN 1993-1-1. The expression given in BS EN 1993-1-1 is quoted incorrectly and has been corrected below to agree with BS 5950.

$$C_n = \frac{12R_{\max}}{[R_1 + 3R_2 + 4R_3 + 3R_4 + R_5 + 2(R_S - R_E)]}$$

where:

$R_1$  to  $R_5$  are the values of  $R$  according to the following equation

$$R = \frac{M_{y,Ed} + aN_{Ed}}{f_y W_{pl,y}}$$

at the ends, quarter points and mid-length (see Figure C.2)  
and only positive values of  $R$  should be included. If  $N_{Ed}$  is a tension, it should be taken as zero.

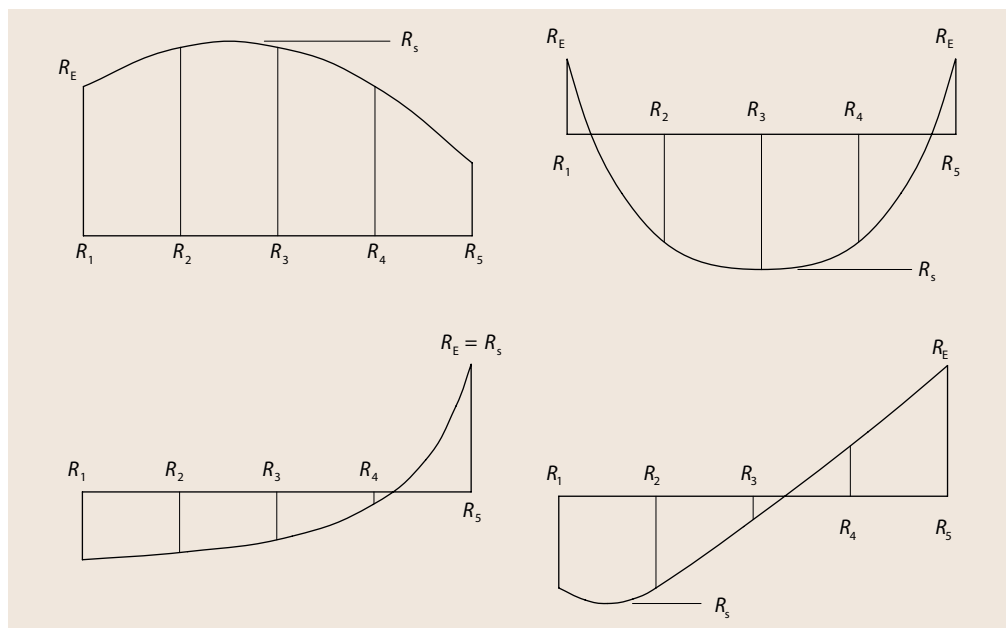


Figure C.2  
Moment values

In addition, only positive values of  $(R_s - R_E)$  should be included, where:

- $R_E$  is the greater of  $R_1$  or  $R_5$   
 $R_s$  is the maximum value of  $R$  anywhere in the length  $L_y$   
 $R_{\max}$  is the maximum of the absolute values of  $R$  anywhere in the length  $L_y$ .

When calculating  $C_m$  (in accordance with Clause BB.3.3.1) or  $C_n$  (in accordance with Clause BB.3.3.2), bending moments that produce compression in the non-restrained flange should be taken as positive. Only positive values of  $R$  should be taken.

When calculating  $C_n$  (in accordance with Clause BB.3.3.2) it is assumed that the axial loads are applied through the shear centre.

### C.5.2 Calculation of $M_{cr0}$

For uniform sections, symmetric about the minor axis, restrained along the tension flange at intervals:

$$M_{cr0} = \frac{1}{2a} \left( \frac{\pi^2 EI_z a^2}{L_t^2} + \frac{\pi^2 EI_w}{L_t^2} + GI_T \right)$$

therefore if the value of  $N_{crT}$  is calculated beforehand,

$$M_{cr0} = \frac{i_s^2}{2a} N_{crT}$$

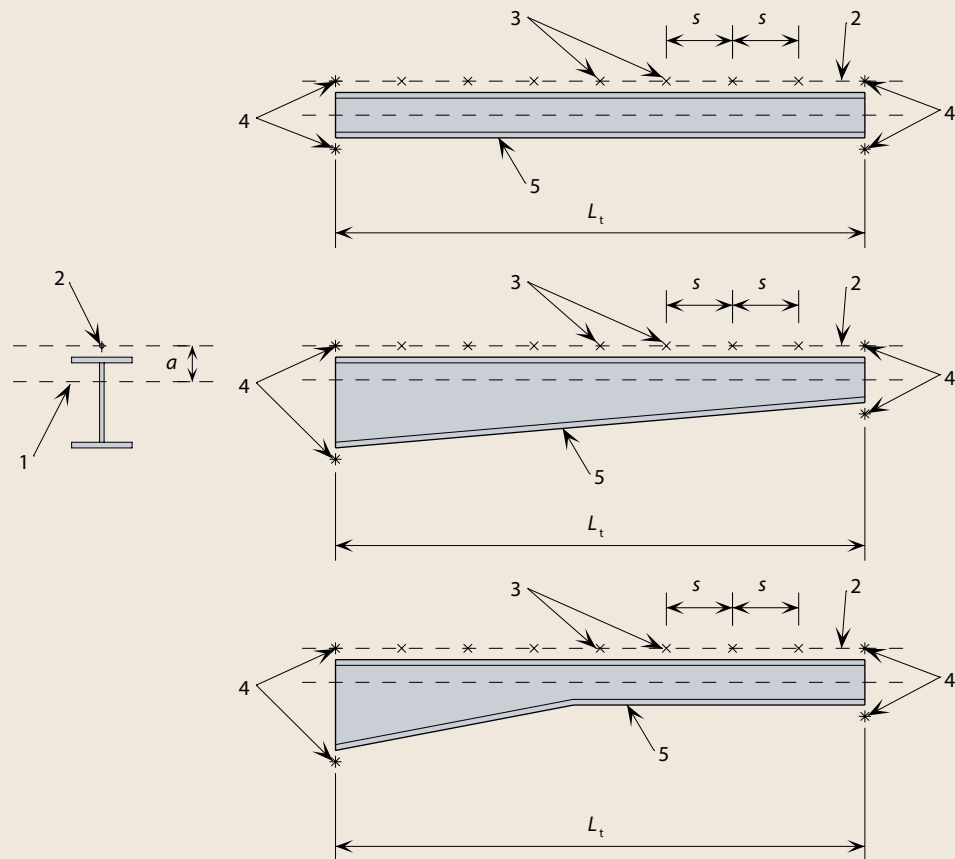
$$\text{but } M_{cr0} \leq \frac{\pi^2 EI_z}{s^2} \sqrt{\frac{I_w}{I_z} + \frac{s^2 GI_T}{\pi^2 EI_z}}$$

where:

- $N_{crT}$  is the elastic critical torsional buckling force as defined in Section C.4 of this document  
 $s$  is the distance between the restraints along the restrained longitudinal axis (e.g. the spacing of the purlins)  
 $I_T$  is the torsional constant of the member  
 $I_w$  is the warping constant of the member  
 $I_z$  is the second moment of area about the weak axis  
 $E$  is the modulus of elasticity (210000 N/mm<sup>2</sup>)  
 $G$  is the shear modulus (81000 N/mm<sup>2</sup>).

For tapered or haunched members,  $M_{cr0}$  is calculated using the section properties at the shallow end.

The parameters  $a$ ,  $L_t$  and  $s$  are shown in Figure C.3.



1. Level of shear centre of the shallowest cross-section
2. Axis where restraint is provided
3. Intermediate lateral restraints to tension flange
4. Lateral restraints to both flanges, providing torsional restraint
5. Compression flange

Figure C.3  
Arrangement  
of tension  
flange restraints





# APPENDIX D - ELASTIC CRITICAL BUCKLING DERIVATION

In the paper Stability of columns supported locally by side rails<sup>[42]</sup> solutions are given for the critical loads of a uniform strut with two axes of symmetry supported at the ends with torsional restraints and with intermediate restraints to one flange on an axis a distance  $a$  from the geometric centroid (the tension flange in the case of uniform moment). Notation is as given in Appendix C with the following additional items:

- $s$  length of element between lateral restraints;
- $l$  length of element between torsional restraints;
- $P_E$  Euler buckling load for element of length  $s$ ;
- $P_T$  torsional elastic critical buckling load for element of length  $l$ ;
- $P_{TC}$  torsional elastic critical buckling load for element of length  $s$ ;
- $M_E$  critical moment for lateral-torsional buckling.

## **Axial load**

For axial load only, a strut with tension flange restraints at spacing  $s$ , will either buckle between restraints at axial compression:

$$P_{cr} = P_E = \frac{\pi^2 EI_z}{s^2}$$

or if  $P_E$  is higher than  $P_T$ , at a value

$$P_{cr} = P_T = \frac{1}{i_s^2} \left[ GI_T + P_E \left( \frac{d^2 + 4a^2}{4} \right) \left( \frac{ns}{L_t} \right)^2 + \text{terms} \right]$$

The additional terms depend on the torsional restraint to the strut that the tension flange restraints provide. If the torsional stiffness provided by the restraints is assumed to be zero, the critical load is given by:

$$P_T = \frac{1}{i_s^2} \left[ GI_T + \frac{\pi^2 EI_z}{s^2} \frac{s^2}{L_t^2} \left( \frac{d^2 + 4a^2}{4} \right) n^2 \right]$$

where  $n$  is the number of half-waves in the buckling mode. The number of half-waves is determined by the torsional stiffness provided by the restraints and the flexural characteristics of the columns but since the torsional stiffness of the restraints is assumed to be zero, the minimum value of  $P_T$  occurs when  $n = 1$ . Making the familiar substitution  $I_w = I_z \frac{d^2}{4}$  for an I or H section with bisymmetry, where  $d$  is the distance between the shear centres of the flanges and using EN 1993 notation gives:

$$N_{crT} = \frac{1}{i_s^2} \left[ GI_T + \frac{\pi^2 EI_z a^2}{L_t^2} + \frac{\pi^2 EI_w}{L_t^2} \right]$$

as given in BS EN 1993-1-1 Appendix BB (Clause BB.3.3.1).

### Pure bending

If the strut is subject to a pure bending moment, it will either buckle by twisting about the restrained longitudinal axis where:

$$M_{crT} = \frac{i_s^2}{2a} N_{crT}$$

or if  $M_E$ , the elastic critical moment for lateral-torsional buckling over the length  $s$ , is lower than  $M_{crT}$ , this will be the buckling mode. The formula for this is given by Horne and Ajmani as:

$$M_E = \sqrt{P_E} \sqrt{GI_T + P_E \frac{d^2}{4}}$$

or in EN 1993 notation,

$$M_{cr} = \sqrt{\frac{\pi^2 EI_z}{s^2}} \sqrt{GI_T + \frac{\pi^2 EI_w}{s^2}}$$

In previous SCI publications P147 and P164, this has been expressed as

$$M_{cr} = \frac{\pi^2 EI_z}{s^2} \sqrt{\frac{I_w}{I_z} + \frac{GI_T s^2}{\pi^2 EI_z}}$$

Rotations are zero at the intermediate restraint positions so buckling of the element between these positions can occur in the same manner as for any beam. Lateral-torsional buckling is therefore independent of the restraint axis about which the torsional buckling occurs. It is also clearly independent of the torsional stiffness provided by the intermediate restraints (because there are no rotations). The modes of buckling are shown in Figure D1.

In summary:

- Under axial load only, a strut will buckle either torsionally about the axis of restraint or flexurally between the lateral supports;
- Under uniform moment only, a strut will buckle either torsionally about the axis of restraint or in a lateral-torsional mode between the lateral supports.

The mode of buckling will depend on which mode has the lowest critical value.

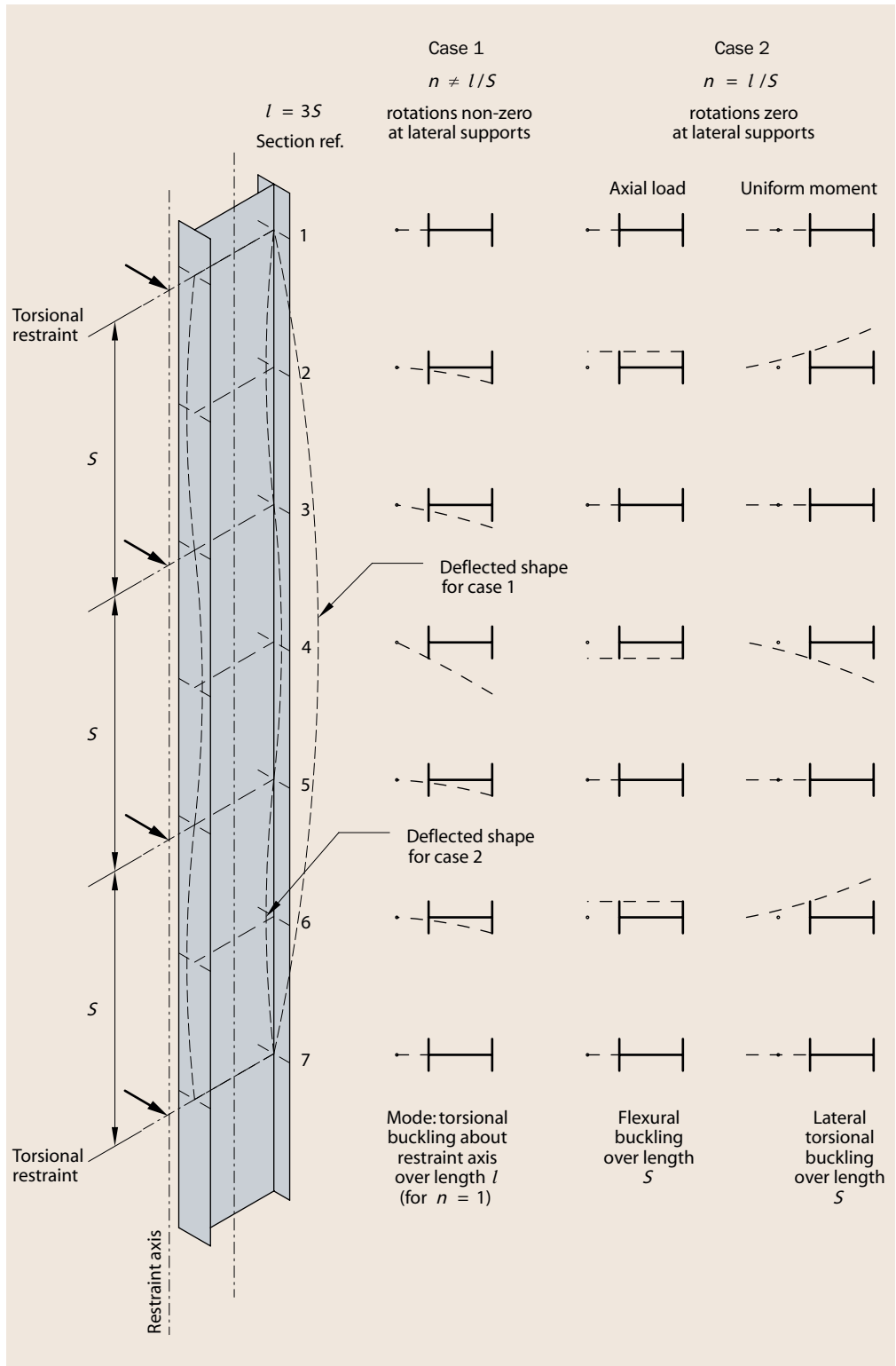


Figure D.1  
Effect of tension  
flange restraint on  
buckled mode shape

### Combined axial load and bending

Horne and Ajmani present a formula for interaction between axial force and pure moment for a strut of length  $s$  without intermediate supports which gives combinations of axial load and end moments at which the strut will buckle. The critical combinations are given by:

$$(P_E - P)(i^2 P_E P - M_E^2) + P_E M^2 = 0$$

If the axial load producing torsional buckling in an unsupported column of length  $s$  is denoted by  $P_{TC}$  then

$$P_{TC} = \frac{1}{i^2} \left[ GI_T + P_E \left( \frac{d^2}{4} \right) \right] = \frac{M_E^2}{P_E i^2}$$

Substituting in the interaction formula gives:

$$\left( \frac{M}{M_E} \right)^2 - \left( 1 - \frac{P}{P_E} \right) \left( 1 - \frac{P}{P_{TC}} \right) = 0$$

For an open section  $P_{TC}$  is bigger than  $P_E$  so if the axial force is increased from zero,  $M$  is zero when  $P = P_E$ . If  $P$  is zero, the equation is satisfied when  $M = M_E$ . For a 610 x 229 UKB 101 over a length of 1.8 m  $M_E = 5575$  kNm,  $P_E = 18640$  kN and  $P_{TC} = 27410$  kN. The interaction curve is as shown in Figure D.2:

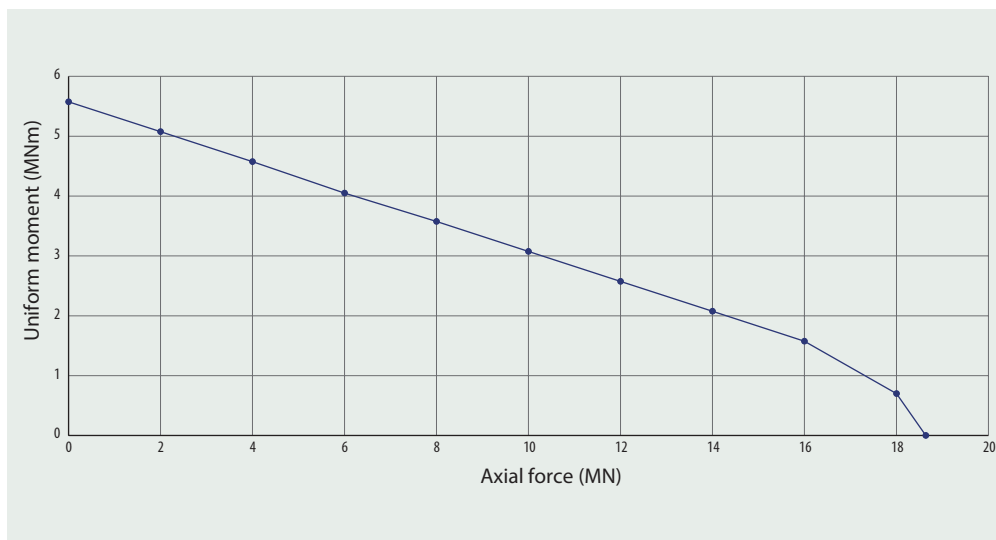


Figure D.2  
Interaction of  
axial load and  
uniform moment

The segment lengths for the buckling modes under axial load and bending are the same and there is a smooth interaction between axial load and bending so it is consistent to use equation 6.62 to verify the resistance of a segment.

### Critical buckling mode

Consider an open section column segment of length  $l$  with torsional restraints at the ends with an intermediate tension flange restraint at a distance  $s$  from one end. The tension flange restraint will result in buckling in a flexural mode over a length shorter

than  $l$ . If  $s = l/2$ , then the column will buckle in two half-waves of length  $s$ . The torsional buckling mode over the full length  $l$  will occur at a lower value of axial load and this will be the critical buckling mode. This is true whether the element is subject to axial load or uniform bending.

If  $s$  is increased and the critical load in flexural buckling over length  $s$  is compared with the critical load in torsional buckling over length  $l$ , at a certain value of  $s$ ,  $P_T = P_E$  and

$$\frac{l}{s} = \sqrt{\frac{1}{i_s^2 I_z} \left[ \frac{I_T l^2}{2.6 \pi^2} + I_W + I_z a \right]}$$

For the UKB section considered above,  $l/s = 1.082$  for a length  $l = 3.6$  m so that  $s$  is 92.4% of  $l$  i.e. about 3.33 m. The position of the intermediate restraint which would result in the flexural mode of buckling becoming critical is less than 300 mm from the end of the segment – a position which is not likely to be adopted. For practical purposes, where there is an intermediate restraint,  $P_E > P_T$  and the buckling mode will always be torsional. A similar result pertains where pure moment is applied.

The element can be checked for combined axial load and bending moment considering the segment length between torsional restraints and using the torsional critical axial load and the torsional critical moment to determine the relevant non-dimensional slenderness. For completeness, the element can also be checked for axial load and moment considering the length between lateral restraints and using the flexural critical axial load (Euler load) and the lateral-torsional critical moment to determine the relevant non-dimensional slenderness.





# APPENDIX E - MEMBER IMPERFECTIONS

It has been normal practice in the UK to neglect the effect of in-plane member imperfections on the in-plane stability of portal frames. However, BS EN 1993-1-1 states that unless all second order effects and imperfections are accounted for in the global analysis, a buckling check of individual members must be completed. Member checks in the plane of bending about the major axis are carried out using Equation 6.61 in Clause 6.6.3. However, the expression requires an in-plane buckling length and a lateral-torsional buckling resistance for the complete member. These values are not easily determined.

Second order effects in overall frame behaviour due to initial frame imperfections can be allowed for by applying equivalent horizontal forces. Frame and member flexibility ( $P-\Delta$  and  $P-\delta$  effects) can also be allowed for in a global analysis in a straightforward manner. If second order effects due to initial member imperfections can be shown to be negligible, once the global analysis has been carried out, only cross-section checks are required in plane to verify members.

In order to determine whether the general provisions of EN 1993 need to be applied, an extensive programme of finite element analysis into the effect of member imperfections has been carried out. A wide range of portal frames has been analysed using second order analysis software to determine the effect of member imperfections on the elastic critical load factor of the frame. The imperfections in the members can be in the opposite sense to the member deflections under load, or in the same sense so imperfections in both senses had to be examined.

The results of the programme of analysis show that elastic critical load factors calculated with and without member imperfections are almost always within 0.3% of each other for all the frames examined. The effect on the plastic collapse factor  $\lambda_p$  was also examined and similarly showed that very little change occurred with the inclusion of member imperfections.

The conclusion of this work is that the effects of member imperfections are small enough to be ignored. The finite element work has therefore justified traditional UK practice.

It is sufficient, therefore, to carry out second order analysis which takes into account:

- Frame imperfections.
- Frame flexibility.
- Member flexibility.

In-plane member imperfections are ignored. Members are verified in plane by carrying out a cross-section check and out of plane by using Equation 6.62 and Annex BB.3.

The amplified forces method proposed in the paper by Lim *et al* (Reference 30) can be used for both elastic and plastic analysis.







# APPENDIX F - WORKED EXAMPLES: COLUMN, RAFTER AND HAUNCH CHECKS

## F.1 Introduction

This worked example has been provided to illustrate the necessary verifications for columns, rafters and haunches next to a plastic hinge. Elastic verifications of adjacent segments are also included. As the examples are illustrative, they do not follow the usual design process where purlin and sheeting rail positions are set out to suit the cladding that has been chosen and the secondary steelwork positions adjusted and stays provided so that the verification of the elements is achieved. The calculations instead indicate where torsional and lateral restraints to the elements must be provided.

Clause references are to BS EN 1993-1-1 unless noted otherwise. Expressions from BS EN 1993-1-1 appear in brackets.

The examples demonstrating verification of the haunch require section property data at various cross-sections. In these examples, the properties have been calculated by spreadsheet, generally ignoring the root radii of the haunch cutting. In some cases, calculated properties exclude all radii. The software *LTBeamN* may be used to verify section property data for haunched sections, but see the note in Appendix B.3 regarding the monosymmetry index.

When selecting buckling curves for flexural or lateral torsional buckling, the haunched section has been described as a rolled section. This is because the haunch is cut from a rolled section and the welding (at the underside of the rafter haunch) is distant from the compression flange. Had a haunch been fabricated from plate, the buckling curves should be selected on the basis of a welded section.

### F1.1 Frame geometry

A frame analysis has been undertaken, including equivalent horizontal forces and allowing for second order effects. All bending moments, shears and axial forces are the design values.

The following sections have been selected:

Column:           762 × 267 × 147 UB, S355

Rafter:             610 × 229 × 101 UB, S355

The basic geometry of the frame is shown in Figure F.1.



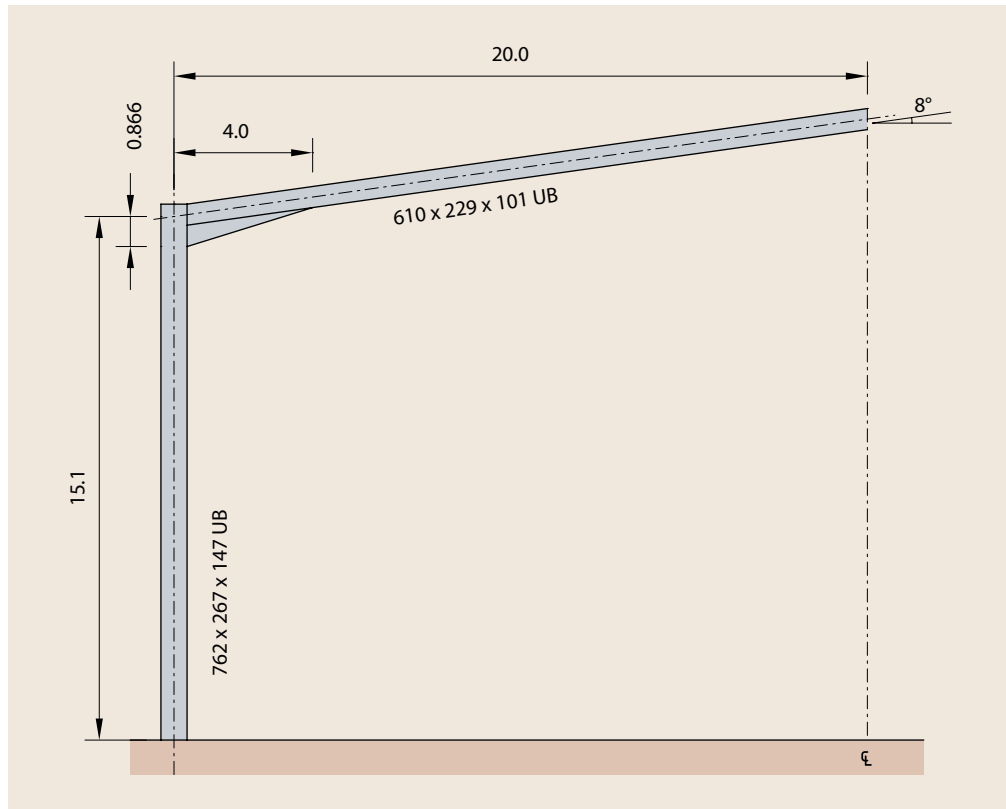


Figure F.1  
Basic frame  
geometry

The design bending moments in the column at the underside of the haunch and in the rafter at the sharp end of the haunch are less than  $M_p$  of the section. Although the plastic resistance of the elements is not in fact reached, the verification of segments adjacent to an assumed plastic hinge is demonstrated in the later sections.

## F.2 Column verification

The column is shown in Figure F.2. A torsional restraint is provided at the underside of the haunch. In all cases (elastic or plastic), a restraint to the inside flange, at the underside of the haunch is required.

Initially, the section is classified and a cross-section check is completed.

Two options are available to verify the design. It has been assumed that there is a plastic hinge at the underside of the haunch and a plastic check used to determine the distance to the next torsional restraint (Section F2.3 below). The remaining section of the column has been verified using elastic checks (Section F2.4). Alternatively, as the column does not contain a plastic hinge, all segments between torsional restraints could be verified using elastic checks.

Advantage of the benefit of restraint to the tension flange (by the sheeting rails on the outside flange) is taken.

*All references are to  
BS EN 1993-1-1  
unless noted  
References to  
expressions are  
given in brackets*

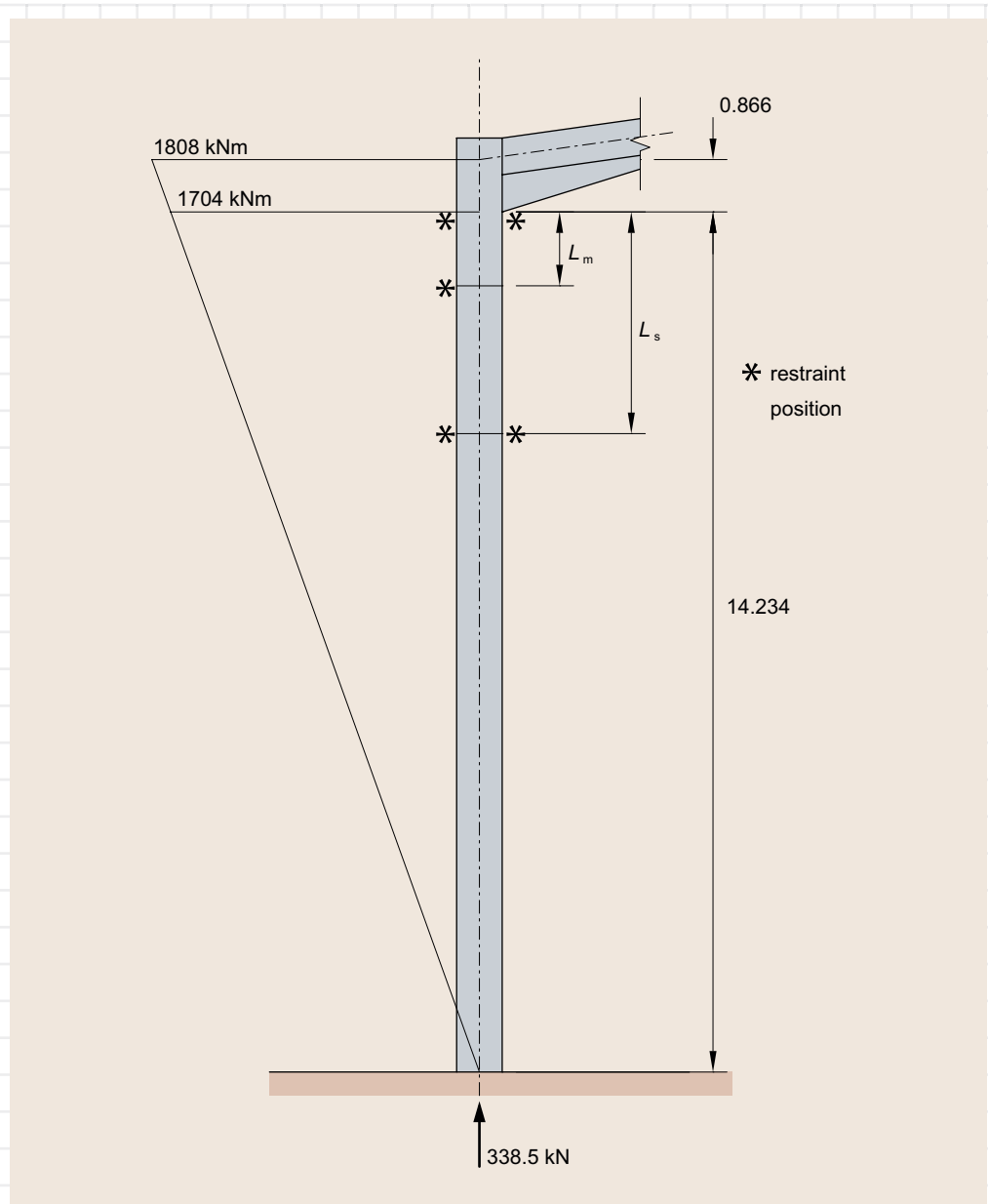


Figure F.2  
Column geometry  
and bending  
moment

## F2.1 Classification of column under axial load and bending

A column section has been chosen following the in-plane plastic analysis of a portal frame.

Section size: 762 x 267 x 147 UB, S355

$$\begin{aligned}
 M_{\max} &= 1808 \text{ kNm at eaves; } M = 1704 \text{ kNm at underside of haunch} \\
 N_{\text{Ed}} &= 338.5 \text{ kN (base)} \\
 V &= 121.3 \text{ kN}
 \end{aligned}$$

### P363 Properties

$$\begin{aligned}
 h &= 754 \text{ mm} & b &= 265.2 \text{ mm} \\
 t_f &= 17.5 \text{ mm} & t_w &= 12.8 \text{ mm} \\
 A &= 187 \text{ cm}^2 & h_w &= d = 686 \text{ mm}
 \end{aligned}$$

$$\begin{aligned}
 I_z &= 5460 \text{ cm}^4 \\
 i_y &= 30 \text{ cm} \\
 i_z &= 5.4 \text{ cm} \\
 W_{pl,y} &= 5160 \text{ cm}^3 \\
 I_w &= 7.4 \text{ dm}^6 = 7.4 \times 10^{12} \text{ mm}^6 \\
 I_T &= 159 \text{ cm}^4
 \end{aligned}$$

BS EN 10025 Yield strength  $f_y = 345 \text{ N/mm}^2$  ( $16 < t_f \leq 40$ )

### Section classification

P363 D-172

$$\begin{aligned}
 N_{pl,Rd} &= 6450 \text{ kN} \\
 N_{Ed} / N_{pl,Rd} &= 338.5 / 6450 = 0.0525 \\
 N_{Ed} / N_{pl,Rd} &< 0.110 \\
 \text{Section is at least Class 2}
 \end{aligned}$$

## F2.2 Cross-section check

The plastic section modulus is used because the section is at least Class 2.

$$M_{pl,y,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} = 5160 \times 10^3 \times 345 = 1.78 \times 10^9 \text{ Nmm}$$

$$M_{pl,y,Rd} = 1780 \text{ kNm}$$

Consider the bending resistance in the presence of axial load.

6.2.9.1(4) No reduction in bending resistance for axial load is necessary if  $N_{Ed} \leq 0.25 N_{pl,Rd}$  and

$$N_{Ed} \leq \frac{0.5 h_w t_w f_y}{\gamma_{M0}}$$

$$0.25 N_{pl,Rd} = 0.25 \times 6450 = 1613 \text{ kN}$$

$$\frac{0.5 h_w t_w f_y}{\gamma_{M0}} = \frac{0.5 \times 686 \times 12.8 \times 345}{1.0} = 1.51 \times 10^6 = 1515 \text{ kN}$$

$$N_{Ed} = 338.5 < \begin{cases} 1613 \\ 1515 \end{cases}$$

No reduction in bending resistance need be made.

$$M_{N,Rd} = 1780 \text{ kNm}$$

6.2.8 By inspection, no reduction in bending resistance need be made for shear as the shear force is low.

$$M_{Ed} = 1704 \text{ kNm} < 1780 \text{ kNm} \text{ so the cross-section is adequate.}$$

## F2.3 Verification to the first torsional restraint

The position of the first torsional restraint is given by  $L_s$  in BB.3.1.2(2) covering members with a linear moment gradient, which demands a restraint at the plastic hinge and one or more intermediate restraints at a maximum spacing of  $L_m$ . Following the determination of the restraint positions, the segment between the lateral restraint and the first torsional restraint is verified using an elastic check.

### F2.3.1 Position of first torsional restraint

A plastic check is being used to determine the critical length: a plastic hinge is assumed at one end of the segment. In fact the maximum bending moment is less than the plastic moment and an elastic check could be carried out. If the plastic check is satisfactory, an elastic check is too.

$$\text{BB.3.1.2(2)B} \quad L_s = \sqrt{C_m} L_k \left( \frac{M_{pl,y,Rk}}{M_{N,y,Rk} + aN_{Ed}} \right)^{0.5} \quad (\text{BB.7})$$

Note: the equation in EN1993 omits the 0.5 power in error. The equation given above is the same form as that in BS 5950 Section G.3.3.1

$$L_k = \frac{\left( 5.4 + \frac{600 f_y}{E} \right) \left( \frac{h}{t_f} \right) i_z}{\sqrt{5.4 \left( \frac{f_y}{E} \right) \left( \frac{h}{t_f} \right)^2 - 1}}$$
$$L_k = \frac{\left( 5.4 + \frac{600 \times 345}{210000} \right) \left( \frac{754}{17.5} \right) \times 54}{\sqrt{5.4 \left( \frac{345}{210000} \right) \left( \frac{754}{17.5} \right)^2 - 1}} = 3780 \text{ mm}$$

### Modification factor $C_m$

$$(\text{BB.13}) \quad C_m = \frac{1}{B_0 + B_1 \beta_t + B_2 \beta_t^2}$$

Trial 1.  $L_s = 4.0 \text{ m}$

A trial length is required because the modification factor depends on the ratio of the bending moments at the ends of the segment.

Bending moment at proposed restraint position:

$$M = 1704 \times \frac{10.234}{14.234} = 1225 \text{ kNm}$$

$$\beta_t = \frac{1225}{1704} = 0.719$$

$$B_o = \frac{1+10\eta}{1+20\eta} \quad B_1 = \frac{5\sqrt{\eta}}{\pi+10\sqrt{\eta}}$$

$$B_2 = \frac{0.5}{1+\pi\sqrt{\eta}} - \frac{0.5}{1+20\eta}$$

$$\eta = \frac{N_{crE}}{N_{crT}}$$

$$N_{crE} = \frac{\pi^2 EI_z}{L_t^2} = \frac{\pi^2 \times 210000 \times 5460 \times 10^4}{4000^2} \times 10^{-3} = 7073 \text{ kN}$$

$$N_{crT} = \frac{1}{i_s^2} \left[ \frac{\pi^2 EI_z a^2}{L_t^2} + \frac{\pi^2 EI_w}{L_t^2} + GI_T \right]$$

$$= \frac{1}{i_s^2} \left[ N_{crE} a^2 + N_{crE} \frac{I_w}{I_z} + GI_T \right]$$

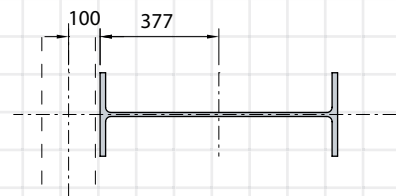
$$i_s^2 = i_y^2 + i_z^2 + a^2$$

$a$  is the offset between the axis of rotation and the centroid of the column section.

Assume centroid of sheeting rail is 100 mm off the flange face.

$$a = \frac{h}{2} + 100 = \frac{754}{2} + 100 = 477 \text{ mm}$$

$$i_s^2 = 300^2 + 54^2 + 477^2 = 320 \times 10^3 \text{ mm}^2$$



Using units of kN and mm:

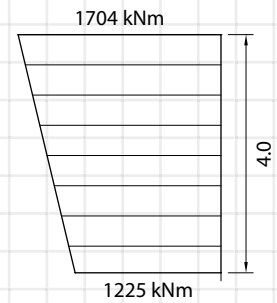
$$N_{crT} = \frac{1}{320 \times 10^3} \left[ 7073 \times 477^2 + 7073 \times \frac{7.4 \times 10^{12}}{5460 \times 10^4} + \frac{210 \times 159 \times 10^4}{2.6} \right] = 8425 \text{ kN}$$

(Note  $E/G = 2 \times (1 + \nu) = 2 \times (1 + 0.3) = 2.6$ )

$$\eta = \frac{N_{crE}}{N_{crT}} = \frac{7073}{8425} = 0.840 \quad \sqrt{\eta} = 0.917$$

$$B_o = \frac{1+10 \times 0.84}{1+20 \times 0.84} = 0.528$$

$$B_1 = \frac{5 \times 0.917}{\pi + 10 \times 0.917} = 0.372$$



$$B_2 = \frac{0.5}{1 + \pi \times 0.917} - \frac{0.5}{1 + 20 \times 0.840} = 0.103$$

$$C_m = \frac{1}{0.528 + 0.372 \times 0.719 + 0.101 \times 0.719^2} = 1.18$$

Substituting values in the formula for  $L_s$

$$L_s = \sqrt{1.18} \times 3780 \times \left( \frac{1780}{1780 + 0.477 \times 338.5} \right)^{0.5} = 3930 \text{ mm}$$

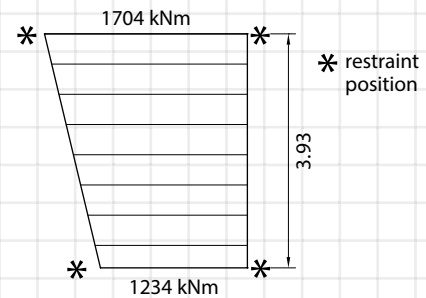
This length is less than the 4.0 m originally assumed so a reduced spacing is required.

Trial 2.  $L_s = 3.93 \text{ m}$

$$M = 1704 \times \frac{10.304}{14.234} = 1234 \text{ kNm}$$

$$\beta_t = 0.724$$

$$N_{crE} = 7327 \text{ kN}$$



$$N_{crT} = \frac{1}{320 \times 10^3} \left[ 7327 \times 477^2 + 7327 \times \frac{7.4 \times 10^{12}}{5460 \times 10^4} + \frac{210 \times 159 \times 10^4}{2.6} \right] = 8714 \text{ kN}$$

$\eta = 0.841$ .  $B_0$ ,  $B_1$  and  $B_2$  are as previously calculated.

$$C_m = \frac{1}{0.528 + 0.372 \times 0.724 + 0.101 \times 0.724^2} = 1.18 \text{ (as before)}$$

$$(BB.7) \quad L_s = \sqrt{1.18} \times 3780 \times \left( \frac{1780}{1780 + 0.477 \times 338.5} \right)^{0.5} = 3930 \text{ mm}$$

Adopt  $L_s = 3.93 \text{ m}$

### F2.3.2 Position of intermediate restraint

$$(BB.5) \quad L_m = \frac{38i_z}{\sqrt{\frac{1}{57.4} \left( \frac{N_{Ed}}{A} \right) + \frac{1}{756C_1^2} \left( \frac{W_{ply}^2}{AI_T} \right) \left( \frac{f_y}{235} \right)^2}}$$

Trial 1.  $L_m = 1.3 \text{ m}$

A trial length is necessary because the moment correction factor depends on the ratio of the bending moments at the ends of the segment being considered.



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Bending moment at proposed restraint position:

$$M = 1704 \times \frac{12.934}{14.234} = 1548 \text{ kNm}$$

Moment correction factor:

$$C_1 = 1.77 - 0.88\psi + 0.11\psi^2$$

$$\psi = \frac{1548}{1704} = 0.908$$

$$C_1 = 1.77 - 0.88 \times 0.908 + 0.11 \times 0.908^2 = 1.06$$

Calculate  $L_m$

$$\frac{N_{Ed}}{57.4 \times A} = \frac{338.5 \times 10^3}{57.4 \times 18700} = 0.315$$

$$\frac{W_{ply}^2}{AI_T} = \frac{(5160 \times 10^3)^2}{18700 \times 159 \times 10^4} = 895$$

$$\left(\frac{f_y}{235}\right)^2 = \left(\frac{345}{235}\right)^2 = 2.16$$

$$L_m = \frac{38 \times 54}{\sqrt{0.315 + \frac{1}{756 \times 1.06^2} \times 895 \times 2.16}} = 1275 \text{ mm}$$

The calculated length is less than the initial guess so a smaller length must be checked.

Trial 2.  $L_m = 1.270 \text{ m}$

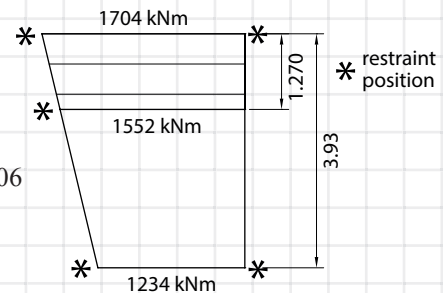
$$M = 1704 \times \frac{12.964}{14.234} = 1552 \text{ kNm}$$

$$\psi = 0.911$$

$$C_1 = 1.77 - 0.88 \times 0.911 + 0.11 \times 0.911^2 = 1.06$$

$$L_m = 1275 \text{ mm (as before)}$$

Adopt  $L_m = 1.27 \text{ m}$



### F2.3.3 Verification between lateral restraint and first torsional restraint

There is no plastic hinge within this length so an elastic check may be used.

$$\text{Length} = 3.93 - 1.27 = 2.66 \text{ m}$$

The spacing of tension flange restraints is verified using Equation 6.62

$$(6.62) \quad \frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

$$6.3.1.2 \quad N_{b,z,Rd} = \frac{\chi_z A f_y}{\gamma_{M1}}; \quad \chi_z = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}}$$

$$\phi = 0.5 \left[ 1 + \alpha (\bar{\lambda}_z - 0.2) + \bar{\lambda}^2 \right]$$

$$\bar{\lambda} = \frac{L_{cr}}{i_z} \frac{1}{\lambda_1} \quad \text{where } \lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

$$6.3.1.3 \quad \lambda_1 = \pi \sqrt{\frac{210 \times 10^3}{345}} = 77.5$$

$$\bar{\lambda} = \frac{2660}{54 \times 77.5} = 0.636$$

Table 6.2

For buckling about z-z axis use curve b, (rolled section:  $h/b > 1.2$ ;  $t_f < 40$ );

Table 6.3

$$\alpha = 0.34$$

$$\phi = 0.5 \times [1 + 0.34 \times (0.636 - 0.2) + 0.636^2] = 0.776$$

$$\chi_z = \frac{1}{0.776 + \sqrt{0.776^2 - 0.636^2}} = 0.819$$

$$6.3.2.1 \quad N_{b,z,Rd} = \frac{0.819 \times 18700 \times 345}{1.0 \times 10^3} = 5284 \text{ kN}$$

$$M_{b,Rd} = \frac{\chi_{LT} W_{pl,y} f_y}{\gamma_{M1}}$$

The plastic section modulus is used because the section is at least Class 2.

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl} f_y}{M_{cr}}} \quad \chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}}$$

$$6.3.2.3 \quad \phi_{LT} = 0.5 [1 + \alpha_{LT} (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \bar{\lambda}_{LT}^2]$$

$$\bar{\lambda}_{LT,0} = 0.4; \beta = 0.75$$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_T}{\pi^2 E I_z}}$$

Determine  $C_1$ .

$$\text{Moment ratio} = \frac{1234}{1552} = 0.795$$

$$C_1 = 1.77 - 0.88\psi + 0.11\psi^2$$

$$C_1 = 1.77 - 0.88 \times 0.796 + 0.11 \times 0.796^2 = 1.14$$

$$M_{cr} = 1.14 \times \frac{\pi^2 \times 210 \times 10^3 \times 5460 \times 10^4}{2660^2} \times \sqrt{\frac{7.4 \times 10^{12}}{5460 \times 10^4} + \frac{2660^2 \times 159 \times 10^4}{2.6 \pi^2 \times 5460 \times 10^4}}$$

$$= 6908 \text{ kNm}$$

(Note  $E/G = 2 \times (1 + \nu) = 2 \times (1 + 0.3) = 2.6$ )

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} f_y}{M_{cr}}} = \sqrt{\frac{5160 \times 10^3 \times 345}{6908 \times 10^6}} = 0.508$$

NA.2.17  
Table 6.3

$$h/b = 2.84$$

For a rolled section where  $2 < h/b < 3.1$ , use buckling curve c therefore  $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5[1 + 0.49 \times (0.508 - 0.4) + 0.75 \times 0.508^2] = 0.623$$

$$\chi_{LT} = \frac{1}{0.623 + \sqrt{0.623^2 - 0.75 \times 0.508^2}} = 0.940$$

Because  $C_1$  is low, the benefit of  $f$  (as given in 6.3.2.3(2)) is conservatively ignored.

$$M_{b,Rd} = \frac{0.940 \times 5160 \times 10^3 \times 345}{1.0} \times 10^{-3} = 1673 \text{ kNm}$$

### Calculate $k_{zy}$

Table B.2

$$k_{zy} = \text{Max} \left[ \left[ 1 - \frac{0.1 \bar{\lambda}_z}{(C_{mLT} - 0.25)} \cdot \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right], \left[ 1 - \frac{0.1}{(C_{mLT} - 0.25)} \cdot \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right] \right]$$

Table B.3

$$C_{mLT} = 0.6 + 0.4\psi \geq 0.4$$

$$\psi = 0.795; C_{mLT} = 0.918$$

$$k_{zy} = \text{Max} \left[ \left[ 1 - \frac{0.1 \times 0.636}{(0.918 - 0.25)} \times \frac{338.5}{5284} \right], \left[ 1 - \frac{0.1}{(0.918 - 0.25)} \times \frac{338.5}{5284} \right] \right]$$

$$= \text{Max} [0.994, 0.990] = 0.994$$

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} = \frac{338.5}{5284} + 0.994 \times \frac{1552}{1673} = 0.986 < 1.0 \quad \text{OK}$$

Note: This is as expected because the critical length check determines  $L_s$  and confirms lateral torsional buckling in the presence of axial load can be ignored over this length.

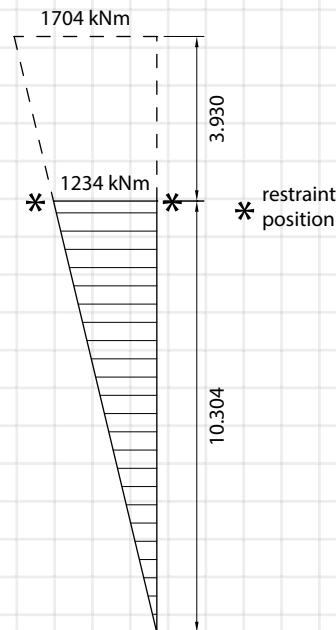
The bending moment diagram is triangular and reduces to zero at the base of the column because pinned feet are assumed in the analysis. This means that lower down the column (below the first torsional restraint) the combined bending moment and axial load on the column are less onerous than those just examined. If the spacing of intermediate restraints is less than 2.66 m in all segments of the column, lateral-torsional buckling will not occur between lateral restraints and tension flange restraint between torsional restraints can be assumed.

## F2.4 Verification from first torsional restraint to base

Initially, the entire length from the torsional restraint at 10.304 m to the base is checked. It is found in Section F2.4.1 that even utilizing the benefit of tension flange restraint, the design moment exceeds the LTB resistance – so no combined check is completed. An additional torsional restraint is introduced in Section F2.4.2, and both segments checked.

### F2.4.1 Initial check 10.304 m to base

It was demonstrated in 2.6 that sheeting rails at no greater than 2.66 m spacing provide tension flange restraint. Equation 6.62 is used to examine if the 10.304 m segment to the base is stable without further torsional restraints.



$$6.3.1.2 \quad \bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}}$$

In this case,  $N_{cr}$  is the elastic torsional buckling force because flexural buckling is prevented by the tension flange restraints.

$$N_{\text{crT}} = \frac{1}{i_s^2} \left( \frac{\pi^2 EI_z a^2}{L_t^2} + \frac{\pi^2 EI_w}{L_t^2} + GI_T \right)$$

$$a = 477 \text{ mm}; \quad i_s^2 = 320 \times 10^3 \text{ mm}^2.$$

$$N_{\text{crT}} = N_{\text{crE}} \left[ \frac{a^2}{i_s^2} + \frac{I_w}{i_s^2 I_z} \right] + \frac{GI_T}{i_s^2}$$

$$N_{\text{crE}} = \frac{\pi^2 \times 210 \times 10^3 \times 5460 \times 10^4}{10304^2} \times 10^{-3} = 1066 \text{ kN}$$

Using units of kN and mm:

$$N_{\text{crT}} = 1066 \times \left[ \frac{477^2}{320 \times 10^3} + \frac{7.4 \times 10^{12}}{320 \times 10^3 \times 5460 \times 10^4} \right] + \frac{210 \times 159 \times 10^4}{2.6 \times 320 \times 10^3}$$

(Note  $E/G = 2 \times (1 + \nu) = 2 \times (1 + 0.3) = 2.6$ )

$$N_{\text{crT}} = 1066 \times (0.711 + 0.424) + 401 = 1611 \text{ kN}$$

$$\bar{\lambda}_T = \sqrt{\frac{18700 \times 345}{1611 \times 10^3}} = 2.0$$

6.3.1.4(3) Note that for torsional buckling use Table 6.2 and the curve relating to the z axis so  $\alpha = 0.34$  as before.

$$\phi = 0.5[1 + 0.34 \times (2.0 - 0.2) + 2.0^2] = 2.806$$

$$\chi = \frac{1}{2.806 + \sqrt{2.806^2 - 2.0^2}} = 0.209$$

$$N_{\text{b,Rd}} = \frac{0.209 \times 18700 \times 345}{1.0} \times 10^{-3} = 1348 \text{ kN}$$

### Determine $M_{\text{crit}}$

For uniform elements with tension flange restraint

$$M_{\text{cr}} = C_m M_{\text{cr0}}$$

$$\text{where } M_{\text{cr0}} = \frac{i_s^2}{2a} \cdot N_{\text{crT}}$$

$$M_{\text{b,Rd}} = \chi_{\text{LT}} \frac{W_{\text{ply}} f_y}{\gamma_{\text{M1}}}$$

$$\text{BB.3.3.1 } C_m = \frac{1}{B_0 + B_1 \beta_t + B_2 \beta_t^2} \quad \beta_t = 0 \text{ (triangular distribution)}$$

$$\eta = \frac{N_{\text{crE}}}{N_{\text{crT}}} = \frac{1066}{1611} = 0.662; \quad \sqrt{\eta} = 0.814$$

$B_0, B_1$  and  $B_2$  are calculated in the same way as in Section 2.3

$$B_0 = 0.535; \quad B_1 = 0.361; \quad B_2 = 0.096$$

$$C_m = \frac{1}{B_0} = \frac{1}{0.535} = 1.87$$

$$M_{cr} = 1.87 \times \frac{320 \times 10^3}{2 \times 477} \times 1611 \times 10^3 \times 10^{-6} = 1010 \text{ kNm}$$

$M_{Ed} = 1234 > 1010 \therefore$  fails even before calculating  $M_{b,Rd}$ . Intermediate torsional restraints are therefore required.

#### F2.4.2 10.304 to intermediate restraint at 5.304

Introduce a torsional restraint 5.0 m below the torsional restraint at 10.304 m, i.e. at 5.304 m above the base.

Moment at this height:

$$M = \frac{1234 \times 5.304}{10.304} = 635 \text{ kNm}$$

Consider the upper segment.

At 10.304 m  $N_{cr,E} = 1066 \text{ kN}$

$$N_{crE} = 1066 \times \frac{10304^2}{5000^2} = 4527 \text{ kN}$$

$$N_{crT} = 4527 \times (0.711 + 0.424) + 401 = 5539 \text{ kN}$$

$$\eta = \frac{N_{crE}}{N_{crT}} = \frac{4527}{5539} = 0.817; \quad \sqrt{\eta} = 0.904$$

$$\bar{\lambda}_T = \sqrt{\frac{18700 \times 345}{5539 \times 10^3}} = 1.08$$

As before, for torsional buckling choose the curve relating to the z axis so  $\alpha = 0.34$ .

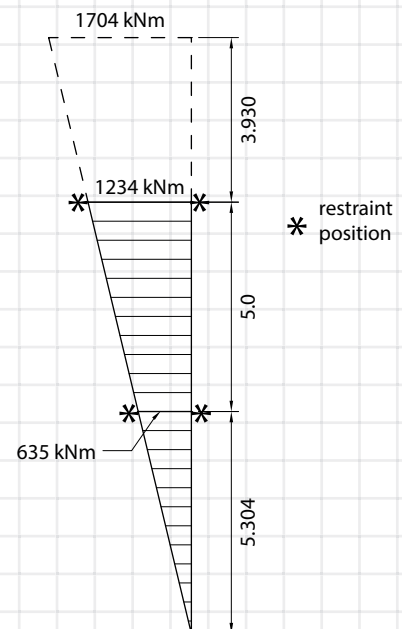
$$\phi = 0.5[1 + 0.34 \times (1.08 - 0.2) + 1.08^2] = 1.23$$

$$\chi = \frac{1}{1.23 + \sqrt{1.23^2 - 1.08^2}} = 0.550$$

$$N_{b,Rd} = \frac{0.550 \times 18700 \times 345}{1.0} \times 10^{-3} = 3548 \text{ kN}$$

Determine  $M_{crT}$

$$M_{crT} = C_m M_{cr0}$$





$$\text{where } M_{cr0} = \frac{i_s^2}{2a} \cdot N_{crT}$$

$$BB.3.3.1 \quad C_m = \frac{1}{B_0 + B_1\beta_t + B_2\beta_t^2}$$

$$\beta_t = \frac{635}{1234} = 0.515$$

$B_0$ ,  $B_1$  and  $B_2$  are calculated in the same way as in Section 2.3.

$$B_0 = 0.529; \quad B_1 = 0.371; \quad B_2 = 0.101$$

$$C_m = \frac{1}{0.529 + 0.371 \times 0.515 + 0.101 \times 0.515^2} = 1.34$$

$$M_{crT} = 1.34 \times \frac{320 \times 10^3}{2 \times 477} \times 5539 \times 10^{-3} = 2490 \text{ kNm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{5160 \times 10^3 \times 345}{2490 \times 10^6}} = 0.845$$

NA 2.17 As before, for a rolled section where  $h/b > 2$ , use buckling curve c therefore

6.3.2.3  $\alpha_{LT} = 0.49$

Table 6.3

$$\phi_{LT} = 0.5 [1 + 0.49 \times (0.845 - 0.4) + 0.75 \times 0.845^2] = 0.877$$

$$\chi_{LT} = \frac{1}{0.877 + \sqrt{0.877^2 - 0.75 \times 0.845^2}} = 0.735$$

Determine  $\chi_{LTmod}$

$$\psi = 0.515;$$

$$C_1 = 1.77 - 0.88 \times 0.515 + 0.11 \times 0.515^2 = 1.35$$

NA 2.18  $k_c = 1/\sqrt{C_1} = 0.861$

6.3.2.3(2)  $f = 1 - 0.5 (1 - 0.861) [1 - 2.0 (0.845 - 0.8)^2] = 0.931$

$$\chi_{LTmod} = \frac{0.735}{0.931} = 0.789$$

$$M_{b,Rd} = 0.789 \times 5160 \times 10^3 \times 345 \times 10^{-6} = 1405 \text{ kNm}$$

Check interaction using Equation 6.62

6.3.3  $\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$

(6.62)

$$C_{mLT} = 0.6 + 0.4\psi \geq 0.4$$

$$\psi = 0.515$$

Table B.3

$$C_{mLT} = 0.806$$

Table B.2

$$k_{zy} = \text{Max} \left[ \begin{array}{l} 1 - \frac{0.1 \times 1.08}{(0.806 - 0.25)} \times \frac{338.5}{3548} \\ 1 - \frac{1}{(0.806 - 0.25)} \times \frac{338.5}{3548} \end{array} \right]$$

Note,  $\bar{\lambda}_T$  has been used in the formula for  $k_{zy}$  because the tension flange restraints result in a torsional buckling mode not a flexural one.

$$= \text{Max} [0.981, 0.983] = 0.983$$

$$\frac{338.5}{3548} + 0.983 \times \frac{1234}{1405} = 0.959 \leq 1.0 \quad \text{OK}$$

Torsional restraint at 5.304 above base: upper segment is adequate.

An intermediate lateral restraint is required at a spacing of less than 2.66 m to provide tension flange restraint as discussed in Section F2.3.3. One intermediate restraint is sufficient. The spacing of sheeting rails would normally be less than this.

#### **F2.4.3 Intermediate restraint at 5.304 m to base**

An intermediate lateral restraint is required at a spacing of less than 2.66 m to provide tension flange restraint as discussed in Section F2.3.3. One intermediate restraint will achieve this.

By inspection, as the maximum moment is lower and  $M_{cr}$  is higher, the lateral torsional resistance is higher, the lower segment is also adequate.

## F2.5 Column verification – Summary

The lateral and torsional restraints provided to the column and cross-references to the checks completed are shown in Figure F.3.

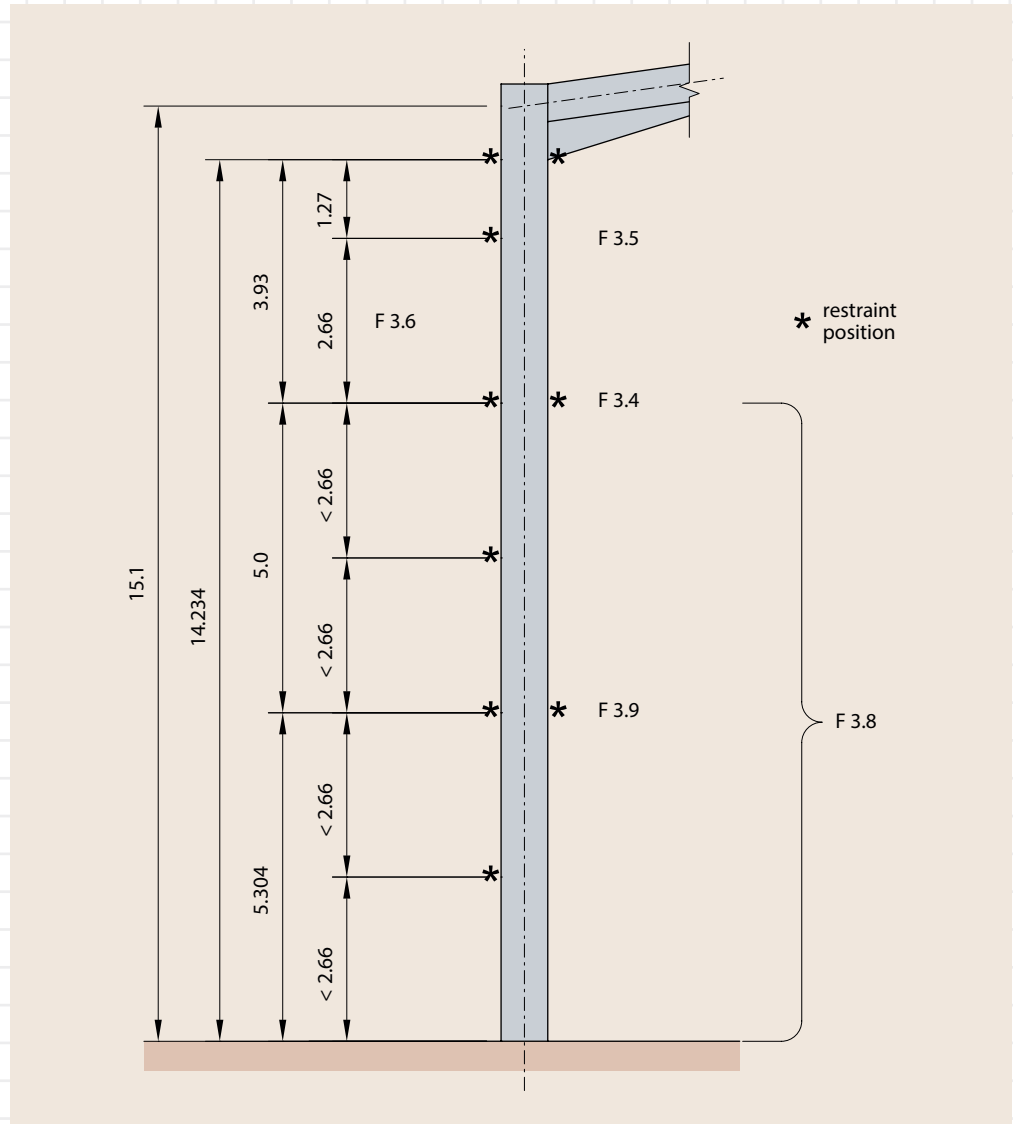


Figure F.3  
Column: summary  
of restraints

### F.3 Rafter verification

The rafter is shown in Figure F.4.

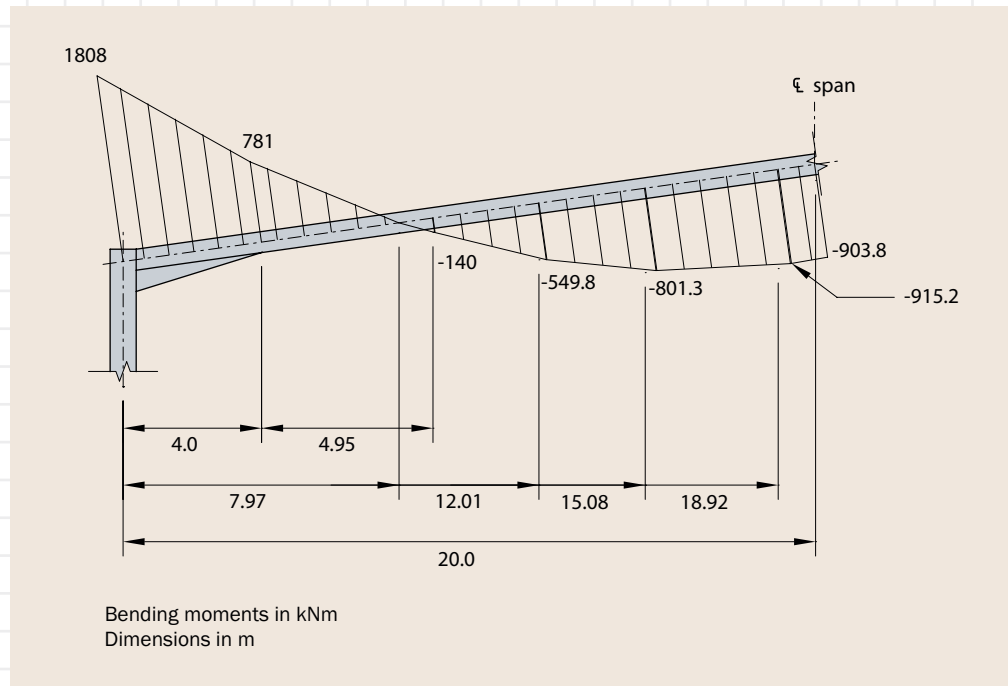


Figure F.4  
Rafter geometry  
and bending  
moment

All references  
are to  
BS EN 1993-1-1  
unless noted

Although the moment at the sharp end of the haunch (781 kNm) is less than  $M_p$ , checks of the rafter haunch adjacent to a plastic hinge are shown in Sections F3.3 and F3.5 to demonstrate the process. If there are no plastic hinges in the rafter, elastic checks should be completed by verifying expression 6.62 of BS EN 1993-1-1; examples are provided in P397.

Firstly the rafter is classified and a cross-sectional check is completed. Then, in Section F3.5.4, assuming there is a plastic hinge at the sharp end of the haunch, the position of the first torsional restraint is established at a distance  $L_s$ .

#### F3.1 Rafter classification under axial load and bending

A rafter section has been chosen following the in-plane plastic analysis of a portal frame.

Section size: 610 × 229 × 101 UB, S355

$N_{Ed}$  = 120.7 kN at apex       $M_{max}$  = 1808 kNm at the rafter/column centre  
line intersection.

$N_{Ed}$  = 163.6 kN at eaves       $M_{y,Ed}$  = 781 kNm at end of haunch (hogging).

$V_{Ed}$  = 287.2 kN at eaves       $M_{y,Ed}$  = 915.2 kNm at 18.918 m (sagging)

Point of contraflexure at 7.97 m.

Rafter slope = 8 degrees.

**Properties**

P363	$h$	$= 602.6 \text{ mm}$	$b$	$= 227.6 \text{ mm}$
	$t_f$	$= 14.8 \text{ mm}$	$t_w$	$= 10.5 \text{ mm}$
	$A$	$= 129 \text{ cm}^2$	$h_w$	$= d = 547.6 \text{ mm}$
	$I_z$	$= 2910 \text{ cm}^4$	$r$	$= 12.7 \text{ mm}$
	$i_y$	$= 24.2 \text{ cm}$		
	$i_z$	$= 4.75 \text{ cm}$		
	$W_{pl,y}$	$= 2880 \text{ cm}^3$	$I_y$	$= 75800 \text{ cm}^4$
	$I_w$	$= 2.52 \text{ dm}^6$		
	$I_T$	$= 77 \text{ cm}^4$		

Yield strength  $f_y = 355 \text{ N/mm}^2$  ( $t \leq 16$ )

Haunch cutting is the same as rafter.

**Section classification**

P363 D-177  $N_{pl,Rd} = 4850 \text{ kN}$   
 $N_{Ed} / N_{pl,Rd} = 163.6 / 4850 = 0.036 < 0.111$

Section is at least Class 2.

**F3.2 Cross-sectional check of rafter**

6.2.5  $M_{pl,Rd} = \frac{W_{pl,y} f_y}{\gamma_{M0}} = \frac{2880 \times 10^3 \times 355 \times 10^{-6}}{1.0} = 1022 \text{ kNm}$

$M_{Ed} = 915.2 \text{ kNm}$

Consider the bending resistance in the presence of axial load.

6.2.9.1(4) No reduction in bending resistance for axial load is necessary if  $N_{Ed} \leq 0.25 N_{pl,Rd}$  and

$$N_{Ed} \leq \frac{0.5 h_w t_w f_y}{\gamma_{M0}}$$

$$0.25 N_{pl,Rd} = 0.25 \times 4850 = 1145 \text{ kN}$$

$$\frac{0.5 h_w t_w f_y}{\gamma_{M0}} = \frac{0.5 \times 547.6 \times 10.5 \times 355}{1.0} \times 10^{-3} = 1020 \text{ kN}$$

$$N_{Ed} = 163.6 < \begin{cases} 1145 \\ 1020 \end{cases}$$

No reduction in bending resistance need be made

$$M_{pl,Rd} = 1022 \text{ kNm}$$

6.2.8 By inspection, no reduction in bending resistance need be made for shear as the shear force is low.

$$M_{Ed} = 915.2 \text{ kNm} < 1022 \text{ kNm} \text{ so the cross-section is adequate}$$

In this example, for illustrative purposes it will be assumed that a hinge has formed in the rafter at the end of the haunch. Determination of the stable lengths between torsional and lateral restraints therefore involves the uniform depth rafter on the higher side of the plastic hinge (Section F3.3.1) and the haunched part of the rafter on the lower side of the plastic hinge (Section F3.5.4).

### F3.3 Verification to the first torsional restraint

The position of the first torsional restraint to the rafter on the higher side of the hinge is given by  $L_s$  in BB.3.1.2(3) covering members with a non-linear moment gradient, which demands a restraint at the plastic hinge and one or more intermediate restraints at a maximum spacing of  $L_m$ .

#### F3.3.1 Position of the first torsional restraint, $L_s$

Consider the rafter segment adjacent to the haunch (Figure F.5). Determine the stable length under a non-linear bending moment, adjacent to a plastic hinge.

$$(BB.8) \quad L_s = \sqrt{C_n} L_k$$

$$(BB.6) \quad L_k = \frac{\left(5.4 + \frac{600 f_y}{E}\right) \left(\frac{h}{t_f}\right) i_z}{\sqrt{5.4 \left(\frac{f_y}{E}\right) \left(\frac{h}{t_f}\right)^2 - 1}}$$

$$L_k = \frac{\left(5.4 + \frac{600 \times 355}{210000}\right) \left(\frac{602.6}{14.8}\right) \times 47.5}{\sqrt{5.4 \left(\frac{355}{210000}\right) \left(\frac{602.6}{14.8}\right)^2 - 1}} = \frac{12405}{3.759} = 3300 \text{ mm}$$

$$(BB.14) \quad C_n = \frac{12 R_{\max}}{[R_1 + 3R_2 + 4R_3 + 3R_4 + R_5 + 2(R_5 - R_E)]}$$

Note this formula is incorrectly presented in EN1993 and has been corrected here to conform to that printed in BS 5950.

$$(BB.15) \quad R = \frac{M_{y,Ed} + a N_{Ed}}{f_y W_{pl,y}}$$

The values of  $R$  are presented in Table F.1.



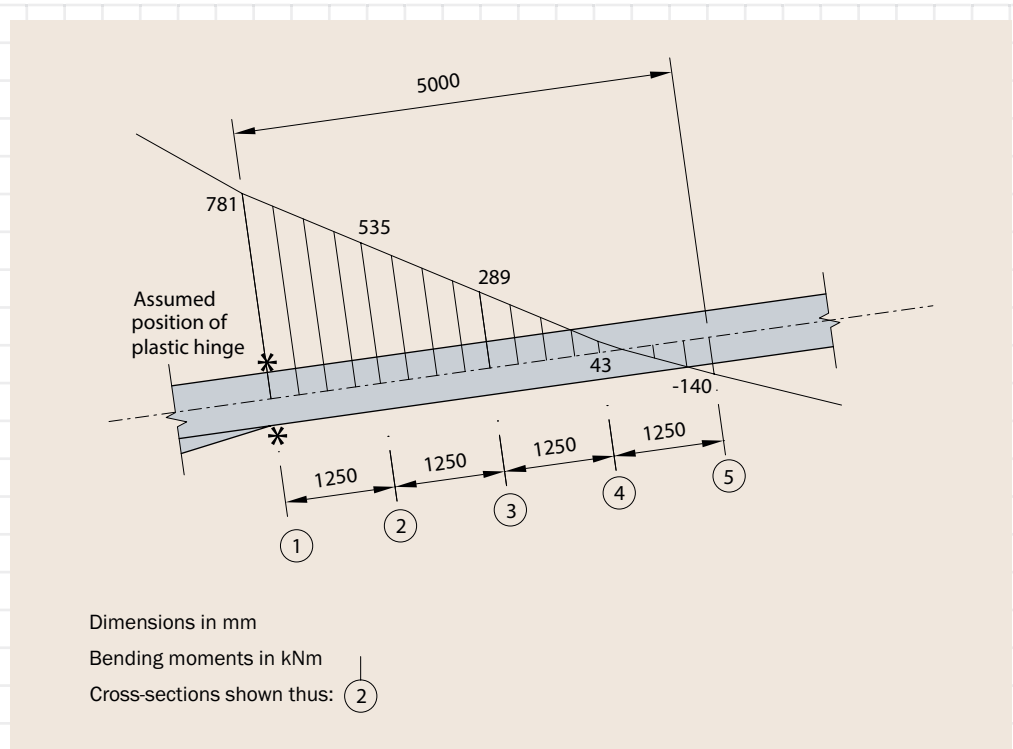


Figure F.5  
 Bending moments in rafter near haunch

Adopt a trial length of 5 m.

Only positive values of  $R$  are included because when  $R$  is negative, the flange force in the segment switches from compression to tension.

Assume the centroid of the purlins is 100 mm above the rafter flange.

$$a = \frac{602.6}{2} + 100 = 401 \text{ mm}$$

$$aN_{Ed} = 401 \times 163.6 \times 10^{-3} = 65.6 \text{ kNm}$$

$$f_y W_{pl,y} = 355 \times 2880 \times 10^3 \times 10^{-6} = 1022 \text{ kNm}$$

$$\text{For Section 1 (Figure F.5), } R = \frac{781 + 65.6}{1022} = 0.828$$

SECTION	1	2	3	4	5
$M_{y,Ed}$ (kNm)	781	535	289	43	-140
$a N_{Ed}$ (kNm)	65.6	65.6	65.6	65.6	65.6
$W_{pl,y} f_y$ (kNm)	1022	1022	1022	1022	1022
$R$	0.828	0.588	0.347	0.106	0 (- ve value)

Table F.1  
 Values of  $R$

$$R_S = R_E = 0.828; R_{max} = 0.828$$

$$C_n = \frac{12 \times 0.828}{0.828 + 3 \times 0.588 + 4 \times 0.347 + 3 \times 0.106 + 2(0.828 - 0.828)} = 2.312$$

$$L_s = \sqrt{2.312 \times 3300} = 5018 \text{ mm}$$

$$\text{Adopt } L_s = 5000 \text{ mm}$$

A purlin is required at this position. As the bottom flange is in tension at this location, a rafter stay is not required as long as the following criteria are satisfied:

The rafter is a UB section; the purlin-to-rafter connection has at least two bolts; the depth of the purlin is at least one quarter of the depth of the rafter.

Lateral-torsional buckling effects may be ignored over this segment if intermediate restraints are provided that satisfy  $L_m$ , as demonstrated in Section F3.3.2.

### F3.3.2 Position of intermediate lateral restraint

The position of the first lateral restraint adjacent to a plastic hinge is given by:

$$L_m = \frac{38i_z}{\sqrt{\frac{1}{57.4} \left( \frac{N_{Ed}}{A} \right) + \frac{1}{756C_1^2} \left( \frac{W_{pl,y}^2}{AI_T} \right) \left( \frac{f_y}{235} \right)^2}}$$

Try  $L_m = 1300$  mm from the assumed hinge position at the end of the haunch.

*A trial length is necessary because the moment ratio depends on the length.*

Assume a triangular moment distribution to the point of contraflexure. It is conservative to assume a linear moment distribution because the compression force in the flange is greater if this is assumed.

$M_{max} = 781$  kNm reducing linearly to zero over 4.00 m

By similar triangles  $M_{min} = \frac{4000 - 1300}{4000} \times 781 = 527$

$$\psi = \frac{527}{781} = 0.675$$

$$C_1 = 1.77 - 0.88 \times 0.675 + 0.11 \times 0.675^2 = 1.23$$

$$L_m = \frac{38 \times 47.5}{\sqrt{\frac{1}{57.4} \left( \frac{163.6 \times 10^3}{12900} \right) + \frac{1}{756 \times 1.23^2} \frac{(2880 \times 10^3)^2}{12900 \times 77 \times 10^4} \left( \frac{355}{235} \right)^2}} = 1314 \text{ mm}$$

Adopt  $L_m = 1300$  mm.

Provide restraint to the tension flange.

### F3.4 Verification of purlin spacing

The spacing of other restraints can be checked using equation 6.62 near the apex, using  $M_{y,Ed} = 915.2$  kNm

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

In this case, assume purlins are provided at a spacing of 1800 mm.

$$6.3.1.2 \quad N_{b,z,Rd} = \frac{\chi_z A f_y}{\gamma_{M1}}; \quad \chi_z = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$

$$\phi = 0.5 \left[ 1 + \alpha (\bar{\lambda}_z - 0.2) + \bar{\lambda}^2 \right]$$

$$\bar{\lambda} = \frac{L_{cr}}{i_z} \frac{1}{\lambda_1} \quad \lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

$$6.3.1.3 \quad \lambda_1 = \pi \sqrt{\frac{210 \times 10^3}{355}} = 76.4$$

$$\bar{\lambda} = \frac{1800}{47.5 \times 76.4} = 0.496$$

Table 6.2 For buckling about z-z axis use curve b, (rolled section:  $h/b > 1.2$ ;  $t_f < 40$ );  
Table 6.3

$$\alpha = 0.34$$

$$\phi = 0.5 \times [1 + 0.34 \times (0.496 - 0.2) + 0.496^2] = 0.673$$

$$\chi_z = \frac{1}{0.673 + \sqrt{0.673^2 - 0.496^2}} = 0.887$$

$$N_{b,z,Rd} = \frac{0.887 \times 12900 \times 355}{1.0} \times 10^{-3} = 4060 \text{ kN}$$

$$6.3.2.1 \quad M_{b,Rd} = \frac{\chi_{LT} W_{pl,y} f_y}{\gamma_{M1}}$$

The plastic section modulus is used because the section is at least Class 2.

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} f_y}{M_{cr}}} \quad \chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}}$$

$$\phi_{LT} = 0.5 [1 + \alpha_{LT} (\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}) + \beta \bar{\lambda}_{LT}^2]$$

$$\bar{\lambda}_{LT,0} = 0.4; \quad \beta = 0.75$$

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_T}{\pi^2 E I_z}}$$

Determine  $C_1$ . Assume the worst case occurs near the apex and the moment is approximately uniform.

$$C_1 = 1.0 \text{ for a uniform moment}$$

$$M_{cr} = 1.0 \times \frac{\pi^2 \times 210 \times 10^3 \times 2910 \times 10^4}{1800^2} \times \sqrt{\frac{2.52 \times 10^{12}}{2910 \times 10^4} + \frac{1800^2 \times 77 \times 10^4}{2.6 \pi^2 \times 2910 \times 10^4}}$$

$$= 5580 \times 10^6 \text{ Nmm} = 5580 \text{ kNm}$$

(Note  $E/G = 2 \times (1 + \nu) = 2 \times (1 + 0.3) = 2.6$ )

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{ply} f_y}{M_{cr}}} = \sqrt{\frac{2880 \times 10^3 \times 355}{5580 \times 10^6}} = 0.428$$

NA.2.17  
Table 6.3

$h/b = 2.65$ . For a rolled section where  $2 < h/b < 3.1$ , use buckling curve c therefore  $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5[1 + 0.49 \times (0.428 - 0.4) + 0.75 \times 0.428^2] = 0.576$$

$$\chi_{LT} = \frac{1}{0.576 + \sqrt{0.576^2 - 0.75 \times 0.428^2}} = 0.983$$

$$M_{b,Rd} = \frac{0.983 \times 2880 \times 10^3 \times 355}{1.0} \times 10^{-6} = 1005 \text{ kNm}$$

**Calculate  $k_{zy}$**

Table B.2

$$k_{zy} = \text{Max} \left[ \begin{array}{l} \left[ 1 - \frac{0.1 \bar{\lambda}_z}{(C_{mLT} - 0.25)} \cdot \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right] \\ \left[ 1 - \frac{0.1}{(C_{mLT} - 0.25)} \cdot \frac{N_{Ed}}{\chi_z N_{Rk} / \gamma_{M1}} \right] \end{array} \right]$$

$$C_{mLT} = 0.6 + 0.4\psi \geq 0.4$$

Table B.3

$$\psi = 1.0 \text{ (uniform moment assumed); } C_{mLT} = 1.0$$

$$k_{zy} = \text{Max} \left[ \begin{array}{l} \left[ 1 - \frac{0.1 \times 0.496}{(1.0 - 0.25)} \times \frac{163.6}{4062} \right] \\ \left[ 1 - \frac{0.1}{(1.0 - 0.25)} \times \frac{163.6}{4062} \right] \end{array} \right]$$

$$= \text{Max} [0.997, 0.995] = 0.997$$

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy} \frac{M_{y,Ed}}{M_{b,Rd}} = \frac{163.6}{4062} + 0.997 \times \frac{915.2}{1005} = 0.948 < 1.0 \quad \text{OK}$$

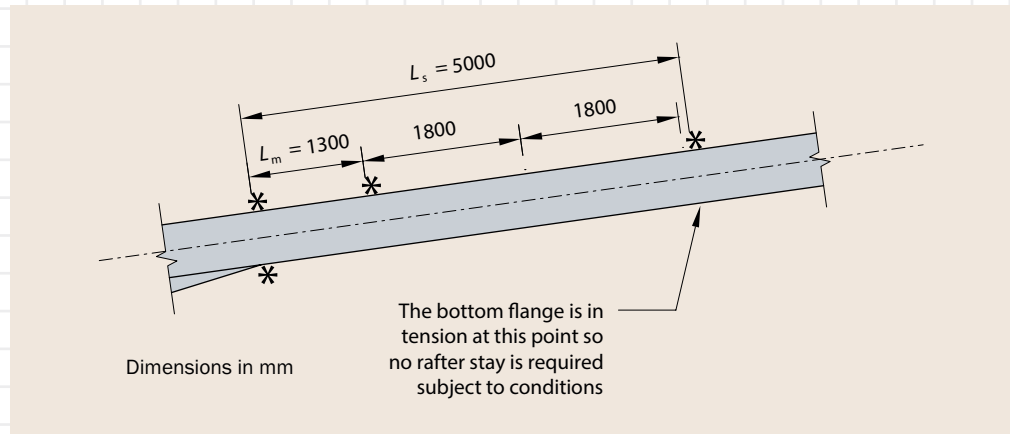
A purlin spacing of 1800 mm is adequate for the gravity load case.

Note that if purlins are provided at 1800 mm centres after the first intermediate purlin, a purlin will be positioned at  $(1300 + 1800 + 1800) = 4900$  from the torsional restraint at the end of the haunch.

A check for uplift is required but this is not provided in this illustration. Publication P397 includes this check in an example.

The resulting restraint locations are shown in Figure F.6 below.

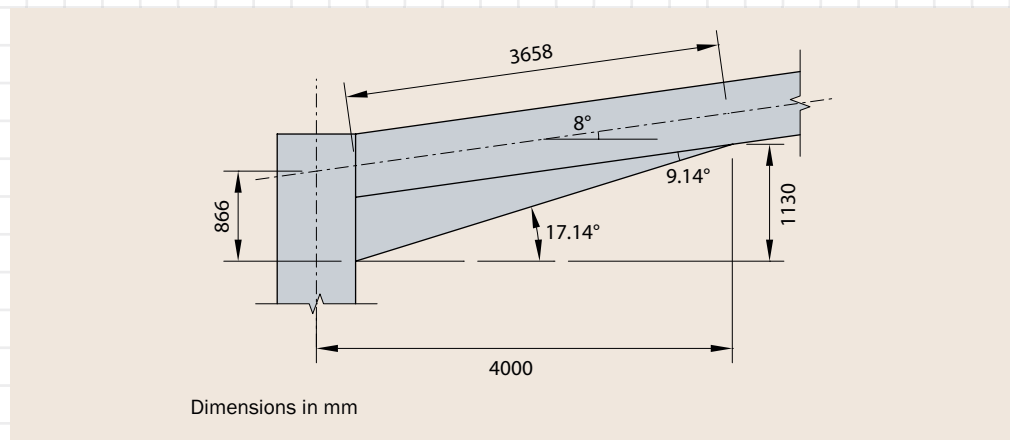
Figure F.6  
Lateral and torsional restraint positions in the rafter adjacent to the haunch



### F3.5 Verification of the haunch

The haunch geometry is shown in Figure F.7. The bending moments at the salient points are shown in Figure F.4. The haunch is firstly classified, and then the compression stress due to axial load and bending is determined at intermediate sections along the haunch. It is shown that the haunch remains elastic at each cross-section so the haunch is verified elastically in Section F3.5.2. For demonstration purposes, plastic checks are demonstrated in Section F3.5.4, assuming a plastic hinge had formed at the sharp end of the haunch.

Figure F.7  
Haunch geometry



#### F3.5.1 Classification of haunch section

##### Cutting flanges

$$t_f = 14.8 < 16 \rightarrow f_y = 355 \text{ N/mm}^2$$

$$\varepsilon = \sqrt{\frac{235}{355}} = 0.814$$

Table 5.2 For Class 1, the limiting  $c/t$  value =  $9\varepsilon = 7.32$

$$c = \frac{b - (2r + t_w)}{2} = \frac{227.6 - (2 \times 12.7 + 10.5)}{2} = 95.9 \text{ mm}$$

$$c/t = \frac{95.9}{14.8} = 6.48 < 7.32$$

Flanges are Class 1

### Cutting web

Assume cutting web is wholly in compression

Table 5.2

Limiting $c/t$ value =	$33 \varepsilon = 26.8$ (Class 1)
	$38 \varepsilon = 30.9$ (Class 2)
	$42 \varepsilon = 34.2$ (Class 3)

At the deepest section of the cutting, the web depth  $\approx (596 - 14.8) = 581$  mm

$$c/t = \frac{581}{10.5} = 55, \text{ i.e. the web would be Class 4}$$

6.2.2.4 (1)

Use Clause 6.2.2.4 (1) to determine the effective web depth.

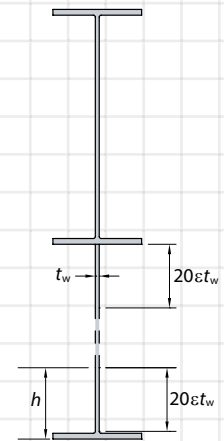
Replace the cutting web by a part adjacent to the cutting flange and a part adjacent to the bottom flange of the rafter.

Assume a depth of web  $= 20 \varepsilon t_w$  at top and bottom of cutting is effective.

$$20 \varepsilon t_w = 20 \times 0.814 \times 10.5 = 171 \text{ mm}$$

Height above u/s of bottom flange:

$$h = 171 + t_f + r = 171 + 14.8 + 12.7 = 198 \text{ mm}$$



When the cutting depth is less than 369 mm, no area is neglected.

### F3.5.2 Elastic verification of the haunched section

6.3.5.3(2)B

Where a rotated plastic hinge occurs immediately adjacent to one end of a haunch, the tapered segment need not be treated as a segment adjacent to a plastic hinge if:

- The restraint at the plastic hinge is placed within a distance  $h/2$  along the tapered segment, not the uniform segment (See Figure F.8).
- The compression flange of the haunch remains elastic.

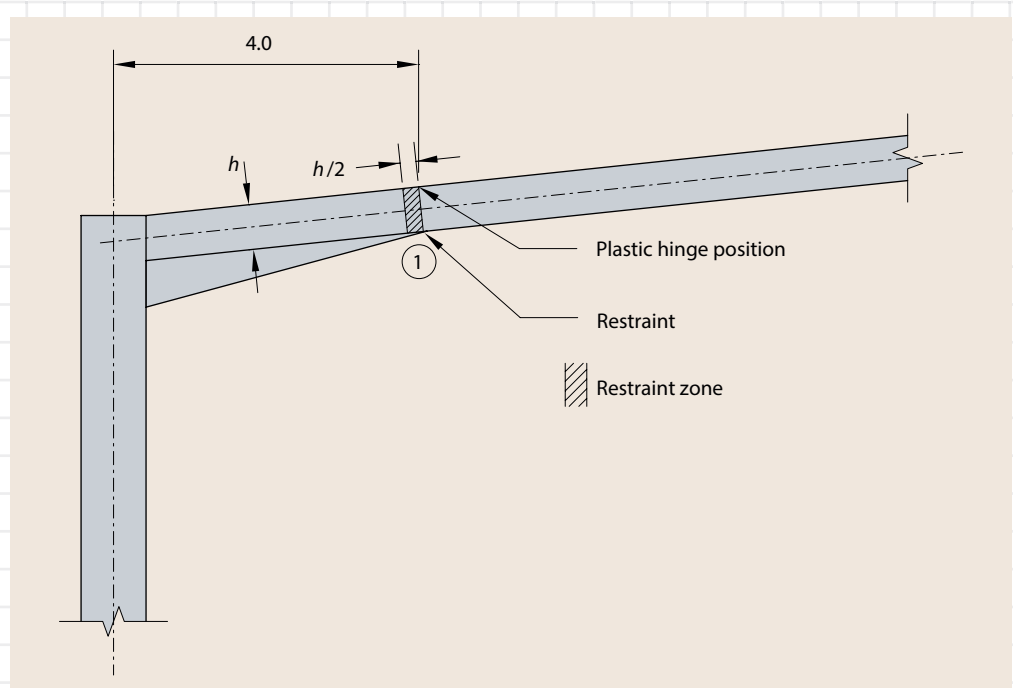


Figure F.8  
Location of restraint at plastic hinge position

**Determine elastic properties**

The elastic properties will be used to assess if the haunch remains elastic at 5 cross-sections throughout its length.

Examine sections 1.1 to 1.5 and the rafter (section 1.6) at the shallow end of the haunch (see Figure F.9). Section properties are given Table F.2.

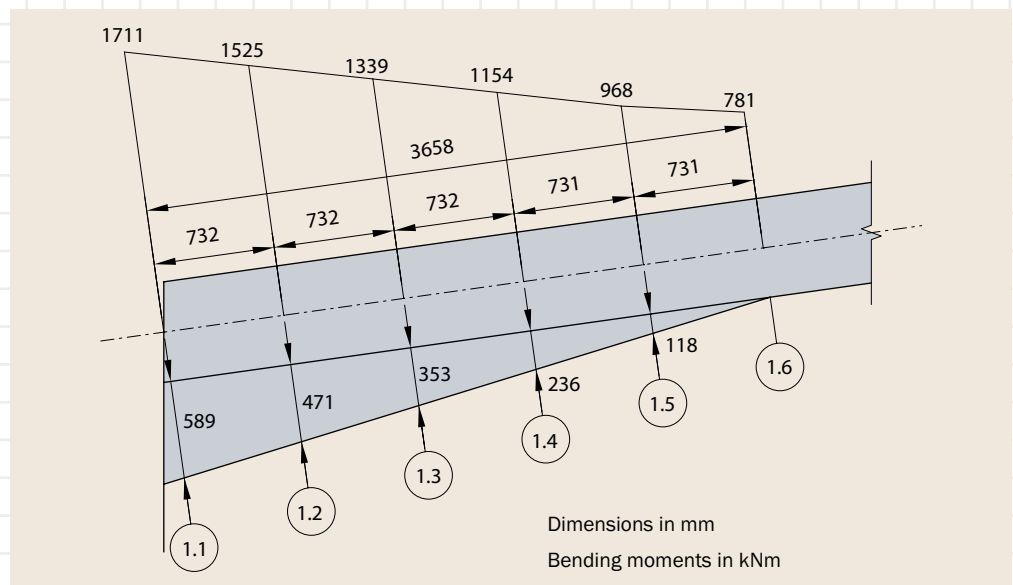


Figure F.9  
Haunch sections 1.1 to 1.6

- $d_c$  = depth of cutting.
- $\bar{y}$  = distance from the centroid of the compound section to the centroid of the rafter section.
- $I_y$  = second moment of area of section.
- $W_{el,bot}$  = Elastic modulus of section relative to the cutting flange.



Table F.2  
Second moment  
of area and elastic  
section modulus

SECTION	1.1	1.2	1.3	1.4	1.5	1.6 (RAFTER)
$d_c$ (mm)	589	471	353	236	118	0
(neglected web)	(220)	(102)				
$\bar{y}$ (mm)	261	229	196	149	103	0
$I_y$ (mm <sup>4</sup> )	$3.49 \times 10^9$	$2.82 \times 10^9$	$2.18 \times 10^9$	$1.70 \times 10^9$	$1.28 \times 10^9$	$7.58 \times 10^8$
$W_{el,bot}$ (mm <sup>3</sup> )	$5.55 \times 10^6$	$5.19 \times 10^6$	$4.76 \times 10^6$	$4.37 \times 10^6$	$4.06 \times 10^6$	$2.52 \times 10^6$

$I_y$  and  $W_{el,bot}$  are calculated by spreadsheet. Properties are calculated ignoring root radii.

$$f_{yc} = M_{y,Ed} / W_{el,bot} \quad f_c = N_{Ed} / \text{gross area}$$

The stress check is shown in Table F.3

Table F.3  
Stress check

SECTION	1.1	1.2	1.3	1.4	1.5	1.6 (RAFTER)
$M_{y,Ed}$ (kNm)	1711	1525	1339	1154	968	781
$f_{yc}$ (N/mm <sup>2</sup> )	308	294	282	264	239	310
$f_c$ (N/mm <sup>2</sup> )	8	8	8	9	10	13
$\Sigma$ (N/mm <sup>2</sup> )	316	302	290	273	249	323

The combined stress is everywhere less than 355 N/mm<sup>2</sup> so the compression flange of the haunch remains elastic throughout its length. The haunch therefore need not be treated as an element adjacent to a plastic hinge.

### F3.5.3 Buckling resistance of haunched segment

Length = 3658 mm

Bending moments:

$$M_{y,Ed,max} = 1711 \text{ kNm}$$

$$M_{y,Ed,min} = 781 \text{ kNm}$$

There is no specific guidance for verifying a haunched segment in BS EN 1993-1-1.

SCI recommends satisfying the following interaction:

$$\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{Ed,i}}{M_{b,Rd,i}} \leq 1.0$$

for all points,  $i$ , in the segment where the bottom flange is in compression (i.e. all points), where  $N_{b,Rd,i}$  is the compression resistance assuming tension flange restraint based on the properties at the minimum depth section and  $M_{b,Rd,i}$  is the buckling

resistance moment at cross-section,  $i$ , based on the section modulus at the cross-section being considered. (This approach is based on that adopted in BS 5950).

The cutting is slender so use effective properties:

$$20 \varepsilon t_w = 20 \times 0.814 \times 10.5 = 171 \text{ mm}$$

$$20 \varepsilon t_w + t_f + r = 198 \text{ mm}$$

When the cutting depth is less than  $171 + 198 = 369$  mm, the section is fully effective.

Assume there is a purlin at 1858 mm from the column face providing tension flange restraint. The length between the purlin and the sharp end of the haunch is 1800 mm.

The haunch will be examined at Sections 1 to 5, as shown in Figure F.9.

Section properties in Table F.4 are calculated by spreadsheet ignoring root radii:

SECTION	1.1	1.2	1.3	1.4	1.5	1.6 (RAFTER)
$d_c$ (mm)	589	471	353	236	118	0
(neglect)	220	102	0	0	0	0
Plastic NA	In flange	In web	In web	In web	In web	In web
$W_{pl,I}$ (mm <sup>3</sup> )	$6.85 \times 10^6$	$6.28 \times 10^6$	$5.57 \times 10^6$	$4.81 \times 10^6$	$4.14 \times 10^6$	$2.88 \times 10^6$
$I_z$ (mm <sup>4</sup> )	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$	$2.91 \times 10^7$
$I_w$ (mm <sup>6</sup> )	$1.01 \times 10^{13}$	$8.18 \times 10^{12}$	$6.57 \times 10^{12}$	$5.23 \times 10^{12}$	$4.16 \times 10^{12}$	$2.52 \times 10^{12}$
$I_T$ (mm <sup>4</sup> )	$1.10 \times 10^6$	$1.10 \times 10^6$	$1.10 \times 10^6$	$1.03 \times 10^6$	$1.00 \times 10^6$	$7.70 \times 10^5$
$I_y$ (mm <sup>4</sup> )	$3.49 \times 10^9$	$2.82 \times 10^9$	$2.18 \times 10^9$	$1.70 \times 10^9$	$1.28 \times 10^9$	$7.58 \times 10^8$

Table F.4  
Section properties

Note: the additional web contributes very little to  $I_z$  hence the values are the same.

$I_y$  and  $I_T$  values are calculated neglecting the ineffective portion of cutting web.

$I_w$  values are based on gross properties; the contribution from the web is negligible.

BB.3.3.3  
(BB.16) Determine the taper factor  $c$

$$c = 1 + \frac{3}{\left(\frac{h}{t_f} - 9\right)} \left(\frac{h_{\max}}{h_{\min}} - 1\right)^{2/3}$$

$$h/t_f = \frac{602.6}{14.8} = 40.7$$

$$c = 1 + \frac{3}{(40.7 - 9)} \left(\frac{1192}{602.6} - 1\right)^{2/3} = 1.093$$

### Properties for the minimum depth section

The minimum depth section in this case is the rafter section. The haunch is restrained on the tension flange by purlins. The value of  $N_{b,Rd}$  is given by:

$$6.3.1.2 \quad N_{b,Rd} = \chi_z \frac{Af_y}{\gamma_{M1}} \quad \chi_z = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$

$$\phi = 0.5 \left[ 1 + \alpha (\bar{\lambda}_z - 0.2) + \bar{\lambda}^2 \right]$$

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}}$$

In this case,  $N_{cr}$  is the elastic torsional buckling force because flexural buckling is prevented by the tension flange restraints.

$$N_{crT} = \frac{1}{i_s^2} \left( \frac{\pi^2 EI_z a^2}{L_t^2} + \frac{\pi^2 EI_w}{L_t^2} + GI_T \right) = \frac{1}{i_s^2} \left[ N_{crE} a^2 + N_{crE} \frac{I_w}{I_z} + GI_T \right]$$

$$i_s^2 = i_y^2 + i_z^2 + a^2$$

$a$  is the offset between the axis of rotation and the centroid of the rafter section.

Assume centroid of sheeting rail is 100 mm off the flange face.

$$a = \frac{h}{2} + 100 = \frac{603}{2} + 100 = 401 \text{ mm}$$

$$i_s^2 = 242^2 + 47.5^2 + 401^2 = 222 \times 10^3 \text{ mm}^2$$

$$N_{crE} = \frac{\pi^2 \times 210 \times 10^3 \times 2910 \times 10^4}{3658^2} \times 10^{-3} = 4510 \text{ kN}$$

$$N_{crT} = \frac{1}{222 \times 10^3} \left[ 4510 \times 401^2 + 4510 \times \frac{2.52 \times 10^{12}}{2910 \times 10^4} + \frac{210 \times 77 \times 10^4}{2.6} \right]$$

$$N_{crT} = 4510 \times (0.724 + 0.390) + 280 = 5300 \text{ kN}$$

(Note  $E/G = 2 \times (1 + \nu) = 2 \times (1 + 0.3) = 2.6$ )

$$\bar{\lambda}_T = \sqrt{\frac{12900 \times 355}{5300 \times 10^3}} = 0.930$$

Note that for torsional buckling use Table 6.2 and the curve relating to the z axis

so  $\alpha = 0.34$ .

$$\phi = 0.5 [1 + 0.34 \times (0.930 - 0.2) + 0.930^2] = 1.06$$

$$\chi = \frac{1}{1.06 + \sqrt{1.06^2 - 0.930^2}} = 0.638$$

$$N_{b,Rd} = 0.638 \times \frac{129 \times 10^2 \times 355}{1.0} \times 10^{-3} = 2920 \text{ kN}$$

6.3.1.4(3)  
Table 6.2  
Table 6.1

**Determine  $C_n$**

BB.14 
$$C_n = \frac{12 R_{max}}{[R_1 + 3R_2 + 4R_3 + 3R_4 + R_5 + 2(R_S - R_E)]}$$

Note this formula is incorrectly presented in EC3 and has been corrected here to conform to that printed in BS 5950.

$$R = \frac{M_{y,Ed} + aN_{Ed}}{f_y W_{pl}}$$

**R values**

Bending moments are determined by linear interpolation assuming a trapezoidal diagram from the column face to the end of the haunch. Values are required at quarter points as shown in Figure F.10. R values are calculated in Table F.5.

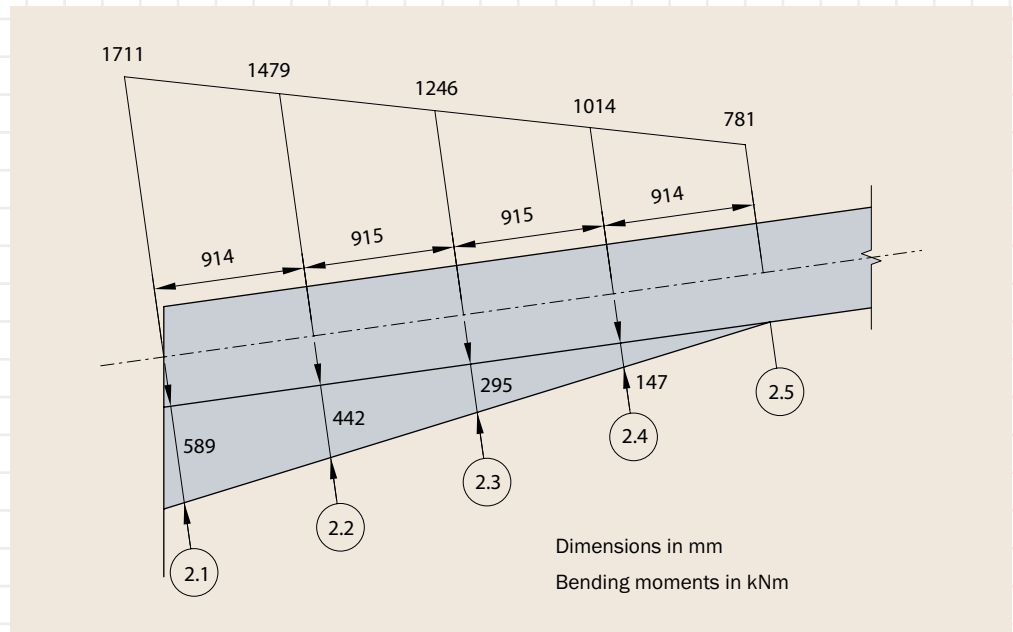


Figure F.10  
Sections for  
R values

SECTION	2.1	2.2	2.3	2.4	2.5 (RAFTER)
Cutting depth (mm)	589	442	295	147	0
$W_{pl,y}$ (mm <sup>3</sup> )	$6.85 \times 10^6$	$6.13 \times 10^6$	$5.18 \times 10^6$	$4.3 \times 10^6$	$2.88 \times 10^6$
$M_{y,Ed}$ (kNm)	1711	1479	1246	1014	781
$aN_{Ed}$ (kNm)	65.6	65.6	65.6	65.6	65.6
$f_y W_{pl,y}$ (kNm)	2432	2176	1838	1526	1022
R	0.731	0.710	0.714	0.708	0.828

Table F.5  
R values at  
Sections 2.1 to 2.5

$$R_{\max} = 0.828; \quad R_S = R_E = 0.828$$

Determine  $C_n$

$$C_n = \frac{12 \times 0.828}{0.731 + 3 \times 0.710 + 4 \times 0.714 + 3 \times 0.708 + 0.828} = 1.15$$

Note: the final term in the denominator:  $2(R_S - R_E) = 0$  because both  $R$  values equal 0.828).

### **Elastic critical moment**

The elastic critical moment can be determined from the elastic torsional buckling force  $N_{cr,T}$ , as indicated in the following expression:

Ref. 42 and  
Appendix C

$$M_{cr0} = \frac{i_s^2}{2a} N_{cr,T}; \quad M_{cr} = \frac{C_n}{c^2} M_{cr0}$$

$$M_{cr0} = \frac{222 \times 10^3}{2 \times 401} \times 5300 \times 10^{-3} = 1470 \text{ kNm}$$

$$M_{cr} = \frac{1.15}{1.093^2} \times 1470 = 1420 \text{ kNm}$$

The elastic critical moment is less than the maximum bending moment in the section so the haunch will buckle unless further torsional restraints are provided.

*In a realistic design case, the next step would be to provide a torsional restraint at a purlin position part-way along the haunch. The segments between the torsional restraints would then be checked, repeating the above procedure where a segment has intermediate purlins providing restraint to the tension flange of the haunch. This procedure is illustrated further in Section F3.5.6.*

### **F3.5.4 Plastic verification of the haunch**

This section illustrates the necessary verification if there was a plastic hinge at the sharp end of the haunch.

The position of the first torsional restraint is given by  $L_s$  in BB.3.2.2 (covering non-uniform members), which demands a restraint at the plastic hinge and one or more intermediate restraints at a maximum spacing of  $L_m$ .

$L_s$  is determined first followed by  $L_m$ . An initial estimate of the length is required because the resistance depends on the shape of the bending moment diagram over the chosen length. This length is then verified. The segment of the haunch beyond the torsional restraint is then verified elastically. As an alternative, a torsional restraint can be provided at  $L_m$  and the remainder of the haunch verified elastically. This is demonstrated in Section F3.5.5. Using the benefit of restraints to the tension flange demands that the restraints are spaced sufficiently close together to be effective.

In a uniform member, this is demonstrated using Expression 6.62. For the non-uniform haunch, the verification is demonstrated in Section F3.5.7.

**Position of the first torsional restraint**

BB.3.2.2  
(BB.11) 
$$L_s = \frac{\sqrt{C_n} L_k}{c}$$

**Taper factor  $c$**

BB.3.3.3(1)  $h = 602.6 \text{ mm}$   
 $b = 227.6 \text{ mm}$        $h/b = 2.65 > 1.2$       Therefore OK  
 $t_f = 14.8 \text{ mm}$        $h/t_f = 40.7 > 20$       Therefore OK  
 Try  $L_s = 3.3\text{m}$

Segment is tapered

$h_{\min} = \text{rafter depth} = 602.6 \text{ mm}$   
 $h_{\max} = 3300 \tan 9.14^\circ + 602.6 = 531 + 602.6 = 1134 \text{ mm}$

Taper factor  $c = 1 + \frac{3}{\left(\frac{h}{t_f} - 9\right)} \left(\frac{h_{\max}}{h_{\min}} - 1\right)^{2/3} = 1.09$

**Determine  $C_n$**

Take the centroid of the purlin (restraining member) at 100 mm from the face of the rafter flange as before.

$a = h/2 + 100 = 401 \text{ mm}$

$$R = \frac{M_{y,Ed} + a N_{Ed}}{f_y W_{ply}}$$

Consider 5 sections over a 3.3 m length (see Figure F.11).

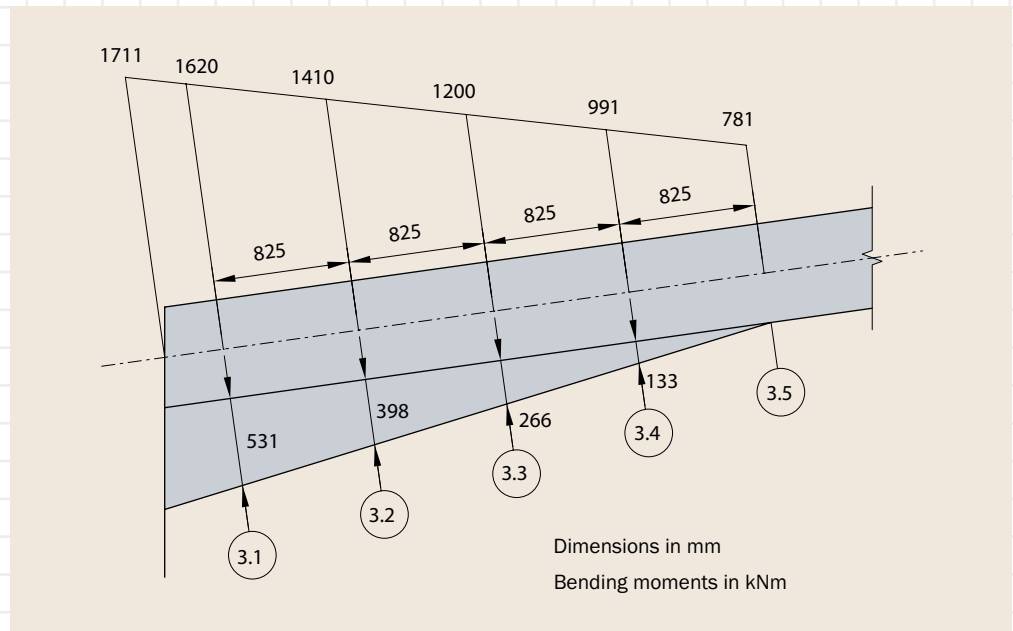


Figure F.11  
Sections for  $R$   
values over  
3.3 m length

Dimensions in mm  
Bending moments in kNm

**Plastic section properties**

SECTION	3.1	3.2	3.3	3.4	3.5 (RAFTER)
$d_c$ (mm)	531	398	266	133	0
(neglect) (mm)	162	29	0	0	0
Plastic NA	In flange	In flange	In web	In web	In web
$W_{pl,y}$ (mm <sup>3</sup> )	$6.54 \times 10^6$	$5.89 \times 10^6$	$4.99 \times 10^6$	$4.22 \times 10^6$	$2.88 \times 10^6$

Table F.6  
Plastic section  
properties at  
Sections 3.1 to 3.5

$R$  values in segment are shown in Table F.7.

SECTION	3.1	3.2	3.3	3.4	3.5 (RAFTER)
$M_{y,Ed}$ (kNm)	1620	1410	1200	991	781
$W_{pl,y}$ (mm <sup>3</sup> )	$6.54 \times 10^6$	$5.89 \times 10^6$	$4.99 \times 10^6$	$4.22 \times 10^6$	$2.88 \times 10^6$
$aN_{Ed}$ (kNm)	65.6	65.6	65.6	65.5	65.6
$f_y W_{pl,y}$ (kNm)	2322	2091	1771	1498	1022
$R$	0.726	0.706	0.715	0.705	0.828

Table F.7  
 $R$  Values at  
Sections 3.1 to 3.5

$$R_S = R_E = 0.828; R_{\max} = 0.828$$

$$(BB.14) \quad C_n = \frac{12 R_{\max}}{[R_1 + 3R_2 + 4R_3 + 3R_4 + R_5 + 2(R_S - R_E)]}$$

Note this formula for  $C_n$  is a correction of the formula in EN 1993-1-1.

$$C_n = \frac{12 \times 0.828}{0.726 + 3 \times 0.706 + 4 \times 0.715 + 3 \times 0.705 + 0.828} = 1.15$$

Note: the final term in the denominator  $2(R_S - R_E) = 0$  because both  $R$  values equal 0.828.

$$L_s = \frac{\sqrt{1.15} \times 3300}{1.09} = 3247 \text{ mm.} \quad \text{Adopt } L_s = 3240 \text{ mm}$$

$L_s$  is slightly less than the value assumed initially and used to determine  $c$  (the taper factor) and  $C_n$ , but judged to be satisfactory.

If a plastic hinge was present at the sharp end of the haunch and the haunch was not elastic over its full length, a torsional restraint would be needed no further than 3240 mm from the sharp end.



**Position of intermediate lateral restraint,  $L_m$**

The position of first lateral restraint  $L_m$  within the length  $L_s$ .

$$L_m = \frac{38 i_z}{\sqrt{\frac{1}{57.4} \left( \frac{N_{Ed}}{A} \right) + \frac{1}{756 C_1^2} \left( \frac{W_{ply}^2}{A I_T} \right) \left( \frac{f_y}{235} \right)^2}}$$

BB.3.2.2

$\frac{W_{ply}^2}{A I_T}$  is the maximum value in the segment.

$A$  is the cross-sectional area (mm<sup>2</sup>) at the location where  $\frac{W_{ply}^2}{A I_T}$  is a maximum of the tapered segment.

$I_T$  is the torsional constant.

$i_z$  is the minimum value of radius of gyration  $\sqrt{\frac{I_z}{A}}$  in the segment.

SECTION	3.1	3.2	3.3	3.4	3.5 (RAFTER)
$A$ (mm <sup>2</sup> )	19843	19843	18760	17363	12753
$W_{ply}$ (mm <sup>3</sup> )	$6.54 \times 10^6$	$5.89 \times 10^6$	$4.99 \times 10^6$	$4.22 \times 10^6$	$2.88 \times 10^6$
$I_T$ (mm <sup>4</sup> )	$1.10 \times 10^6$	$1.10 \times 10^6$	$1.06 \times 10^6$	$1.01 \times 10^6$	$0.77 \times 10^6$
$W_{ply}^2/A I_T$	1959	1589	1252	1015	845

Table F.8  
Values of  
 $W_{ply}^2/A I_T$

The maximum value of  $\frac{W_{ply}^2}{A I_T}$  occurs at Section 3.1, the deepest cross-section in the segment.

Assume a linear moment distribution over a length of 3658 mm with  $M_{max} = 1711$  kNm.

Assume a trial length  $L_m = 1000$  mm

Over a 1000 mm length,  $M = 781 + \frac{(1711 - 781)}{3658} \times 1000 = 1035$

$M_{min} = 781$  kNm

$\psi = \frac{781}{1035} = 0.755$

$C_1 = 1.77 - 0.88 \times 0.755 + 0.11 \times 0.755^2 = 1.17$

$$L_m = \frac{38 \times 47.5}{\sqrt{\frac{1}{57.4} \left( \frac{163600}{19843} \right) + \frac{1959}{756 \times 1.17^2} \left( \frac{355}{235} \right)^2}} = 855 \text{ mm}$$

Try 840 mm:  $M = 995$  kNm;  $\psi = 0.785$ ;  $C_1 = 1.15$ ;  $L_m = 840$  mm

Adopt  $L_m = 840$  mm Provide restraint to the tension flange.

The remaining length of the haunch  $(3240 - 840) = 2400$  mm should be checked as in Section F3.5.7 for lateral torsional buckling between purlins providing tension flange restraint, to ensure that the tension flange restraints are sufficiently close together.

### F3.5.5 Verification of elastic segment of the haunch

The torsional restraint to the haunched segment is 3.24 m from the sharp end of the haunch. The distance between the face of the column and the torsional restraint is  $3658 - 3240 = 418$  mm. This portion of the haunched rafter must be verified elastically. In this particular case, the length of the haunch requiring justification is short and the buckling resistance is clearly adequate. A cross-section check at each end of the segment will be sufficient

An alternative approach to stabilising the member is to provide a torsional restraint a distance,  $L_m$ , from a plastic hinge at the end of the haunch and verify the remainder of the haunched rafter elastically.

In this case,  $L_m$  determines the length of the segment so the relevant section properties are determined for a trial length.

#### Stable length $L_m$

$$BB.3.2.2 \quad L_m = \frac{38 i_z}{\sqrt{57.4 \left( \frac{163600}{A} \right) + \frac{1}{756 C_1^2} \left( \frac{W_{ply}^2}{A I_T} \right) \left( \frac{355}{235} \right)^2}}$$

Trial 1.  $L_m = 1250$  mm                      Cutting depth = 201 mm

$$\frac{W_{ply}^2}{A I_T} = 1140; A = 18077 \text{ mm}^2 \text{ (calculated by spreadsheet)}$$

Bending moment at end of segment

$$M_{max} = 781 + \frac{(1711 - 781)}{3658} \times 1250 = 1099 \text{ kNm}$$

$$\psi = 781/1099 = 0.711$$

$$C_1 = 1.77 - 0.88 \times 0.711 + 0.11 \times 0.711^2 = 1.20$$

$$L_m = \frac{38 \times 47.5}{\sqrt{57.4 \left( \frac{163600}{18077} \right) + \frac{1140}{756 \times 1.20^2} \left( \frac{355}{235} \right)^2}} = 1130 \text{ mm (not ok)}$$

Trial 2.  $L = 1135$  m                      Cutting depth =  $\frac{201 \times 1135}{1250} = 183$  mm

$$M_{max} = 781 + \frac{(1711 - 781)}{3658} \times 1135 = 1070$$

$$\psi = 781/1070 = 0.730$$

$$C_1 = 1.77 - 0.88 \times 0.730 + 0.11 \times 0.730^2 = 1.19$$

$$\frac{W_{ply}^2}{A I_T} = 1100; A = 17900 \text{ mm}^2$$

$$L_m = 1137 \text{ mm} \quad \text{Adopt } L_m = 1135 \text{ mm}$$

Provide torsional restraints at this location.

### F3.5.6 Elastic check of the remaining haunched segment

The remaining part of the haunch (length = 3658 – 1135 = 2523 mm) should be justified in a similar way to that shown in F3.5.3 and F3.5.7. The minimum depth section is in this case monosymmetric because it falls in the haunched portion of the rafter.

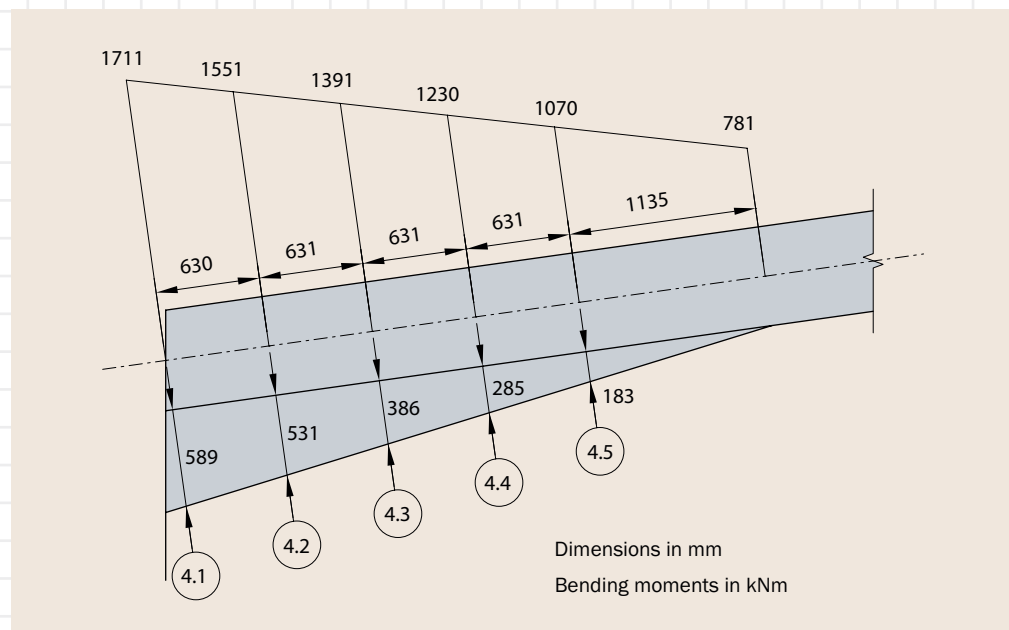


Figure F.12  
Sections 4.1 to 4.5

### Bending moments

$$M_{\max} = 1711 \text{ kNm}$$

$$M_{\min} = 1070 \text{ kNm}$$

Satisfy the interaction:

$$\frac{N_{Ed}}{N_{b,Ed}} + \frac{M_{Ed,i}}{M_{b,Rd,i}} \leq 1.0$$

for all points,  $i$ , in the segment where the bottom flange is in compression (i.e. all points).

$N_{b,Rd,i}$  is the compression resistance assuming tension flange restraint based on the properties at the minimum depth and  $M_{b,Rd,i}$  is the buckling resistance moment at cross-section,  $i$ , based on the section modulus at the cross-section considered.

When the cutting depth is less than  $171 + 198 = 369$  mm, the section is fully effective.

Neglect the portion of the web where the depth is greater than 369 mm. The section properties are shown in Table F.9.

SECTION	4.1	4.2	4.3	4.4	4.5
$d_c$ (mm)	589	488	386	285	183
(neglect) (mm)	(220)	(119)	(17)		
$A_{TOT}$ (mm <sup>2</sup> )	19843	19843	19843	18959	17888
$W_{pl}$ (mm <sup>3</sup> )	$6.85 \times 10^6$	$6.38 \times 10^6$	$5.82 \times 10^6$	$5.11 \times 10^6$	$4.50 \times 10^6$
$I_z$ (mm <sup>4</sup> )	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^{7*}$
$I_w$ (mm <sup>6</sup> )	$1.01 \times 10^{13}$	$8.43 \times 10^{12}$	$6.99 \times 10^{12}$	$5.76 \times 10^{12}$	$4.72 \times 10^{12}$
$I_T$ (mm <sup>4</sup> )	$1.10 \times 10^6$	$1.10 \times 10^6$	$1.10 \times 10^6$	$1.07 \times 10^6$	$1.02 \times 10^6$
$I_y$ (mm <sup>4</sup> )	$3.49 \times 10^9$	$2.91 \times 10^9$	$2.39 \times 10^9$	$1.91 \times 10^9$	$1.50 \times 10^9$

Table F.9  
Section properties at  
Sections 4.1 to 4.5

Note: Properties are calculated excluding root radius fillets

$R$  values are shown in Table F.10.

SECTION	4.1	4.2	4.3	4.4	4.5
$M_{y,Ed}$	1711	1551	1391	1230	1070
$aN_{Ed}$	65.6	65.6	65.6	65.6	65.6
$f_y W_{pl,y}$	2432	2263	2066	1815	1598
$R$	0.731	0.714	0.705	0.714	0.711

Table F.10  
 $R$  values at  
Sections 4.1 to 4.5

**Determine  $C_n$**

$$(BB.14) \quad C_n = \frac{12 \times 0.731}{0.731 + 3 \times 0.714 + 4 \times 0.705 + 3 \times 0.714 + 0.711}$$

Note: the final term in the denominator:  $2(R_s - R_E) = 0$  because both  $R$  values equal 0.731.

$$C_n = 1.03$$

Determine the taper factor  $c$

$$c = 1 + \frac{3}{(h/t_f - 9)} \left( \frac{h_{\max}}{h_{\min}} - 1 \right)^2$$

$$h = h_{\min} = 602.6 + 183 = 785.6$$

$$h/t_f = \frac{785.6}{14.8} = 53.1$$

$$h_{\max} = 602.6 + 589 = 1192 \text{ mm}$$

$$c = 1 + \frac{3}{(53.1-9)} \left( \frac{1192}{785.6} - 1 \right)^{2/3} = 1.04$$

Determine the compression resistance of the segment, based on the properties of the minimum cross-section.

From Reference 43:

$$N_{\text{crT}} = \frac{1}{\frac{1}{A} (I_y + I_z + A(h_z^2 - z_o^2))} \times \left( \frac{\pi^2 E I_w}{l^2} + \frac{\pi^2 E I_z}{l^2} (z_o - h_z)^2 n^2 + GI_T + \frac{k_\phi l^2}{n^2 \pi^2} \right)$$

for a monosymmetric section where  $h_z$  is the coordinate of the axis of rotation and  $z_o$  is the coordinate of the shear centre, relative to the geometric centroid of the section.

Assume  $n = 1$  (lowest value) and  $k_\phi = 0$  (conservative)

This is the same formula as in BB.3.3.1. In this formula,  $a$  is the distance from the shear centre of the compound section to the axis of rotation.

It is conservative to take the usual value of  $i_s$  as follows:

$$\frac{1}{\frac{1}{A} (I_y + I_z + A(h_z^2 - z_o^2))} \approx \frac{1}{i_y^2 + i_z^2 + a^2} = \frac{1}{i_s^2}$$

$$\text{and } N_{\text{crT}} = \frac{1}{i_s^2} \left[ N_{\text{crE}} (z_o - (-a))^2 + N_{\text{crE}} \frac{I_w}{I_z} + GI_T \right]$$

### Properties for the minimum-depth section

The geometry of the minimum-depth section is shown in Figure F.13.

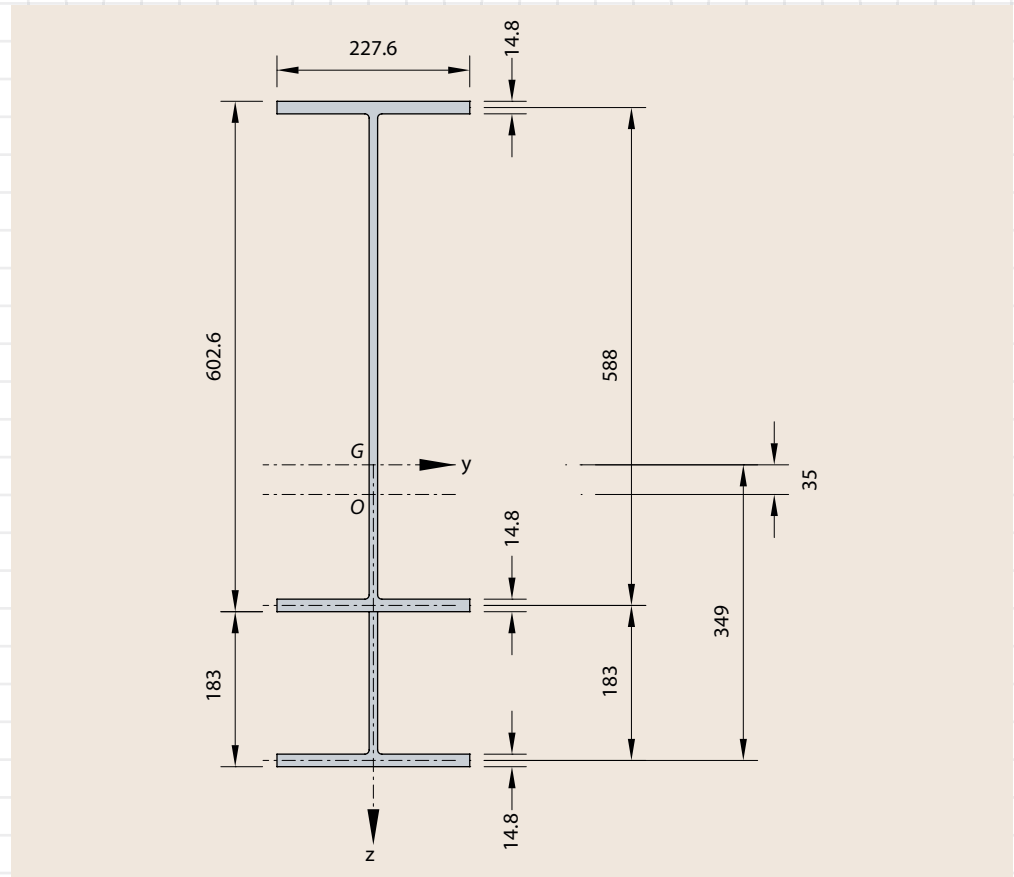


Figure F.13  
Geometry  
of minimum-  
depth section

G is the geometric centroid – the inter-section of the major and minor neutral axes.

O is the position of the shear centre.

Section properties have been calculated by spreadsheet.

Position of shear centre:  $y_o = 314$  mm above centre-line of cutting flange.

Warping constant:  $I_w = 4.72 \times 10^{12}$  mm<sup>6</sup>

$$I_y = 1.50 \times 10^9 \text{ mm}^4$$

$$I_z = 4.36 \times 10^7 \text{ mm}^4$$

$$I_T = 1.02 \times 10^6 \text{ mm}^4$$

$$A = 17900 \text{ mm}^2$$

$$z_o = +35 \text{ mm}$$

$$h_z = a = 401 \text{ mm}$$

**Calculate  $N_{crT}$**

$$\frac{1}{i_s^2} = \frac{1}{\frac{1.50 \times 10^9 + 4.36 \times 10^7}{17900} + 401^2} = \frac{1}{2.47 \times 10^5}$$

$$\frac{I_w}{I_z} = \frac{4.72 \times 10^{12}}{4.36 \times 10^7} = 1.08 \times 10^5$$

$$N_{crE} = \frac{\pi^2 E I_z}{l^2} = \frac{\pi^2 \times 210 \times 4.36 \times 10^7}{2523^2} = 14200 \text{ kN}$$

$$\frac{GI_T}{i_s^2} = \frac{210}{2.6} \times \frac{1.02 \times 10^6}{2.47 \times 10^5} = 334 \text{ kN}$$

$$N_{crT} = 14200 \times \left[ \frac{1.08 \times 10^5 + 1.90 \times 10^5}{2.47 \times 10^5} \right] + 334 = 17500 \text{ kN}$$

6.3.1.2(4) Note: Clause 6.3.1.2(4) states that for  $N_{Ed}/N_{cr} \leq 0.04$  buckling effects may be ignored.

$N_{Ed} = 163.5 \text{ kN}$ .  $0.04 \times 14200 = 568 \text{ kN}$  so flexural buckling can be ignored.

$$\bar{\lambda}_T = \sqrt{\frac{17900 \times 355}{17500 \times 10^3}} = 0.603$$

6.3.1.4(3) Use buckling curve b therefore  $\alpha = 0.34$

Table 6.2

Table 6.1

6.3.1.2

$$\phi = 0.5 [1 + 0.34 \times (0.603 - 0.2) + 0.603^2] = 0.750$$

$$\chi = \frac{1}{0.750 + \sqrt{0.750^2 - 0.603^2}} = 0.836$$

$$N_{b,Rd} = 0.836 \times 17900 \times 355 = 5310 \text{ kN}$$

### Elastic critical moment

$$M_{cr0} = \frac{i_s^2}{2a} N_{crT}$$

$$= \frac{2.47 \times 10^5 \times 17500}{2 \times 401} \times 10^{-3} = 5390 \text{ kNm}$$

$$M_{cr} = \frac{C_b}{c^2} M_{cr0} = \frac{1.03}{1.04^2} \times 5390 = 5130 \text{ kNm}$$

Non-dimensional slenderness for torsional buckling

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_u}{M_{cr}}} = \sqrt{\frac{W_{pl,y} \times 355}{5130 \times 10^6}}$$

$W_{pl,y}$  varies at each cross-section.  $\bar{\lambda}_{LT}$  is computed at different cross-sections in

Table F.11

NA.2.17

Table 6.3

$$h/b = 2.65.$$

For a rolled section where  $2 < h/b < 3.1$ , use buckling curve c therefore  $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5 \left( 1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0.4) + \beta \bar{\lambda}_{LT}^2 \right)$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \bar{\lambda}_{LT}^2}}$$



SECTION	4.1	4.2	4.3	4.4	4.5
$W_{ply}$ (mm <sup>3</sup> )	$68.5 \times 10^5$	$63.8 \times 10^5$	$58.2 \times 10^5$	$51.1 \times 10^5$	$45.0 \times 10^5$
$\bar{\lambda}_{LT}$	0.688	0.664	0.635	0.595	0.558
$\chi_{LT}$	0.833	0.847	0.865	0.889	0.910
$M_{b,Rd}$ (kNm)	2020	1920	1790	1610	1450
$M_{y,Ed}$ (kNm)	1711	1551	1391	1230	1070
$M_{y,Ed}/M_{b,Rd}$	0.845	0.809	0.778	0.762	0.736

Table F.11  
Buckling  
resistance moment at  
Sections 4.1 to 4.5

$$\frac{N_{Ed}}{N_{b,Rd}} = \frac{163.6}{5310} = 0.031$$

Limiting ratio =  $1 - 0.031 = 0.969$  which exceeds  $M_{y,Ed}/M_{b,Rd}$  in every case.

Torsional buckling capacity of the segment with tension flange restraint is adequate.

It is assumed that there is at least one purlin between the torsional restraints, dividing the segment into two lengths of about 1260 mm.

### F3.5.7 Elastic check of tapered segment between purlins

There is no guidance in EN 1993 concerning the verification of haunched members in elastic segments. The procedure set out in BS 5950 will be followed to check the resistance of the section between purlins. It is assumed the buckling mode will be lateral-torsional buckling between the torsional restraints provided at the column and the shallow end of the haunch and the intermediate purlin.

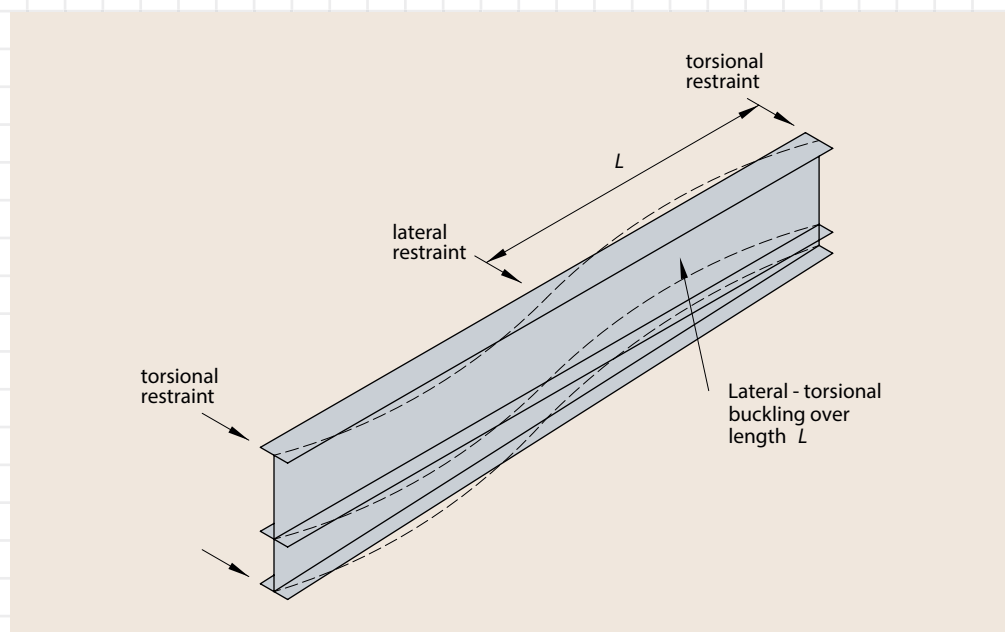


Figure F.14  
Buckling  
mode shape of  
haunched section

**Check adequacy with purlin spacing  $L = 1260$  mm**

Satisfy  $\frac{N_{Ed}}{N_{b,Rd}} + \frac{M_{Ed}}{M_{b,Rd}} \leq 1.0$

Where:

$M_{b,Rd}$  is given by  $W_{pl}$  at the section being considered and  $M_{cr}$  and  $\bar{\lambda}_{LT}$  are determined at the point of maximum bending moment in the segment.

The elastic critical moment  $M_{cr}$  for lateral-torsional buckling of a monosymmetric section is given by:

(Timoshenko & Gere Ref 43)

$$M_{cr} = \frac{-N_{cr}\beta_1}{2} \pm \sqrt{\left(\frac{N_{cr}\beta_1}{2}\right)^2 + \frac{I_0}{A} N_{cr} N_{crT}}$$

where  $\beta_1 = \frac{1}{I_y} \left( \int_A z^3 dA + \int_A y^2 z dA \right) - 2z_0$  and is the monosymmetry index for a cross-section, the value of which can be calculated using the formulae presented in Appendix B3.

Note that *LTBeamN* calculates the monosymmetry index of a section but the value given is  $(\beta_1/2)$ . This must be remembered when substituting in the formula for  $M_{cr}$  if the value is determined from this source.

$$I_0 = I_y + I_z + A (y_0^2 + z_0^2)$$

$(y_0, z_0)$  are the coordinates of the shear centre with respect to the centroid of the section and  $y_0 = 0$  for a monosymmetric section.

$$N_{cr} = \frac{\pi^2 E I_z}{L^2} \quad N_{crT} = \frac{A}{I_0} \left( GI_T + \frac{EI_w \pi^2}{L^2} \right)$$

Substituting:

$$M_{cr} = -\frac{\pi^2 E I_z}{L^2} \frac{\beta_1}{2} \pm \sqrt{\left(\frac{\pi^2 E I_z \beta_1}{2L^2}\right)^2 + \left(GI_T + \frac{EI_w \pi^2}{L^2}\right) \frac{\pi^2 E I_z}{L^2}}$$

$$= -\frac{\pi^2 E I_z}{L^2} \frac{\beta_1}{2} \pm \left(\frac{\pi^2 E I_z}{L^2}\right) \sqrt{\frac{\beta_1^2}{4} + \frac{GI_T L^2}{\pi^2 E I_z} + \frac{I_w}{I_z}}$$

**Calculate  $N_{b,Rd}$**

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}}$$

$$N_{cr} = \frac{\pi^2 E I_z}{L^2} = \frac{\pi^2 \times 210 \times 10^3 \times 4.36 \times 10^7}{1260^2} \times 10^{-3} = 56900 \text{ kN}$$

Table F.9 Over the length of the haunch, the value of  $I_z$  is approximately constant and results from the presence of three flanges. The additional web area as the haunch deepens hardly affects the value of  $I_z$ .

$$A = 19800$$

$$\bar{\lambda} = \sqrt{\frac{19800 \times 355}{56900 \times 10^3}} = 0.351$$

Assume a rolled section  $t_f < 40$  mm

Use curve b therefore  $\alpha = 0.34$

$$\phi = 0.5 \times [1 + 0.34 \times (0.351 - 0.2) + 0.351^2] = 0.587$$

$$\chi = \frac{1}{0.587 + \sqrt{0.587^2 - 0.351^2}} = 0.946$$

$$N_{b,Rd} = 0.946 \times 19800 \times 355 \times 10^{-3} = 6650 \text{ kN}$$

Consider 5 cross-sections as shown in Figure F.15.

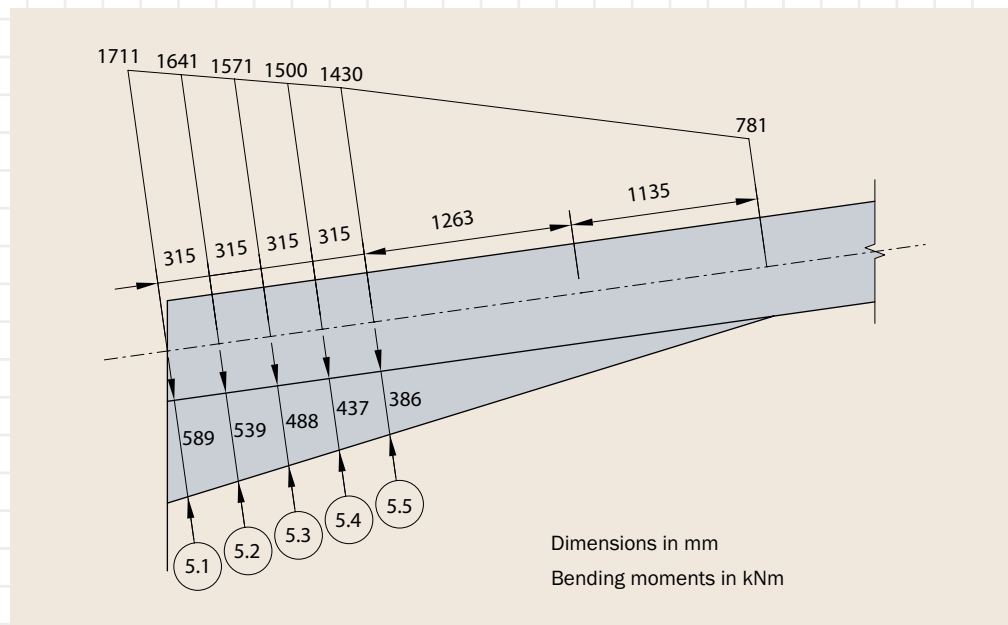


Figure F.15  
Sections 5.1 to 5.5

Section properties are shown in Table F.12 and have been determined ignoring root radii.

SECTION	5.1	5.2	5.3	5.4	5.5
$d_c$ (mm)	589	539	488	437	386
$A_{TOT}$ (mm <sup>2</sup> )	19800	19800	19800	19800	19800
$W_{pl}$ (mm <sup>3</sup> )	$6.85 \times 10^6$	$6.65 \times 10^6$	$6.38 \times 10^6$	$6.10 \times 10^6$	$5.82 \times 10^6$
$I_z$ (mm <sup>4</sup> )	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$	$4.36 \times 10^7$
$I_w$ (mm <sup>6</sup> )	$1.01 \times 10^{13}$	$9.24 \times 10^{12}$	$8.44 \times 10^{12}$	$7.69 \times 10^{12}$	$6.99 \times 10^{12}$
$I_T$ (mm <sup>4</sup> )	$1.18 \times 10^6$	$1.16 \times 10^6$	$1.14 \times 10^6$	$1.12 \times 10^6$	$1.11 \times 10^6$
$I_y$ (mm <sup>4</sup> )	$3.49 \times 10^9$	$3.19 \times 10^9$	$2.91 \times 10^9$	$2.64 \times 10^9$	$2.39 \times 10^9$
$\beta_1$ (mm)	4.59	-15.6	-36.3	-56.9	-77.2
$z_o$ (mm)	4.16	8.70	13.1	17.4	21.3

Table F.12  
Section properties at  
Sections 5.1 to 5.5

Calculation of  $M_{b,Rd}$  at maximum depth section:

$$M_{cr} = -\frac{\pi^2 \times 210 \times 10^3 \times 4.36 \times 10^7}{1260^2} \times \frac{4.59}{2}$$

$$\pm \left( \frac{\pi^2 \times 210 \times 10^3 \times 4.36 \times 10^7}{1260^2} \right) \times \sqrt{\frac{4.59^2}{4} + \frac{1.18 \times 10^6 \times 1260^2}{\pi^2 \times 2.6 \times 4.36 \times 10^7} + \frac{1.01 \times 10^{13}}{4.36 \times 10^7}}$$

$$= 27400 \times 10^6 \text{ Nmm} = 27400 \text{ kNm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{pl,y} f_y}{M_{cr}}} = \sqrt{\frac{6.85 \times 10^6 \times 355}{27400 \times 10^6}} = 0.298$$

NA.2.17  
Table 6.3  
6.3.2.3

Adopt curve b;  $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5 \times [1 + 0.49 \times (0.298 - 0.4) + 0.75 \times 0.298^2] = 0.508$$

$$\chi_{LT} = \frac{1}{0.508 + \sqrt{0.508^2 - 0.298^2}} = 1.088 \quad \text{but } \leq 1.0$$

$$M_{b,Rd} = \chi_{LT} W_{pl,y} f_y = 1.0 \times 6.85 \times 10^6 \times 355 \times 10^{-6} = 2430 \text{ kNm}$$

SECTION	5.1	5.2	5.3	5.4	5.5
$M_{cr}$ kNm	27300	n/a	n/a	n/a	n/a
$\bar{\lambda}_{LT}$	0.298	n/a	n/a	n/a	n/a
$\alpha_{LT}$ (curve c)	0.34	n/a	n/a	n/a	n/a
$\phi_{LT}$	0.508	n/a	n/a	n/a	n/a
$\chi_{LT}$	1.0	1.0	1.0	1.0	1.0
$M_{b,Rd}$ (kNm)	2430	2360	2260	2160	2070
$M_{Ed}$	1711	1641	1571	1500	1430
Utilisation	0.704	0.695	0.695	0.694	0.691

Table F.13  
Buckling  
resistance moment at  
Sections 5.1 to 5.5

$$\frac{N_{Ed}}{N_{b,Rd}} = \frac{163.6}{6650} = 0.025$$

Limiting ratio =  $1 - 0.025 = 0.975$  which exceeds  $M_{y,Ed}/M_{b,Rd}$  in every case.

Therefore the section is adequate.

A similar check is required for the adjacent segment. A summary of the restraints for the checks carried out in Sections F3.5.5, F3.5.6 and F3.5.7 is shown in Figure F.16.

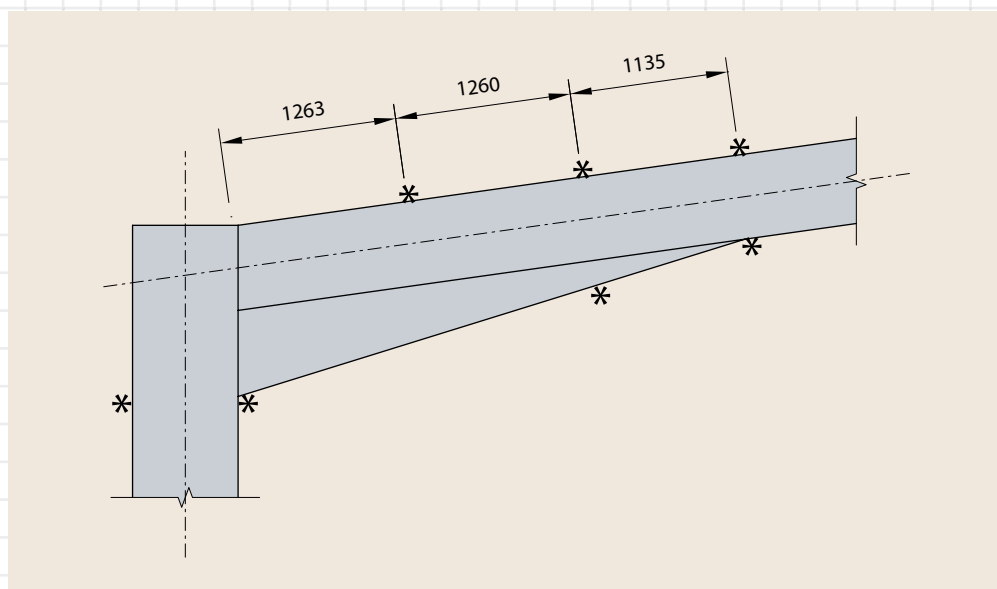


Figure F.16  
Summary of  
haunch restraints

The positions of the restraints to the column, rafter and haunch have been determined and the verification of the elements completed for the gravity load combination. Similar verifications are also necessary for the other load combinations.











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## DESIGN OF STEEL PORTAL FRAME BUILDINGS TO EUROCODE 3

Steel portal frame buildings account for the majority of the market for single storey buildings in the UK. Such buildings are frequently designed using plastic methods and this approach is set out in the present publication in detail, in accordance with the Eurocodes. Recommendations are made where the design Standard is unclear. The publication includes a worked example covering the verification of the primary elements in a portal frame, namely column, rafter and rafter haunch.

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