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In-plane Stability of Portal Frames to BS 5950-1:2000

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FOREWORD

The checking of the in-plane stability of single-storey portal frames requires different approaches to those commonly used for multi-storey buildings. BS 5950-1:2000 introduces more rigorous recommendations for the stability checks for portal frames than the 1990 version. This is necessary because portal frames have proved to be such a successful structural form that more frames are being constructed with geometries that are beyond the range foreseen when the recommendations in BS 5950-1:1990 were prepared.

This document is intended for the design of portal frames used for single-storey buildings loaded predominantly with roof loading that cause large bending moments in the rafters and the external columns. It is not intended for portals used to stabilise buildings, such as used where cross-bracing is not possible, but the principles described are applicable to the design of such frames.

This publication was written by Mr Charles King of The Steel Construction Institute.

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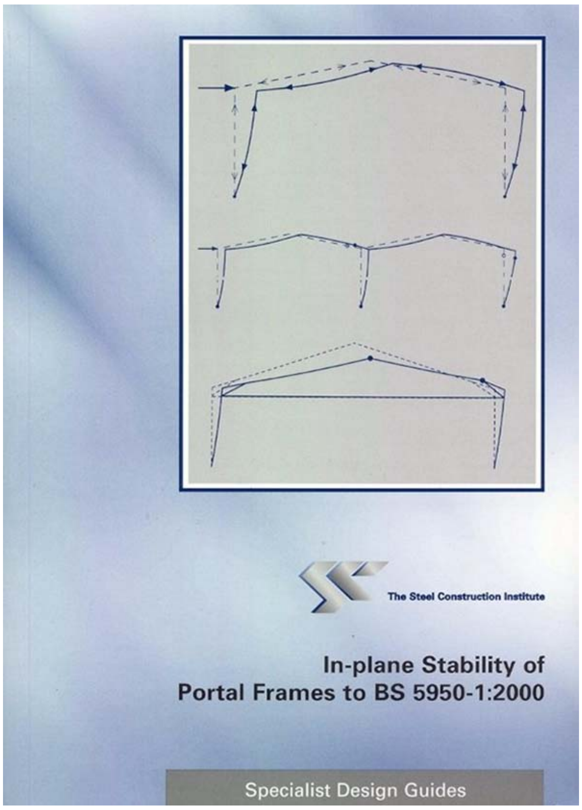
In-plane Stability of Portal Frames to BS 5950-1:2000



- Comprehensive guidance on the in-plane stability checks for portalframes
- Advice on second-order methods
- Worked examples of ordinary and tied portals

See over for details
and order form

CONTENTS

SUMMARY

This document introduces designers to the in-plane stability calculation methods in BS 5950-1:2000 for single-storey portal frames designed using either elastic or plastic analysis. These calculations are an essential part of the Ultimate Limit State (ULS) verifications of portal frames. In addition to a review of all these methods, it shows how second-order calculations can be performed even when second-order software is not available.

This document includes:

- An introduction to the in-plane stability of single-storey portal frames.
- A commentary on the three methods of checking the in-plane stability of portal frames given in BS 5950-1:2000, that is:
    - (a) The Sway-check method
    - (b) The Amplified Moment method
    - (c) Second-order analysis
- Worked examples of a simple method for second-order calculations that can be used where second-order analysis software is not available.

The instances in which individual members need to be checked for in-plane buckling are also explained. Second-order analysis by application of the energy method is explained in a form that can be applied in hand calculations, and this is illustrated by four worked examples.

Stabilité en Plan des Portiques selon la Norme BS 5950-1:2000

Résumé

Ce document présente aux calculateurs les méthodes de calcul de stabilité en plan selon la norme BS 5950-1:2000 pour des portiques à un niveau calculés en utilisant soit une analyse élastique, soit une analyse plastique. Ces calculs sont une partie essentielle des vérifications aux Etats Limites Ultimes (ELU) des portiques. En complément de la présentation de toutes ces méthodes, il est montré comment des calculs au second ordre peuvent être effectués sans avoir recours à un logiciel d'analyse au second ordre.

Ce document comprend :

- *Une introduction à la stabilité en plan des portiques à un niveau.*
- *Des commentaires sur les trois méthodes de vérification de la stabilité en plan des portiques données dans la norme BS 5950-1:2000, c'est-à-dire:*
    - *(a) La méthode de vérification avec longueurs de flambement à noeuds déplaçables.*
    - *(b) La méthode par amplification des moments.*
    - *(c) L'analyse au second ordre.*

- *Des exemples d'application d'une méthode simple de calcul au second ordre qui peut être utilisée en l'absence de logiciel d'analyse au second ordre.*

Les cas pour lesquels les barres doivent être vérifiées vis-à-vis du flambement dans le plan sont aussi explicités. L'analyse au second ordre par application de la méthode de l'énergie est décrite de façon à ce qu'elle puisse être appliquée manuellement et est illustrée par quatre exemples.

Ebene Stabilität von Rahmentragwerken nach BS 5950-1:2000

Zusammenfassung

Dieses Dokument führt Tragwerksplaner in die Berechnungsmethoden der Stabilität in derTragwerksebene von eingeschossigen Rahmentragwerken nach BS 5950-1:2000 ein, die entweder elastisch oder plastisch berechnet wurden. Diese Berechnungen sind ein wichtiger Teil der Überprüfung des Grenzzustands der Tragfähigkeit von Rahmentragwerken. Zusätzlich zum Überblick dieser Methoden wird gezeigt, wie Berechnungen nach Theorie II. Ordnung durchgeführt werden können, auch wenn entsprechende Software nicht verfügbar ist.

Dieses Dokument enthält:

- *eine Einführung in die Stabilität von eingeschossigen Rahmentragwerken in ihrer Ebene,*
- *einen Kommentar zu den drei Methoden der Überprüfung der Stabilität in Tragwerksebene von Rahmentragwerken nach BS 5950-1:2000, welche sind:*
    - *(a) Überprüfung der Seitensteifheit/-weichheit,*
    - *(b) Methode der mit einem Vergrößerungsfaktor erhöhten Momente,*
    - *(c) Berechnung nach Theorie II. Ordnung.*
- *Berechnungsbeispiele einer einfachen Methode für Berechnungen nach Theorie II. Ordnung, die benutzt werden kann, wenn entsprechende Software nicht verfügbar ist.*

Die Fälle, in welchen für einzelne Bauteile ein Knick- / Biegeknicknachweis erforderlich ist, werden erklärt. Die Berechnung nach Theorie II. Ordnung durch Anwendung der Energiemethode wird in einer weise erklärt, daß sie von Hand durchgeführt werden kann; dies wird anhand von vier Berechnungsbeispielen illustriert.

Estabilidad de pórticos en su plano según BS 5950-1:2000

Resumen

Con este documento se describen a los proyectistas los métodos de cálculo de estabilidad de la BS 5950-1:2000 para pórticos sencillos de una planta calculados según métodos elásticos o plásticos que son una parte esencial de las comprobaciones de estados límite últimos (ELU). Además de revisar esos métodos se muestra cómo se pueden llevar a cabo cálculos de segundo orden incluso sin software de segundo orden.

La publicación incluye:

- *Una introducción a la estabilidad en su plano de pórticos de una planta.*
- *Comentarios sobre los tres métodos de comprobación incluidos en BS 5950-1:2000, esto es:*
  - *(a) El método de la comprobación de la deriva*
  - *(b) El método de amplificación de momentos*
  - *(c) Cálculo en segundo orden.*
- *Ejemplos desarrollados de un método sencillo para cálculos de segundo orden utilizable sin software de segundo orden.*

También se explican los casos en que piezas individuales deben comprobarse a pandeo. Los métodos basados en la Energía se explican de forma que puedan ser aplicados manualmente, lo que se ilustra con cuatro ejemplos totalmente resueltos.

Stabilità nel piano di portali in accordo alla BS 5920-1:2000

Sommario

Questa pubblicazione affronta il problema della stabilità nel piano di portali in acciaio e, rivolta prevalentemente ai progettisti, riporta i metodi di calcolo per la progettazione sia elastica sia plastica in accordo alla BS 5950-1:2000. I calcoli effettuati in accordo a tali metodi costituiscono una parte essenziale della verifica agli Stati Limite Ultimi (S.L.U.) di portali in acciaio. In aggiunta ad una presentazione generale di questi metodi, viene mostrato come effettuare analisi del secondo ordine anche quando non si siano disponibili specifici strumenti software in grado di effettuare automaticamente tale tipo di analisi. Questa pubblicazione include:

- *un'introduzione alla stabilità nel piano di portali in acciaio.*
- *un commentario ai tre metodi di verifica per l'instabilità nel piano dei portali in accordo alla BS 5950-1:2000; cioè:*
  - *(a) il metodo di controllo dello spostamento trasversale;*
  - *(b) il metodo di amplificazione dei momenti;*
  - *(c) l'analisi del secondo ordine.*
- *esempi applicativi di un metodo semplificato per i calcoli del secondo ordine da usare quando non sono disponibili metodi più raffinati in grado di tenere direttamente in conto gli effetti del secondo ordine.*

Viene anche trattato il caso in cui le verifiche di stabilità nel piano debbano essere condotte sui singoli elementi. In aggiunta, è proposta l'analisi del secondo ordine sulla base dei metodi energetici in una forma anche applicabile manualmente, con esplicito riferimento ai quattro esempi applicativi riportati nella pubblicazione.

1 THE IN-PLANE STABILITY CHECKS IN BS 5950-1:2000

1.1 Checks for portal frames

Single-storey portal frames of economic proportions need to be checked to ensure that they have adequate in-plane stability, whether designed by elastic or plastic methods. This type of frame cannot be checked by the simple methods for multi-storey frames in BS 5950-1[1] Clauses 2.4.2.6 and 2.4.2.7 because axial compression in the rafter is not considered in that method. The structural phenomena involved in in-plane stability of single-storey frames are described in Section 2 together with a comparison with multi-storey frames.

BS 5950-1:2000 gives three methods for checking the in-plane stability of single-storey frames:

- The Sway-check method
- The Amplified Moment method
- Second-order analysis

The methods apply to portal frames designed either by elastic design (see Clause 5.5.2 of BS 5950-1) or by plastic design (see Clause 5.5.3 of BS 5950-1).

It will almost always be preferable to perform these checks by software. It is possible to perform the checks by 'hand', but the results will almost invariably be less economical. The only benefit of the 'hand' method of second-order analysis is to gain a greater understanding of the response of the frame to the second-order (P-delta) effects and the loss of stiffness resulting from the formation of plastic hinges.

1.2 The methods in brief

1.2.1 The Sway-check method

Range of application

The Sway-check method may be used for portals that are not tied portals and which satisfy the following geometrical limitations:

- Span/height to eaves is not more than 5.
- Rise of apex above column tops is not more than span/4 for symmetrical spans or a value given by a formula for asymmetric rafters.
- Either the notional sway deflection from notional forces (calculated by first-order analysis) is not more than h/1000, or the span/depth ratio of the rafters is within a limit given by a formula. The stiffness of the cladding is not to be considered in calculating the notional sway deflection for predominantly gravity load cases (e.g. Combination 1).

Advantages and disadvantages

The Sway-check method is the simplest method and gives economical designs if the frame is sufficiently stiff to satisfy either the $h/1000$ check or the formula check because of the section sizes selected either to give the necessary strength or to satisfy the Serviceability Limit State (SLS) requirements. This method will often give the most economical designs for single span portals that tend to be relatively stiff. Economy is achieved because there is no reduction in frame strength for the gravity load cases (Load combination 1 of Clause 2.4.1.2 and Crane combination 1 of Clause 2.4.1.3) that are generally the critical design load cases. Many multi-span frames will not satisfy the notional sway requirements without increasing the size of the members above the size required for strength or for SLS requirements. When using the Sway-check method, the steel strength (e.g. S275 or S355) has no effect on the in-plane stability calculation.

The design steps for this method and further details of the method are given in Section 3.

1.2.2 The Amplified Moment method

Range of application

The Amplified Moment method is a method that may be used where the frame does not meet the limitations of the sway-check method. It may be used for portals that are not tied portals and which have an elastic critical buckling ratio, λ_{cr}, not less than 4.6. The elastic critical buckling ratio, λ_{cr}, is described in Section 2.3

Advantages and disadvantages

The Amplified Moment method is a simple method to apply when the value of λ_{cr} is known. If easy-to-use software is available, the method is easy to use. When software is not available, then the formulae in Section 4 may be used, but they are complex and several formulae need to be applied for a multi-span frame. The method gives reasonably economical designs if the frame is relatively stiff because of the section sizes required either to give the required strength or to satisfy the SLS requirements. In particular, where $\lambda_{cr} \geq 10$, there is no reduction in frame strength. Thus, the method will give economical designs for most single span portals because they tend to be relatively stiff. It will also give reasonably economical designs for multi-span frames that are relatively stiff. However, many multi-span frames will not satisfy the requirement that $\lambda_{cr} \geq 4.6$, unless the size of the members is increased above the size required for strength or SLS requirements. The method does recognise the improvement in in-plane stability of the frame resulting from the use of higher strength steel (grade S355 steel). This improvement comes from an increase in λ_p, not from λ_{cr}, which is independent of the change of steel grade.

The design steps for this method and further details of this method are given in Section 4.

1.2.3 Second-order analysis

Range of application

Second-order analysis is another alternative method where the frame does not meet the limitations of the Sway-check method. It may be used for all portals including tied portals. Tied portals must be designed using second-order analysis. For tied portals, the analysis method must also be able to calculate the non-linear behaviour of the apex drop, a capability that may not be included in all packages that describe themselves as 'second-order'.

Advantages and disadvantages

Second-order analysis is simple to apply if there is easy-to-use software available. It will give the most economical designs for more flexible frames such as multi-span frames. It may give less economical designs than the other methods for stiffer frames because it will always calculate a reduction of frame strength from second-order (P-delta) effects. The other methods have threshold stiffness values above which the strength is not reduced. The Second-order method does recognise the improvement in in-plane stability of the frame resulting from the use of higher strength steel (grade S355 steel).

Further details of this method are given in Section 5.

1.3 Selecting methods for different types of frames

1.3.1 Single-span frames (not tied portals)

Single-span frames may be designed by any of the three methods described above. Where the frames are within the geometrical limitations of the Sway-check method and pass either the $h/1000$ check or the formula check (see Section 1.2.1 above), the method does not give any reduction of frame strength for the gravity load cases. Where the frames are outside the geometrical limitations of the Sway-check method or fail the checks, another method must be used. For frames slightly outside the geometrical limitations, it may be worth making minor alterations to the scheme to fit into the limitations, such as an increase in stiffness of the frame to satisfy the deflection check, or setting the bases deeper to suit the span to height ratio or a change of rafter geometry. Where the Sway-check method is not satisfied, either the Amplified Moment method or Second-order analysis should be used.

1.3.2 Multi-span frames (not tied portals)

Multi-span frames often have relatively low stiffness. Although some multi-span frames might be sufficiently stiff for the Sway-check method, many will not. Where the frames are too flexible and have slender internal columns, the most efficient way to improve the frame stiffness will often be to increase the internal column stiffness.

The amplified moment method may give an economical frame where the frame has a value of $\lambda_{cr} \geq 4.6$. Where the value of $\lambda_{cr} \geq 10$, there is no reduction of design strength in this method. However, many multi-span frames will have a value of λ_{cr} less than 4.6, so this method cannot be applied. This leaves the choice between stiffening the frame and using second-order analysis.

1.3.3 Tied portals

Tied portals should always be designed using second-order analysis. The solution method for this analysis is not specified in BS 5950-1, leaving freedom to choose a suitable routine. It should be noted that for tied portals with low roof slopes, there is an important non-linearity in the apex deflections. This arises because the compression of the rafter and the stretching of the tie reduce the height of the apex, which reduces the vertical component of the rafter force. To maintain equilibrium, an increased rafter force is required, which increases the apex deflection until either equilibrium is reached or the apex snaps through.

Therefore, whatever routine is selected, it must take account of the non-linear behaviour of the rafter and tie system, which will almost certainly involve an iterative procedure.

Tied portals of economical proportions will normally have very high axial forces in the rafters. These forces often cause a significant reduction in the stability of the frame. Therefore, rafters will often need to be made significantly stiffer than the section that would satisfy a first-order analysis.

1.3.4 Stability portals or 'wind portals'

Stability portals are outside the scope of this document. Stability portals are portals used to stabilise structures where cross-bracing is not acceptable. Such frames have little vertical loading distributed along the beam element, so have small axial loads in the beam. The dominant failure mode is by sway. Second-order analysis, the Amplified Moment method or the Sway-check method (lateral load case) would be appropriate for checking stability frames, but the gravity load case of the Sway-check method should not be used. Alternatively, where the axial force in the beam is very low, it is reasonable to design such frames according to the rules for multi-storey sway-frames rather than the rules for ordinary portal frames.

1.4 Required load factor, λ_r

BS 5950-1 Clauses 5.5.2 and 5.5.3 introduce the required load factor λ_r. This is a factor to allow for P-delta effects where these have not been calculated in the global analysis. For elastic design of portal frames, the output from a first-order global analysis with ULS loads must be multiplied by λ_r before the member resistances are checked. For plastic design, the plastic collapse factor, λ_p, calculated by first-order global analysis with ULS loads must not be less than λ_r. Member strength and stability calculations should be made at λ_r x ULS rather than 1.0 x ULS.

1.5 Base stiffness

BS 5950-1 Clause 5.1.3 gives guidance on the base stiffness that may be assumed in design. This may be summarised as follows for the cases most frequently occurring in portal frame design.

Base with a pin or rocker

The base stiffness should be taken as zero

Nominally pinned base

If the base moment is assumed to be zero, the base should be assumed to be pinned in the global analysis used to calculate the moments and forces around the frame. However, the base stiffness may be assumed to be equal to 10% of the column stiffness when checking frame stability or determining in-plane effective lengths, which form part of the ULS process. When using elastic-plastic design, an appropriate base capacity must also be specified.

For calculating deflections at SLS, the base stiffness may be assumed to be 20% of the column stiffness, but this should not be used for in-plane stability checks.

Other types of base

BS 5950-1 Clause 5.1.3 also gives guidance for the use of nominally rigid bases and nominal semi-rigid bases.

Application of these provisions for base stiffness to the different methods of checking frames is given in Section 3.3.4 for the Sway-check method, in Section 4.3.5 for the Amplified Moment method and in Section 5.3.4 for Second-order methods. The application to the hand method of second-order calculations is given in Appendix A.2.4 for common portals and Appendix B.2.4 for tied portals.

1.6 Notional horizontal forces

1.6.1 General

BS 5950-1 uses notional horizontal forces, which are taken as 0.5% of the factored vertical dead and imposed loads. They may be applied at the tops of the columns for simplicity, or at the point of application of load, as shown in Figure 1.1.

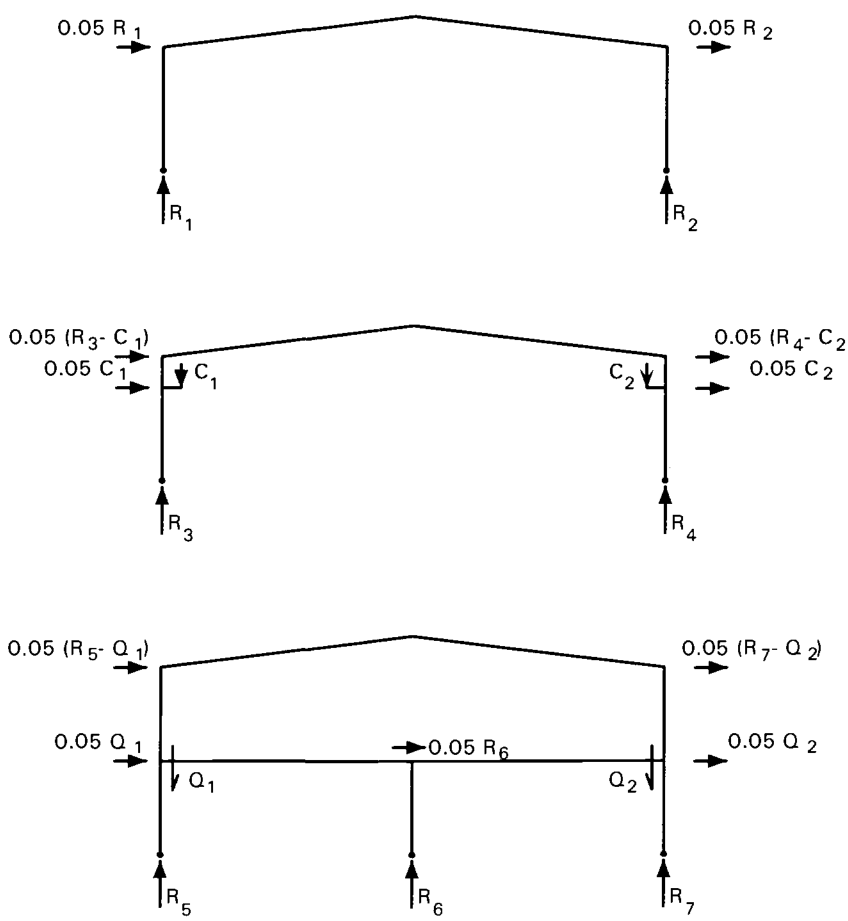


Figure 1.1 *Notional horizontal forces (for mezzanines etc, see Section 1.6.2)*

These notional horizontal forces are used for two completely different purposes:

(i) For checking frame strength
The notional horizontal forces are applied as a design horizontal load to allow for the effects of practical imperfection such as a lack of verticality, as given in Clause 2.4.2.4. The notional horizontal forces are applied in Load combination 1 of Clause 2.4.1.2, which is combination of dead load plus imposed loads (gravity loads).

(ii) For checking frame stiffness
The notional horizontal forces are applied as the loading used in a stiffness check of frames such as in Clause 5.5.4.2.1. In this application, the notional horizontal forces are applied to the frame without any other loading to assess the stiffness of the frame by calculating the horizontal deflections of the column tops assuming linear elastic behaviour. Clause 5.5.4.2.1 says that the forces should be equal to 0.5% of the vertical reaction at the base of the respective column. This assumes that the column reactions are known exactly before the notional horizontal forces are defined. In practice, the deflections are not sensitive to the distribution of the notional horizontal forces. Thus, some approximation may be made in the distribution of these loads. The most important point is that notional horizontal forces must be calculated from all the vertical loads on the building and this is most conveniently calculated by considering the vertical reactions of the columns.

Although the magnitude of the forces in both (i) and (ii) above is the same, at 0.5% of factored loads, there is an important difference in the loads to be applied in the case of crane loads. In Clause 2.4.2.4, it is clear that the vertical crane loads need not be included when calculating the notional horizontal forces for checking frame strength. By contrast, all vertical loads must be applied when checking the frame stiffness, hence in the stiffness check, the notional horizontal forces must include 0.5% of the vertical crane loads. However, the in-plane stability of the frame is not affected by dynamic loading, so the notional horizontal force should be taken as 0.5% of the factored crane load without dynamic or impact effects.

1.6.2 Mezzanines and other connected structures

Where a mezzanine floor or other structure is connected to the portal frame, the stability of the connected structure must be considered when checking both the strength and the stiffness of the portal. Where a connected structure contains its own stability system (e.g. cross-bracing, stability portal or rigid moment connections) that makes the connected structure at least as stiff as the portal frame, then the portal need not resist notional horizontal forces from the connected structure. Where the connected structure is not restrained by any stability system, the sum of the notional horizontal forces from the connected structure must be applied to the portal frame. In the intermediate condition, where the connected structure provides some stability but is not as stiff as the portal frame, the notional horizontal forces from the connected structure may be shared.

The stiffness of the connected structure and the portal frame may be calculated in terms of the slope of the columns induced by the notional horizontal forces. Alternatively, it may be calculated in terms of the deflection at the connection points induced by the notional horizontal forces. It is rare to find these slopes or deflections uniform throughout a structure, so the mean or median of the calculated values may be used.

1.7 Local concentrated lateral loads in buildings

Building structures are often subject to local concentrated loads, such as crane loads. Where these cause sway deflections (e.g. crane surge loads or notional horizontal forces), these loads may be shared by the adjacent frames in buildings with metal roof sheeting or with continuous bracing.

2 INTRODUCTION TO IN-PLANE STABILITY

2.1 Why are there in-plane stability checks?

All slender members resisting axial compression would buckle if the applied axial force were large enough. Stability checks calculations verify that the resistance to buckling is greater than the applied forces. When checking the stability of a column, the buckling resistance is calculated for buckling about both the major axis and the minor axis.

In frames, the stability checks must also verify the adequacy of the buckling resistance about both the major axis and the minor axis. In normal portal frames, buckling out of the plane of the frame is checked in the same way as for any other beam-column, considering buckling between lateral restraints and between torsional restraints provided by bracings etc. These bracings make the effective lengths of each element easily identifiable. However, buckling in the plane of the frame is more complicated than in normal beam column elements. This is because there is normally no bracing in the plane of the frame, and thus the restraint to any column depends on the stiffness of the rafters and the other columns. Equally, the restraint to any rafter depends on the stiffness of the columns and the other rafters. Therefore, checks for the stability of the frame must consider the entire frame stiffness. Although engineers are accustomed to checking the buckling resistance of columns using effective lengths, the effective lengths of portal frames can only be defined correctly if the stiffness of the entire frame is considered.

The in-plane stability checks for portal frames in BS 5950-1 differ from those for beam and column buildings. This is because the axial loads in portal rafters have a much greater effect on the stability of the frame than the axial loads that might occur in the beams of common beam and column buildings.

2.2 Axial compressive forces in frames

2.2.1 General

In-plane stability depends on the magnitude of the axial compression in the members, so it is important to understand the relative magnitude of these forces in the rafters and columns.

Most frames have axial compressive forces in some of the members. The distribution of forces depends not only on the applied loads, but also on the structural form of the frame and the bending moments throughout the frame. The magnitude of the second-order buckling effects depends not only on the magnitude of the force, but also on the elastic critical buckling load of the members and the elastic critical buckling load of the entire frame. This is discussed in Section 2.4.2 and Section 2.4.3. The lower the elastic critical buckling loads, the greater will be the second-order effects from a given axial compressive force.

Where there is axial tension in the members, the second-order effects increase the stiffness of the frame, so no reduction in frame capacity need be considered.

2.2.2 Ordinary portals

A typical bending moment diagram for an ordinary portal frame under vertical loading is shown in Figure 2.1. There is a horizontal reaction at the bases of the columns to maintain equilibrium with the bending moments in the columns. To maintain the horizontal equilibrium of these horizontal reactions, the rafters carry an axial compression as shown in Figure 2.2. These axial compressive forces are not large in magnitude, but they may be significant compared with the elastic critical buckling load of the rafters, because the rafters are relatively long. This effect is considered in Section 2.4.2

The axial compressive force in the rafter is seriously affected by the ratio of the portal span to the column height. This is because the bending moment at the column top depends on the span and the horizontal reaction at the column base depends on the moment at the column top and the height of the column. The moment at the column top is given approximately by:

Column top moment, $M \approx \dfrac{wL^2}{12}$

where:

w is the distributed load on the rafter

L is the span of the portal.

The horizontal reaction for a pinned base is then given by:

$$H = \frac{M}{H} \approx \frac{wL^2}{12h}$$

where:

h is the height of the column.

Therefore, for a given loading and span, the axial compression in the rafters is less for a high portal frame than for a low frame.

The axial compression in the rafters produces second-order effects in the rafters, which reduces the in-plane stability of the frame in addition to the second-order effects from the axial compression in the columns.

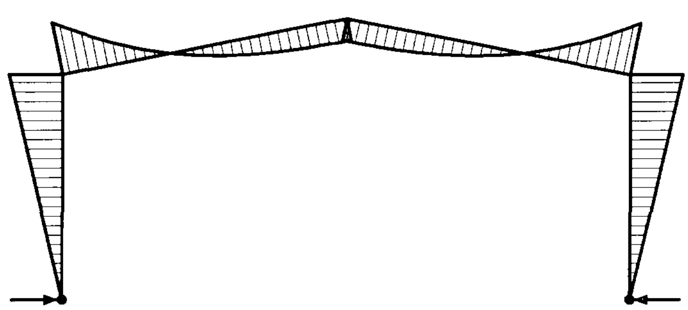


Figure 2.1 *Bending moments in a typical frame*

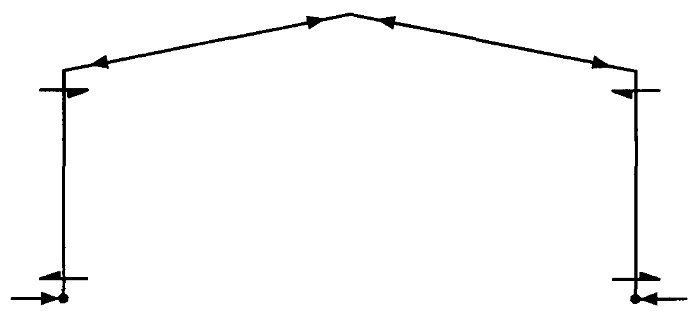


Figure 2.2 *Horizontal reactions and rafter axial force*

2.2.3 Tied portals

Tied portals in which the tie is near the eaves level behave very differently from ordinary portals. The structural behaviour is more like that of a rigidly-jointed truss on posts. The axial compressive forces in the rafters are much higher than in ordinary portals, especially for portals with low roof slopes.

The bending moments for a tied-portal are illustrated in Figure 2.3. The bending moment diagram is similar to a pair of fixed ended beams, each with a span from eaves to apex. Therefore, the bending moments both in the rafters ad in the columns are approximately a quarter of the bending moments in an ordinary portal. This reduction in the bending moment allows the use of a rafter with a much smaller bending resistance. The reduced bending moments are a consequence of the truss action of the tied portal. The axial loads are shown in Figure 2.4.

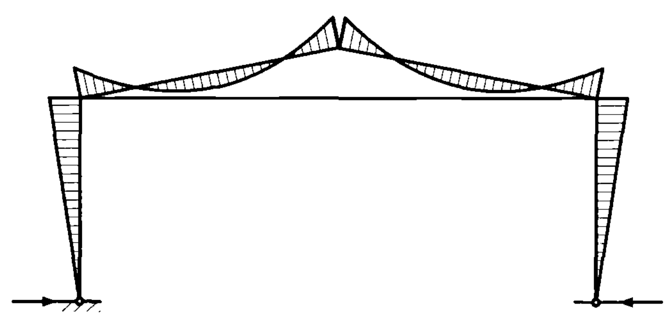


Figure 2.3 *Bending moments in a tied portal*

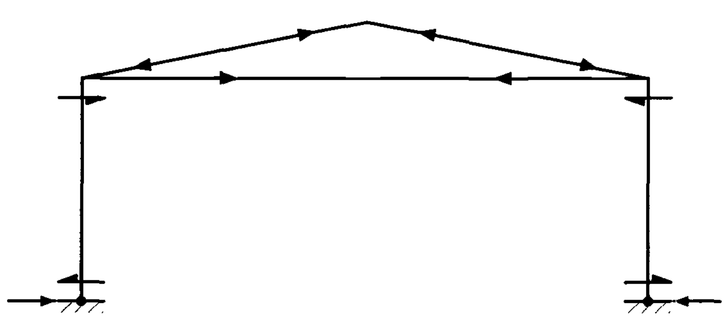


Figure 2.4 *Column shears and rafter and tie axial forces*

The high axial loads on rafters that require only relatively small bending resistance means that the rafters are seriously affected by second-order effects. For this reason, it is recommended that tied-portals are always checked by second-order analysis. However, if this is to be done, the checks should be no less rigorous than the in-plane checks on a truss rafter. In addition, the calculations must allow for the increase in axial forces arising from a reduction in the height between the apex and the tie. This reduction in height is a consequence of the strains in the rafters and tie. A convenient method of avoiding this reduction in height is to install a strut between the apex and the tie to maintain a constant height between the apex and the tie. This must be properly restrained against out-of-plane displacements of the frames at both ends.

2.3 Elastic critical buckling of frames

Struts have a theoretical elastic critical buckling load, or Euler load, which could only be reached if the strut has an infinitely high strength. The buckling load, or Euler load, for a pin-ended strut is given by:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

where:

E is the Young's modulus

I is the inertia of the strut

L is the length of the strut.

The critical buckling load is a theoretical load and exceeds the actual failure load of a real strut as shown in Figure 2.5. In the figure, both P_{cr} and the squash load P_y (= Area × yield stress) are shown.

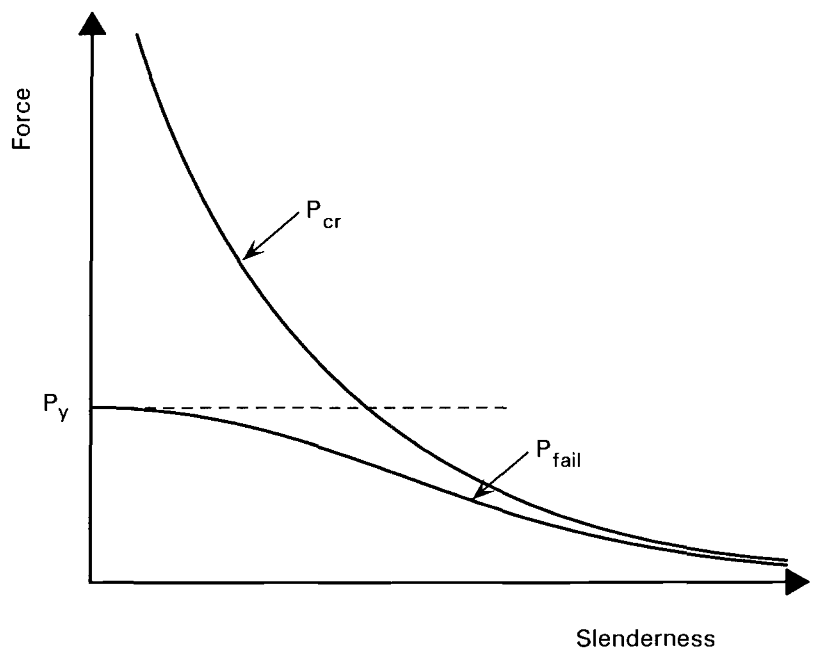


Figure 2.5 *Elastic critical buckling load of a strut*

Similarly, frames have a theoretical elastic critical buckling load, which could only be reached if the frame has an infinitely high strength. This will be referred to in this document as V_{cr}. This is commonly expressed in a ratio called 'lambda crit', λ_{cr}, which is defined as:

$$\lambda_{cr} = \frac{V_{cr}}{V_{ULS}}$$

where:

V_{cr} is the elastic critical buckling load

V_{ULS} is the applied loading at ULS.

The value of V_{cr} depends on the distribution of load on the frame, so λ_{cr} should be calculated from values of V_{cr} and V_{ULS} that have proportionately the same distribution of load.

The value of λ_{cr} varies according to the magnitude of the applied ULS loading, V_{ULS}. A large value of λ_{cr} indicates that the loading on the frame is well below the buckling resistance. A value of λ_{cr} just above unity indicates that the frame is near to its failure load. It must be remembered that failure will usually occur well below V_{cr} due to bending stresses in the frame, initial imperfections and the finite value of yield stress. However, λ_{cr} is a very useful ratio, both as an indicator of the sensitivity of the frame to buckling and in calculating amplification factors.

2.4 Second order (P-delta) effects

2.4.1 General

The strength checks for any structure are valid only if the global analysis gives a good representation of the behaviour of the actual structure.

When any frame is loaded, it deflects and its shape under load is different from the undeformed shape. The deflection causes the axial loads in the members to act along different lines from those assumed in the analysis, as shown diagrammatically in Figure 2.6 and Figure 2.7. If the deflections are small, the consequences are very small and a first-order analysis (neglecting the effect of the deflected shape) is sufficiently accurate. However, if the deflections are such that the effects of the axial load on the deflected shape are large enough to cause significant further deflection, the frame is said to be sensitive to second-order effects. These second-order effects, or P-delta effects, can be sufficient to reduce the resistance of the frame.

Second-order effects are geometrical effects and should not be confused with material-non-linearity.

There are two categories of second order effects:

(i) Effects of deflections within the length of members, sometimes called $P.\delta$ (P-little delta) effects.

(ii) Effects of displacements of the intersections of members, sometimes called $P.\Delta$ (P-big delta) effects.

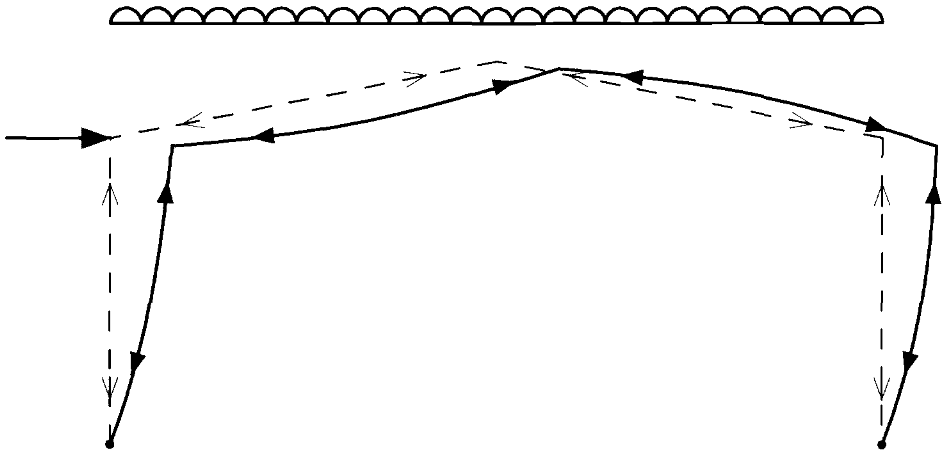


Figure 2.6 *Asymmetric or sway mode of deflection*

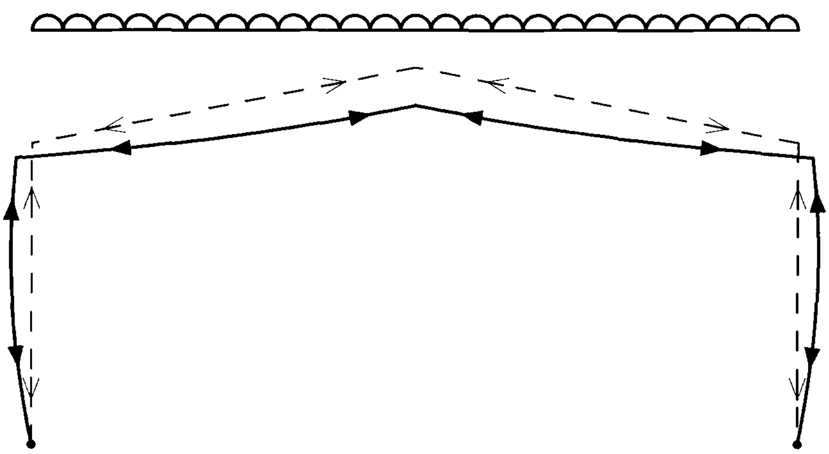


Figure 2.7 *Symmetric mode of deflection*

The practical consequence of $P.\delta$ and $P.\Delta$ effects is to reduce the stiffness of the frames below that calculated by first-order analysis. Single-storey portals are sensitive to the effects of the axial compression forces in the rafters. These forces are commonly of the order of 10% of the elastic critical buckling load (or Euler load) of the rafters, around which level the reduction in effective stiffness becomes important. Tied portals are especially sensitive to the effects because the axial compression forces in the rafters are commonly many times higher than in ordinary portals.

Because of the second-order effects due to the rafter compression, the simple check for λ_{cr} of multi-storey buildings in Clause 2.4.2.6 of BS 5950-1 is unconservative for portal frames.

2.4.2 $P.\delta$ (P–little delta) effects

$P.\delta$ effects on member behaviour are due to displacements at right-angles to a straight line between the ends of the member. Typical displacements are shown in Figure 2.8

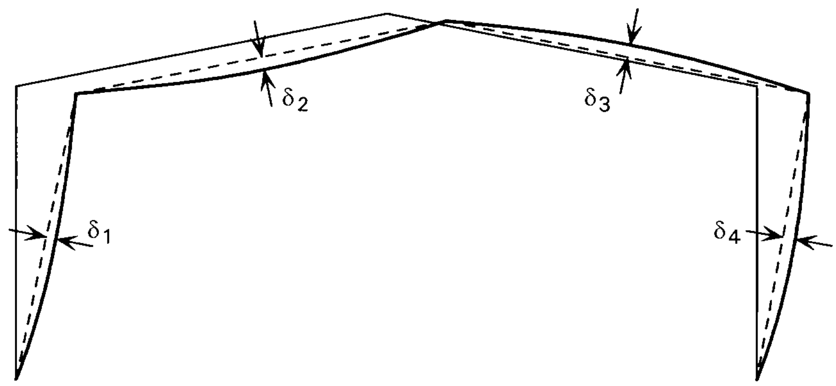


Figure 2.8 *Typical displacements d (little delta)*

These displacements may be the result of an external load or moment, or may be the result of the natural tendency to buckle under pure axial load. The displacements are the sum of the initial deformation of the member and the deflection due to loading. The result of the second-order effects is to increase the bending moment when the axial load is compressive (see Figure 2.9). This increase in the bending moment increases the curvatures, which reduces the effective stiffness of the members. Conversely, when the axial load is tensile, it increases the effective stiffness, though the effect will generally be minimal in common single-storey portal frames.

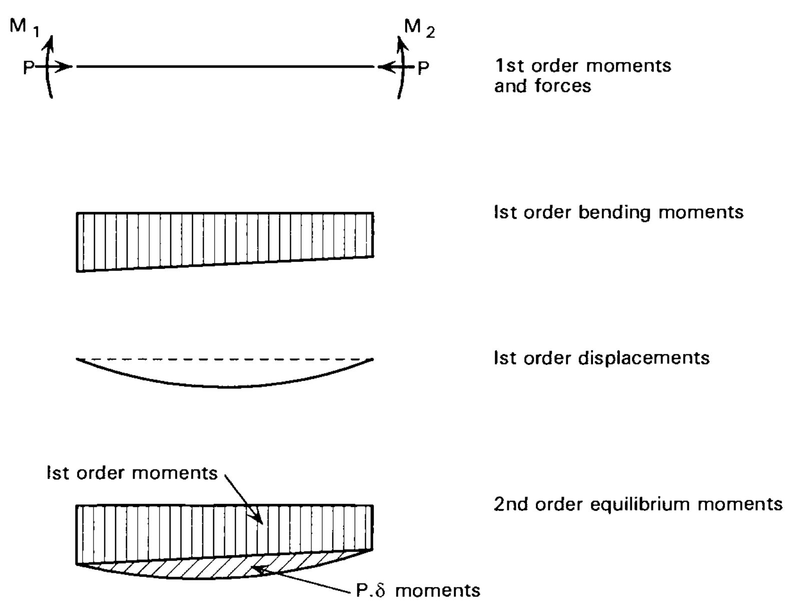


Figure 2.9 *P.δ (P – little delta) effects*

A simple illustration of $P.\delta$ effects is the behaviour of a simply supported beam carrying a distributed load that varies as a sine curve, as shown in Figure 2.10. The deflected shape is also a sine curve. The central deflection when there is no axial force is defined as δ_0.

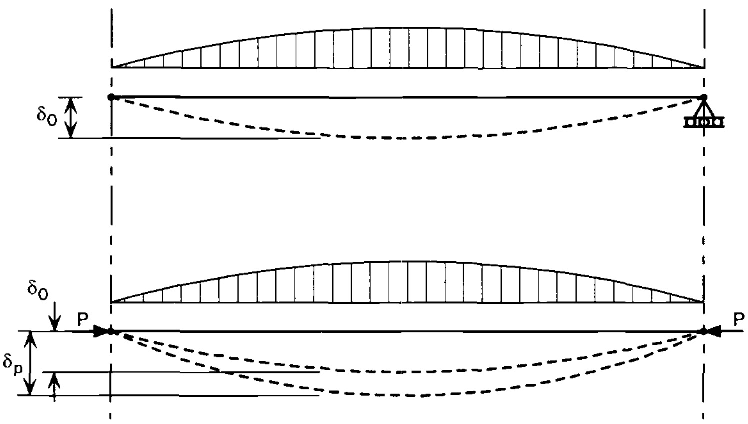


Figure 2.10 *P.δ effects in a simply supported beam*

Elastic theory[2] shows that when an axial compression, P, is applied, the central deflection increases to δ_p where:

$$\delta_P = \frac{\delta_0}{1 - P / P_{cr}}$$

where:

P_{cr} $= \dfrac{\pi^2 EI}{L^2}$ the Euler buckling load

E is Young's modulus

I is the inertia

L is the length.

As P increases, $\left[1 - \dfrac{P}{P_{cr}}\right]$ decreases so δ_p will increase. The stiffness, EI, of the beam affects not only the deflection δ_0, but also it affects the increase of the deflection $\{1/[1 - (P/P_{cr})]\}$.

The difference in bending moment between the first-order analysis and the second-order analysis, δM, is:

$$\delta M = P(\delta_P - \delta_0)$$

$$= \left(P\delta_0 \frac{P/P_{cr}}{1 - P/P_{cr}} \right)$$

Writing $\dfrac{P_{cr}}{P} = \lambda_{cr}$ then:

$$\delta M = P\delta_0 \left(\frac{1}{\lambda_{cr} - 1} \right)$$

2.4.3 P.Δ (P-big delta) effects in purely elastic frames

P.Δ effects are the effects on overall frame behaviour due to displacements of the ends of members at right-angles to their lengths. *P.Δ* effects are shown in their simplest form in Figure 2.11. A vertical load *P* is applied to the top of a cantilever column in which the column top is offset by a distance Δ from a vertical line through the column base. Therefore, the column must not only resist the axial load *P* but also a moment that increases along its length to a value of *P.Δ* at the base.

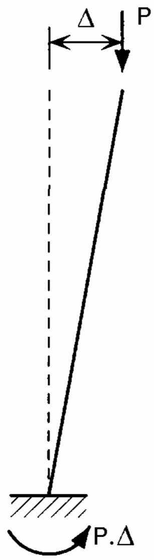


Figure 2.11 *P. Δ effects on a cantilever column*

The displacements, Δ, are the sum of the initial deformation of the frame and the deflection due to loading. For pitched roof portals, the principal modes of deflection are lowering of the apex and sway, as shown in Figure 2.12 and Figure 2.13.

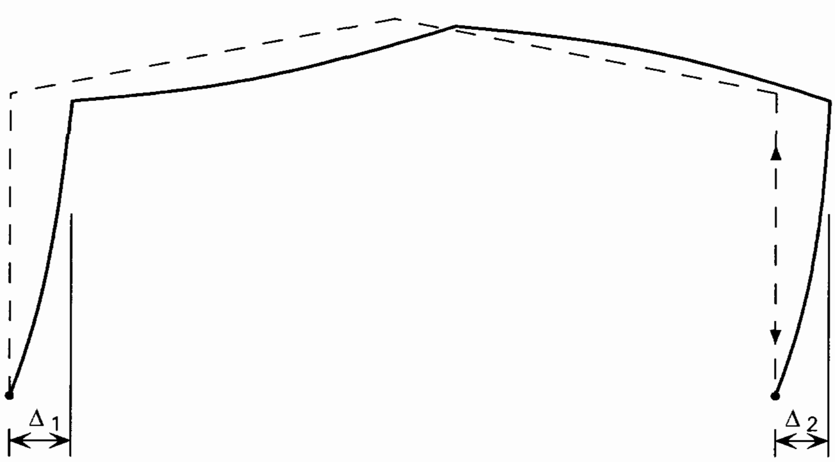


Figure 2.12 *Typical displacements Δ (big delta) in a sway mode*

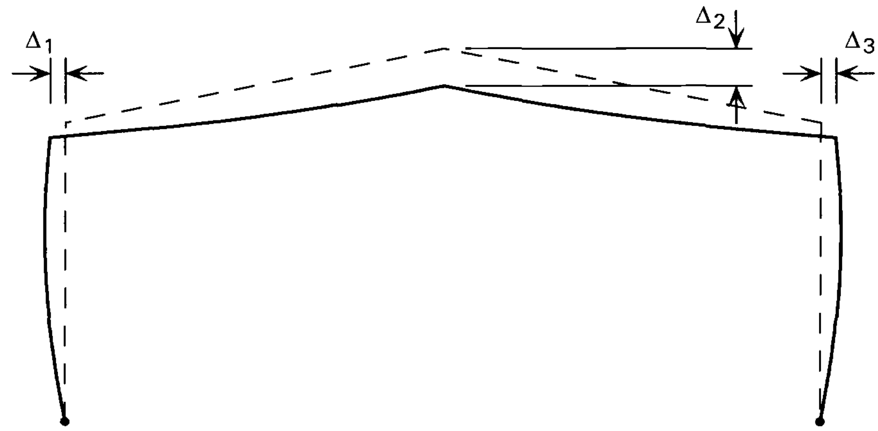


Figure 2.13 *Typical displacements Δ (big delta) in a symmetrical mode*

Another possible mode of failure that is sensitive to $P.\Delta$ effects is 'arching failure' or 'snap-through' of a pair of rafters, see Figure 2.14. In this form of failure, the spread of the valleys allows the apex of the roof to drop, so reducing the arching effect and increasing the bending moments in the rafters and columns.

Tension forces tend to increase the effective stiffness, but this is rarely significant in common structures.

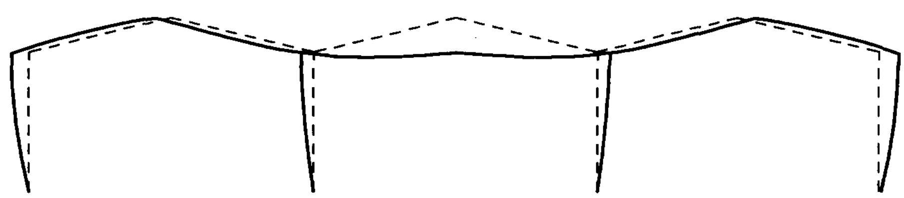


Figure 2.14 *Arching failure or snap-through*

Frames have critical buckling loads, V_{cr}, similar in concept to the critical buckling loads, P_{cr}, for struts as described in Section 2.3. The ratio of the elastic critical buckling load, V_{cr} , to the ULS, load V_{ULS} , is expressed as λ_{cr}, the critical buckling ratio.

In most practical single-storey portal frames, the first mode and second mode of buckling are the most important. This is because the first mode of buckling, shown in Figure 2.15, is similar in shape to the typical sway deflections shown in Figure 2.6. Also, the second mode of buckling, shown in Figure 2.16, is similar in shape to the typical symmetrical deflections shown in Figure 2.7.

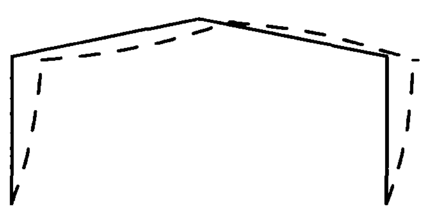


Figure 2.15 *First mode of buckling (buckling load V_{cr1})*

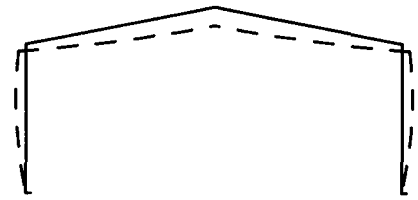


Figure 2.16 *Second mode of buckling (buckling load V_{cr2})*

The deflected form of a frame can be considered as the sum of a number of component deflected forms, where each component is in the shape of one of the buckling modes. Each component of the deflection will be increased according to the λ_{cr} for that mode. Therefore, if a particular deflection, δ, is made up of components δ_1 from buckling mode 1, and δ_2 from buckling mode 2, then the actual deflection, δ_v, including second-order effects, will be given by:

$$\delta_V = \delta_1 \left(\frac{\lambda_{cr1}}{\lambda_{cr1} - 1} \right) + \delta_2 \left(\frac{\lambda_{cr2}}{\lambda_{cr2} - 1} \right)$$

Normally in portal frames, the buckling load in the second mode, V_{cr2}, is at least twice the buckling load in the first mode, V_{cr1}, so the following conclusions can be drawn:

(i) If $V_{cr2} >> V_{cr1}$, then $\lambda_{cr2} >> \lambda_{cr1}$, so that $\left(\frac{\lambda_{cr2}}{\lambda_{cr2} - 1} \right) < \left(\frac{\lambda_{cr1}}{\lambda_{cr1} - 1} \right)$

This means that deflections similar to the first mode of buckling will introduce bigger P-delta effects than deflections similar to the second mode of buckling.

(ii) If the deflections from a load case are almost entirely similar to one mode of buckling, the P-delta effects will be dominated by

$\left(\frac{\lambda_{cr}}{\lambda_{cr} - 1} \right)$ for that mode of buckling.

In many practical frames, the critical load case for ULS is the mainly gravity load case:

1.4 x dead load + 1.6 x imposed load + NHF

where:

NHF is the sum of the notional horizontal forces (which is the very small load of 0.5% of the factored vertical loads).

For this load case, the deflection form is similar to the second mode of buckling, the symmetrical mode shown in Figure 2.16, up to the formation of the first plastic hinge. This buckling mode normally has a relatively high critical buckling load, V_{cr}, giving a relatively high value for λ_{cr}. Thus, this load case commonly has only small magnifications of P-delta effects up to the load level at which the first hinge forms.

Load cases involving lateral loads, such as lateral wind loads or crane horizontal loads, deflect into a shape similar to the first mode of buckling, the asymmetrical mode shown in Figure 2.15. This buckling mode often has a relatively low critical buckling load, V_{cr}, giving a relatively low value for λ_{cr}. Therefore, this load case commonly has significant magnification of P-delta effects.

2.4.4 P.Δ effects in frames with plastic hinges

When a plastic hinge has formed such that the frame becomes asymmetrical, there will be very significant sway deflections as the vertical load is increased, as shown in Figure 2.14. This sway occurs because asymmetric frames deflect horizontally when vertical loads are applied and the plastic hinge changes even symmetric frames to being asymmetric in terms of stiffness.

In addition, the reduction of frame stiffness due to the formation of hinges changes the buckling modes and reduces the value of V_{cr}, so the magnification of P-delta effects is increased.

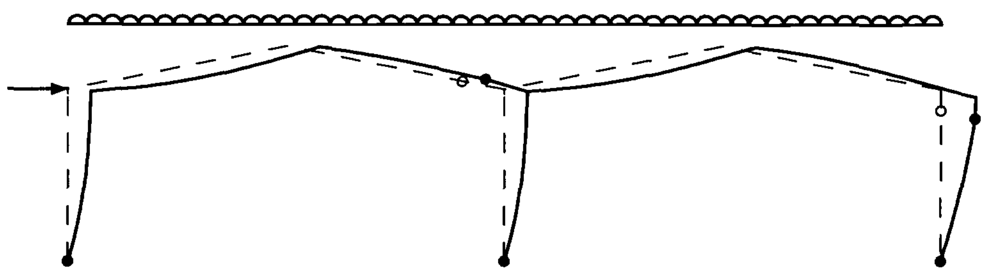


Figure 2.17 *Significant sway deflections due to plastic hinge formation in a 'gravity load' combination (the lateral load is the very small notional horizontal force)*

2.4.5 Different load cases on the same structure

The magnitude of the P-delta effects determines whether these effects can be neglected in the verification of a frame, or whether they must be explicitly included in the verification.

It is the magnitude of the deflection, combined with the magnitude of the axial load that is important. Therefore, the same frame may be insensitive to P-delta effects in one load case, but sensitive to P-delta effects in another load case. For example, a frame loaded so that it deflects symmetrically, such as the frame in Figure 2.7, might be relatively insensitive to P-delta effects because the deflection of the apex does not affect the forces and moments much. This is because the column spread is equal and opposite, so there is not a tendency to fall over sideways. However, the same frame loaded so that it deflects asymmetrically, such as the frame in Figure 2.7, might be relatively sensitive to P-delta because the sway causes a tendency to fall over sideways. This difference in sensitivity for symmetric and asymmetric load cases is common in portal structures with either single-span or multi-span frames.

2.4.6 Differences between portals and multi-storey frames

The differences between the stability checks for portals and the stability checks for multi-storey frames often cause confusion. The reason for different checks is because of the difference between the P-delta effects of the axial compression in the beams of multi-storey frames and the P-delta effects of axial compression in portal rafters.

The bending moment diagram for a multi-storey frame is shown in Figure 2.18. The bending moments in the columns induce shear forces in the columns that act in opposite directions above and below each beam. These opposing shear forces tend to cancel out, so the axial force induced in the floor beams to maintain horizontal equilibrium is small. These forces are shown in Figure 2.19. In addition, the span/depth ratio of floor beams is normally much less than the span/depth ratio of portal rafters. This is because floor loading is much greater than normal portal roof loading and floors generally have greater stiffness requirements to limit deflections or vibrations. Therefore the second-order effects in the floor beams in multi-storey frames of modest spans are usually so small that they do not affect the stability of the frame.

The formula for calculating λ_{cr} for multi-storey buildings in Clause 2.4.2.6 of BS 5950-1 is acceptable for multi-storey buildings, but not acceptable for calculating λ_{cr} for single-storey portal frames because it ignores any second-order effects in the beams.

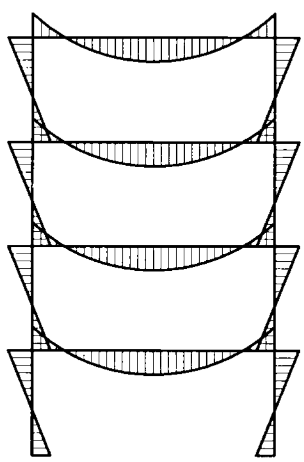


Figure 2.18 *Typical bending moments in a rigidly-jointed multi-storey frame*

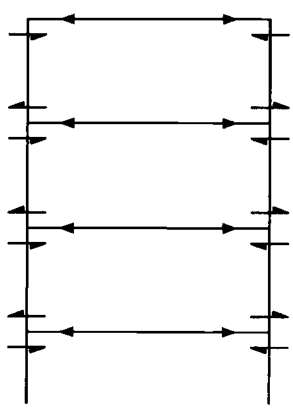


Figure 2.19 *Column shear and beam axial forces in a rigidly-jointed multi-storey frame*

3 SWAY-CHECK METHOD

3.1 Introduction

The Sway-check method for checking the in-plane stability of a portal frame requires only simple analysis techniques. The method is derived from the in-plane stability checks of the 1990 version of BS 5950-1. It applies to pitched-roof, monopitch and flat-roofed portal frames. The check identifies frames in which the second-order effects in the gravity load case (1.4 Dead Load + 1.6 Live Load) are sufficiently small that they may be ignored. This restriction is achieved by the geometrical limitations described in Section 3.2 and by a check on sway stiffness.

The method may be applied either by:

- the $h/1000$ check (Section 3.3) or
- the Formula method (Section 3.4).

For multi-span frames, the rafters in internal spans must be checked by the snap-through check (Section 3.5).

This method is not suitable for tied portals (see Section 5.3.5).

3.2 Geometrical limitations

The Sway-check method of BS 5950-1 is only valid when applied to frames in which the spans comply with the following limitations, shown in Figure 3.1. These limits are defined in BS 5950-1 Section 5.5.4.2.1.

- $L \leq 5h$
- $h_r \leq L/4$ and
- $(h_r/s_a)^2 + (h_r/s_b)^2 \leq 0.5$ for asymmetric rafters

where:

L is the span, taken as between centre-lines of the columns

h is the column height, taken as the height from the top of the foundation to the point of intersection of the centre-line of the rafter and the centre-line of the column; and

h_r, s_a and s_b are as defined in Figure 3.1.

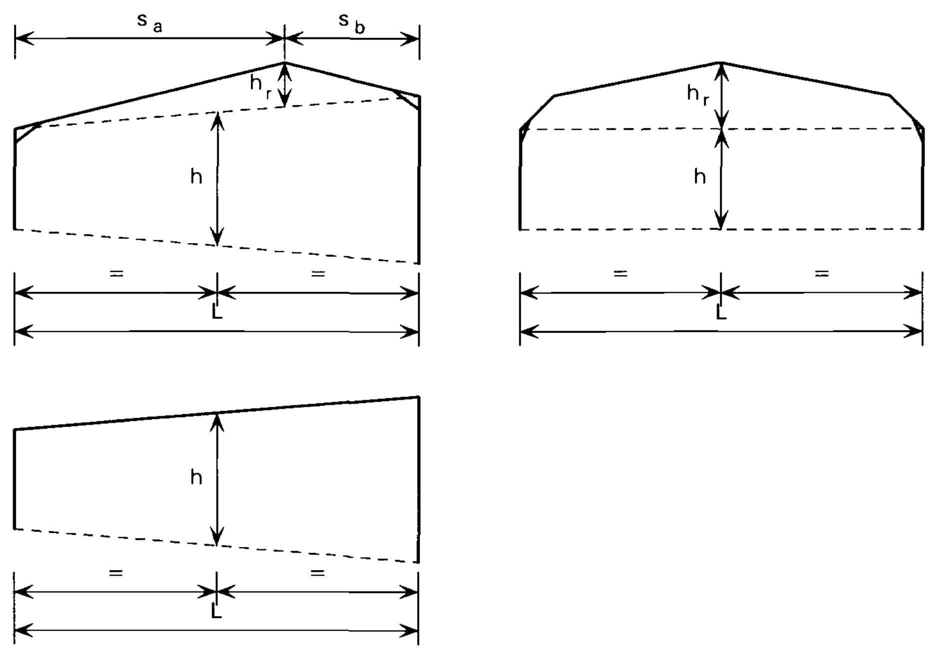


Figure 3.1 *Geometric parameters for single-span frames*

Where the internal columns are of similar stiffness to the external columns, each span should be considered as if it were a separate single span frame.

Where the internal columns are significantly more flexible than the external columns, the height h can be taken from a straight line between the column bases, as shown in Figure 3.2. A typical example would be a frame using UBs for the external columns but UCs for the internal columns.

Where valleys are supported on valley beams, the Sway-check method may be used, provided that the above limitations are observed. Although there is no column, column height must be assumed. This height is the distance from the intersection of the rafters at the valley above the straight line between the column bases (see Figure 3.2).

3.3 The h/1000 check

3.3.1 General

The stiffness of the frame is assessed by a check on the sway deflection due to the notional horizontal forces.

The design steps for 'gravity load' cases, as defined in BS 5950-1 Clause 5.5.4.2.2, are given in Section 3.3.2. The design steps for 'horizontal load' cases, as defined in BS 5950-1 Clause 5.5.4.2.3, are given in Section 3.3.3.

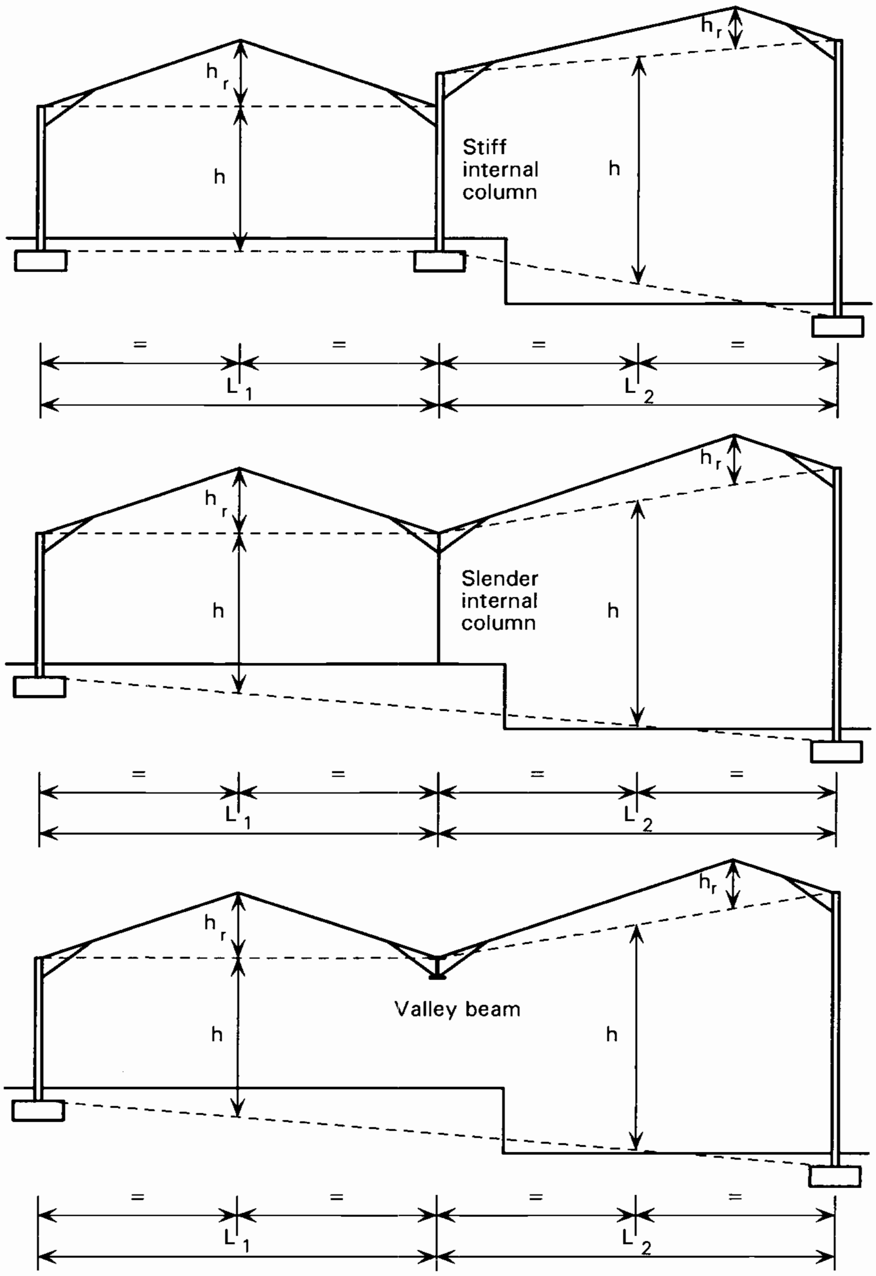


Figure 3.2 *Geometry of frames (h is measured from the top of the foundation)*

3.3.2 Gravity load cases – design steps

This Section gives the steps required to satisfy Clauses 5.5.2 or 5.5.3 of BS 5950-1 using the Sway-check method for gravity loads, as given in Clause 5.5.4.2.2. The loads considered are those in Load combination 1 (see Clauses 2.4.1.2 of BS 5950-1) and Crane combination 1 (see Clause 2.4.1.3 of BS 5950-1).

In the design check, notional horizontal forces are considered. Clause 5.5.4.2.2 states that the notional horizontal deflections, δ, should be calculated using the bare steel frame alone, ignoring any stiffening effects reducing sway, such as plan bracing in the roof or roof sheeting. This is because the sway deflection is acting as an indicator of the sensitivity of the frame to P-delta effects in the symmetric mode of failure shown in Figure 3.3. In this symmetric mode of failure, roof plan-bracing or roof sheeting will give very little assistance to the action of the bare steel frame.

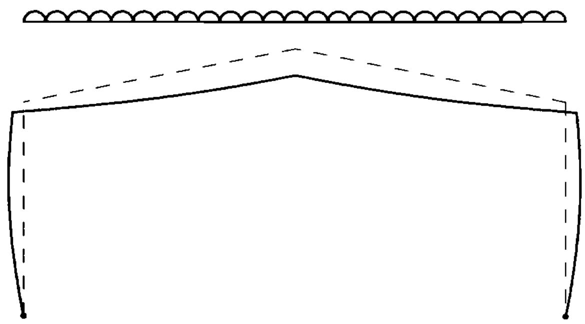


Figure 3.3 *Symmetric mode of failure*

Out-of-plane stability of members must also be checked, as required by BS 5950-1, Chapters 4 and 5, but is outside the scope of this document.

Note that the gravity load case is not suitable for stability portal frames used instead of cross bracing, which should be designed as a lateral load case as Section 3.3.3 or by the Amplified Moment method or Second-order analysis.

An alternative method of checking the frame stiffness using the span to depth ratio of the rafters is given in Section 3.4.

Design steps for plastic design

1. Check that the geometry of the frame is within the geometrical limits (see Section 3.2). If all the spans in the frame are satisfactory, the Sway-check method may be used for this frame.
2. Check the sway stiffness of the frame.
   (a) Calculate the notional horizontal forces (see Section 1.6). For this check (which is a check for the critical buckling ratio, λ_{cr}) 0.5% of vertical crane loads should be included if applicable.
   (b) Apply the notional horizontal forces in-plane (all in one direction) to the bare steel frame and calculate the column top deflections, δ, as shown in Figure 3.4.
   (c) Check that the column top deflections δ do not exceed $h/1000$, where h is the height of the column from the top of the foundation to the point of intersection of the rafter centre-line and the column centre-line. Note that the stiffness of the cladding (or other structure giving sway stiffness not arising from the portal frame) must not be considered when calculating δ.

   If all the column deflections in the frame satisfy the above, the Sway-check method is valid for the frame. In this case, the value of λ_r, the required load factor for frame stability, may be taken as 1.0 for the gravity load case.

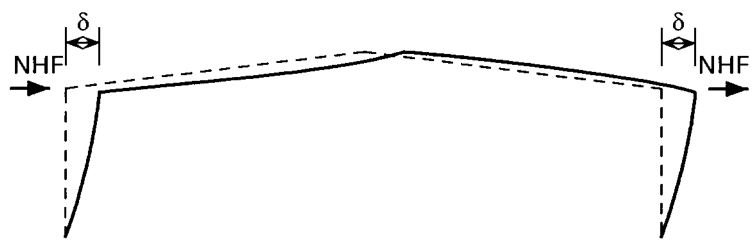


Figure 3.4 *Deflection from notional horizontal forces (NHF)*

3. For frames of three or more bays, check the snap-through stability (see Section 3.5).

4. Carry out a plastic analysis of the frame.

   Apply the gravity loads together with the notional horizontal forces (see Section 1.6) to the frame. In asymmetric frames, it will generally be necessary to apply two load cases, one with the NHF in one direction and the other with the NHF in the other direction to ensure that the most unfavourable load case is applied.

5. Check the strength of the frame.

   Calculate the plastic collapse factor, λ_p (for both directions of NHF where these have been applied as two load cases), and check that $\lambda_p \geq \lambda_r$ (=1.0).

Design steps for elastic design

Design steps 1, 2 and 3 for elastic design are the same as for plastic design.

4. Carry out an elastic analysis of the frame.

   Apply the gravity loads together with the notional horizontal forces to the frame and calculate the forces and moments around the frame. In asymmetric frames, it will generally be necessary to apply two load cases, one with the NHF in one direction and the other with the NHF in the other direction to ensure that the most unfavourable load case is applied.

5. Check the strength of the frame.

   Calculate and check the cross-sectional resistance using Clause 4.8 of BS 5950-1.

3.3.3 Lateral load cases – design steps

This Section gives the steps required to satisfy Clauses 5.5.2 or 5.5.3 of BS 5950-1 using the Sway-check method for horizontal loads, as Clause 5.5.4.2.3. It is applicable to frames where the applicability of the Sway-check method has already been confirmed (Steps 1 and 2 of Section 3.3.2). The loads considered are those in Load combination 2 and Load combination 3 (see Clauses 2.4.1.2 of BS 5950-1) and Crane combination 2 and Crane combination 3 (see Clause 2.4.1.3 of BS 5950-1).

These load cases are those in which there are externally applied horizontal forces acting in the plane of the frame, typified by the loads shown in Figure 3.5. It does not include load cases in which the only horizontal forces are the notional horizontal forces arising from vertical loads on this frame.

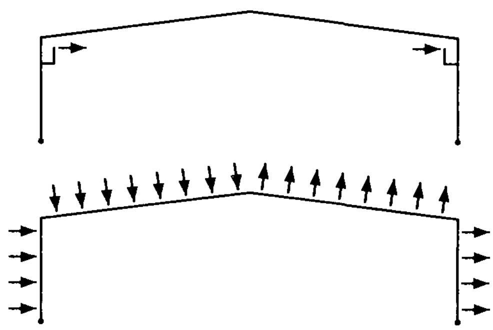


Figure 3.5 *Typical externally applied horizontal forces*

The mode of failure is the sway mode shown in Figure 3.6. In this mode, the sway deflection causes P-delta effects that subject the frame to moments and forces greater than those calculated by first-order analysis. Therefore, the resistance of the frame must exceed the resistance required by first-order analysis.

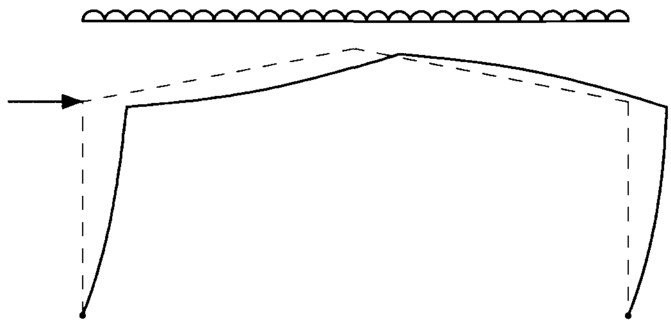


Figure 3.6 *Sway mode of failure*

Clause 5.5.4.2.3 states that, when calculating the deflections for the sway case, the sway stiffness of other structure, plan bracing and roof-sheeting may be included. The inclusion of the stiffness of the cladding, etc. should, however, only be considered if it can be guaranteed to remain throughout the life of the structure. Where sheeting is used to provide stiffness, these structural requirements must be noted in the Health and Safety File required under the CDM regulations.

Member out-of-plane stability must also be checked as required by BS 5950-1 Chapter 4 and 5, but is outside the scope of this document.

Design steps for plastic design

1. Calculate the approximate critical buckling ratio for the Sway-check method, λ_{sc}, for the frame.

   (a) Calculate the notional horizontal forces from the loads applied in the load combination being analysed (see Section 1.6). For this check 0.5% of vertical crane loads should be included.

   (b) Apply the notional horizontal forces in-plane (all in one direction) to the frame and calculate the column top deflections δ. (As noted above, the stiffness of any associated structure or cladding that reduces the column top deflections may be included in the calculation of the column top deflections.)

(c) Calculate $\lambda_{sc} = \dfrac{h}{200\delta}$

where h is the height of the individual column.

λ_{sc} is an approximation to the critical buckling ratio for the sway mode of buckling shown in Figure 2.12. λ_{sc} is calculated from the sway deflection caused by application of the notional horizontal forces derived from this load case.

Where λ_{sc} is less than 5.0, the Sway-check method should not be used.

2. Calculate the required load factor λ_r for frame stability.

$$\lambda_r = \frac{\lambda_{sc}}{\lambda_{sc} - 1}$$

If the loads are such that the axial forces in all the rafters and columns are tensile, then the required load factor λ_r should be taken as 1.0

3. Carry out a plastic analysis of the frame.

Apply the gravity and horizontal loads to the frame, *without* any notional horizontal forces.

4. Check the strength of the frame.

(a) Calculate the plastic collapse factor λ_p and check that $\lambda_p \geq \lambda_r$.

(b) Check the strength and out-of-plane stability at λ_r.

Design steps for elastic design

Design steps 1 and 2 for elastic design are the same as for plastic design.

3. Carry out an elastic analysis of the frame.

Apply the gravity and horizontal loads to the frame without any notional horizontal forces and calculate the forces and moments around the frame.

4. Check the strength of the frame.

(a) Calculate and check the cross-sectional resistance using Clauses 4.8 of BS 5950-1 and using amplified moments and forces, taken as the values given by linear elastic analysis multiplied by λ_r.

(b) Check the out-of-plane stability at λ_r.

3.3.4 Base stiffness for calculation of δ from the notional horizontal forces

BS 5950-1 Clause 5.1.3 gives guidance on the base stiffness that may be assumed in design. The provisions for ULS analysis may be used in the analysis model for the deflection δ caused by the notional horizontal forces. It is important to note that the Sway-check is to check the stiffness of the frame at ULS, so only the ULS base stiffness values may be used, not the SLS values.

Therefore, the following base stiffnesses may be used:

Base with a pin or rocker

The base stiffness should be taken as zero in the calculation of δ.

Nominally pinned base

The base stiffness may be taken as 10% of the column stiffness for the calculation of δ. For the same frame, the base moments transmitted to the foundation may be taken as zero provided the ULS analysis of the frame, from which the moments and forces around the frame are found, assumes that the bases are pinned.

Nominal semi-rigid base

A nominal base stiffness of up to 20% of the column stiffness may be assumed for the ULS calculations, provided that the foundation is designed for the moments and forces obtained in the analysis. Therefore, the base stiffness may be taken as 20% of the column stiffness for the calculation of δ, provided that the foundations are designed to carry the moments from the ULS global analysis for every load case; there is a cost implication.

Nominally rigid base

The base stiffness should be taken as equal to the column stiffness and the foundation must be designed to resist moments from all load cases. Therefore, the base stiffness may be taken as equal to the column stiffness for the calculation of δ. Note that the bases should not be assumed to be rigid for this check.

3.4 The formula method

3.4.1 General

The L_b/D formula is a stiffness check which is approximately equivalent to the Sway-check by calculating deflections in Section 3.3. The stiffness of the frame is assessed by a formula. This formula was derived for regular frames with columns at every valley and with roof load as the only imposed load. Thus the application of this method is restricted to frames that are not subject to loads from crane gantries or other concentrated loads larger than those from purlins.

3.4.2 Gravity load cases – design steps

This check is for load cases with no externally applied horizontal loads other than the notional horizontal forces (NHF).

Note that this check is not suitable for stability portal frames used instead of cross-bracing, which should be designed for a lateral load case (see Section 3.4.3) or by the Amplified Moment method or Second-order analysis.

Design steps for plastic design and elastic design

The design steps for both plastic and elastic design are the same as for the $h/1000$ method (see Section 3.3.2), except that the check on sway stiffness, Step 2, is replaced by a limitation on the span to depth ratio of the rafters, based on an expression that involves the geometry of the frame, the stiffness of the columns and rafters and the strength of the rafters. Step 2 of Section 3.3.2 becomes:

Check that the span to depth ratio of the rafter satisfies:

$$\frac{L_b}{D} \leq \frac{44\,L}{\Omega h}\left[\frac{\rho}{4 + \rho L_r / L}\right]\left[\frac{275}{p_{yr}}\right]$$

in which:

$$L_b = L - \left[\frac{2\,D_h}{D_s + D_h}\right] L_h$$

$$\rho = \left[\frac{2\,I_c}{I_r}\right]\left[\frac{L}{h}\right] \quad \text{for a single-span frame}$$

$$\rho = \left[\frac{I_c}{I_r}\right]\left[\frac{L}{h}\right] \quad \text{for a multi-span frame}$$

Ω is the arching ratio $= W_r / W_o$

where:

- D is the cross-section depth of the rafter
- D_h is the additional depth of the haunch (see Figure 3.7)
- D_s is the depth of the rafter, allowing for its slope (see Figure 3.7)
- h is the main column height
- I_c is the in-plane second moment of area of the column (taken as zero if the column is not rigidly connected to the rafter, or if the rafter is supported on a valley beam)
- I_r is the in-plane second moment of area of the rafter
- L is the span of the bay
- L_b is the effective span of the bay
- L_h is the length of the haunch (see Figure 3.7)
- L_r is the total developed length of the rafters see (Figure 3.8)
- p_{yr} is the design strength of the rafters in N/mm^2
- W_o is the value of W_r for plastic failure of the rafters as a fixed-ended beam of span L (see Figure 3.9)
- W_r is the total factored vertical load on the rafters of the bay (see Figure 3.9).

If the two columns or the two rafters of a bay differ, the mean value of I_c/I_r should be used.

If the haunches at each side of the bay are different, the mean value of L_b should be used.

The strength checks for both plastic and elastic design are carried out in the same way as Steps 3 and 4 of the $h/1000$ method.

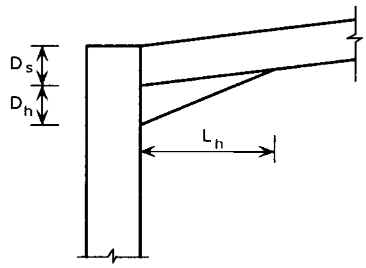


Figure 3.7 *Dimensions of a haunch*

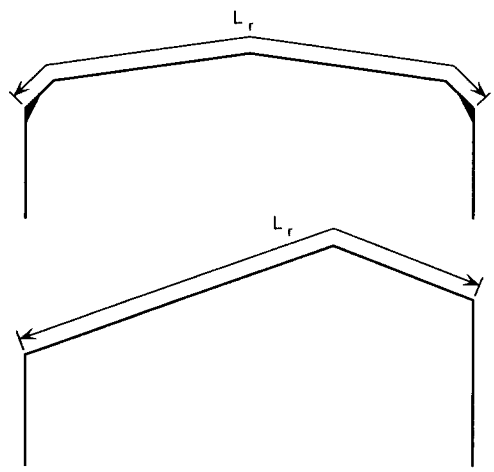


Figure 3.8 *Developed length of rafter*

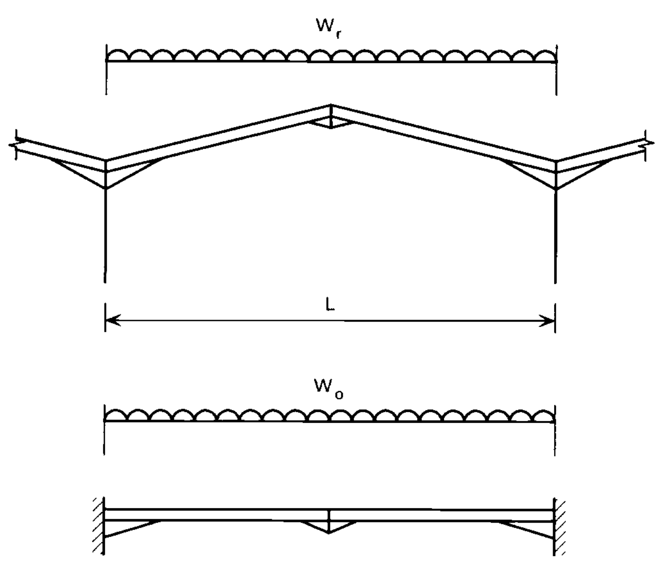


Figure 3.9 *Loads for calculating the arching ratios*

3.4.3 Lateral load cases – design steps

Lateral load cases are load cases in which there are externally applied horizontal forces acting in the plane of the frame. The formula method is not for load cases in which the only horizontal forces are the notional horizontal forces arising from vertical loads applied to the portal. The mode of failure is that shown in Figure 3.6. In this mode, the sway deflection causes P-delta effects as subject the frame to moments and forces greater than those calculated by first-order analysis.

Design steps for plastic design and elastic design

The design steps for both plastic and elastic design are the same as for the $h/1000$ method (see Section 3.3.3), except that approximate critical buckling ratio for the Sway-check method, λ_{sc}, is calculated from a formula that involves the same parameters as those used for the L_b/D formula for the gravity load case (see Section 3.4.2). Step 1 of the method given in Section 3.3.3 becomes:

Calculate the approximate critical buckling ratio

$$\lambda_{sc} = \frac{220\,DL}{\Omega h L_b}\left[\frac{\rho}{4+\rho L_r / L}\right]\left[\frac{275}{p_{yr}}\right]$$

Where λ_{sc} is less than 5.0, the Sway-check method should not be used.

If the wind loads are such that the axial forces in the rafters are tensile, then the required load factor λ_r should be taken as 1.0 because tensile forces cause no additional destabilising forces.

3.5 Snap-through check

The snap-through check, in BS 5950-1 Clause 5.5.4.3, is to check that internal spans of a multi-span frame are adequately modelled in a first-order analysis.

In pitched-roof portals, gravity loads applied to the rafters cause the ends of the rafters to spread as the rafter deflects downwards, see Figure 3.10. In multi-span frames, the internal spans are restricted from spreading by the stiffness of the external spans. The horizontal reaction from the external spans coupled with the rise of the rafters in the internal span causes an arching action in the internal span. This arching action means that the vertical load capacity is greater than the capacity due to bending alone of the rafters. However, this increased capacity depends on the restraint from the external spans. This restraint will not be available if the stiffness of the frame is too low.

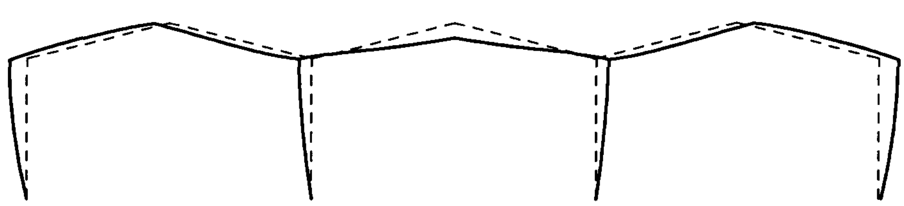


Figure 3.10 *Rafter spread in multi-span frames*

The formula in BS 5950-1 Clause 5.5.4.3 defines a limit to the span to depth ratio of the rafter to ensure adequate stiffness, expressed as:

$$\frac{L_b}{D} \leq \frac{22(4 + L/h)}{4(\Omega - 1)}\left(1 + \frac{I_c}{I_r}\right)\frac{275}{p_{yr}}\tan 2\theta$$

in which the symbols are as defined in Section 3.4, except for θ which is defined below.

θ is the slope of the rafters for a symmetrical ridged span.

$\theta = \tan^{-1}(2h_r/L)$ for other roof shapes

where:

h_r is defined in Figure 3.1 and Figure 3.2.

Where the arching ratio Ω is less than 1.0, no limit need be placed on L_b/D because the vertical load capacity from bending alone is more than sufficient.

The L_b/D formula in the 2000 issue of BS 5950, given above, differs slightly from the 1990 issue in that Ω appears only once in the formula in 2000 issue. This change has been made to ensure that the elastic critical buckling factor, λ_{cr}, remains equal to or greater than 10, to ensure that the second-order effects are insignificant.

The Snap-through check is most likely to be significant where the rafters in internal spans have a lower plastic moment of resistance than required for external spans. The lower moment resistance would cause more of the vertical loads to be carried by arching action, which creates significant axial thrusts in the rafters and could cause snap-through (see Section 2.4.3).

4 AMPLIFIED MOMENTS METHOD

4.1 Application – design steps

The amplified moment method is appropriate where the frame does not meet the Sway-check limitations. It permits the calculation of the load factor for frame stability, λ_r, which is used to account for the deflections of the frame under load. The method requires the determination of the lowest critical buckling ratio, λ_{cr}, for the particular load case on the frame. No method of determining λ_{cr} is given in BS 5950-1.

There is a limit on the application of the method. If $\lambda_{cr} < 4.6$, the frame is too flexible to be designed using this method. If $\lambda_{cr} \geq 10$, the frame is considered to be 'stiff' and λ_r is taken as 1.0.

This method is not suitable in general for tied portals because it does not account for non-linearity in the rafter-tie system (see Section 5.3.5).

This Section gives the steps required to satisfy Clauses 5.5.2 or 5.5.3 of BS 5950-1, using the Amplified moments method, as in Clause 5.5.4.4 of BS 5950-1.

In BS 5950-1, the critical buckling ratio, λ_{cr} from the lowest mode of buckling is required, as it is possible to produce unconservative designs if higher modes are used. The lowest mode is usually the sway mode. The Amplified Moments Method is most accurate when the collapse mode of the frame is the same as the buckling mode from which λ_{cr} is derived. Therefore, the Amplified Moments method is most accurate for load cases causing deflection in a sway mode, as exemplified by Figure 2.6. For load cases causing deflection in the symmetrical mode, the Amplified Moment method is relatively conservative because the deflection mode is similar to a higher mode of buckling.

The Snap-through check of Clause 5.5.4.3 of BS 5950-1 does not need to be applied when using the Amplified moment method.

Out-of-plane stability members must also be checked as required by BS 5950-1 Chapters 4 and 5, but this is outside the scope of this document.

Design steps for plastic design

1. Calculate the critical buckling ratio, λ_{cr}, for the lowest buckling mode from the load case being analysed. See Section 4.3.3 or Section 4.3.4.

2. Calculate the required load factor for frame stability, λ_r:

   if $\lambda_{cr} \geq 10$ $\quad \lambda_r = 1.0$

   if $10 > \lambda_{cr} \geq 4.6$ $\quad \lambda_r = \dfrac{0 \cdot 9\lambda_{cr}}{\lambda_{cr} - 1}$

   Note: if $\lambda_{cr} < 4.6$ the method is not applicable.

3. Carry out a plastic analysis of the frame.

Apply the loads to the frame. For load combinations other than Load combinations 1 (see Clause 2.4.1.2 of BS 5950-1), the notional horizontal forces need not be applied (see Clause 2.4.2.4 of BS 5950-1). Where NHF are applied to asymmetric frames or symmetric frames with asymmetric loading, it will generally be necessary to apply two load cases, one with the NHF in one direction and the other with the NHF in the other direction to ensure that the most unfavourable load case is applied.

4. Check the strength of the frame

   (a) Calculate λ_p and check that $\lambda_p \geq \lambda_r$.

   (b) Check the member strength and out-of-plane stability at λ_r.

Design steps for elastic design

Design steps 1 and 2 for elastic design are the same as for plastic design.

3. Carry out an elastic analysis of the frame, applying the loads as for plastic design.

   Calculate the forces and moments around the frame using linear elastic analysis (first-order analysis).

4. Check the strength of the frame

   (a) Where $\lambda_r > 1.0$, calculate amplified moments and forces, taken as the values given by linear elastic analysis multiplied by λ_r.

   (b) Check the cross-sectional resistances using the amplified moments and forces using BS 5950-1 Clauses 4.8.

4.2 Background to method

The Amplified moment method is based on the Merchant-Rankine[3,4,5,6,7] equation as modified by Wood[8]. It is the same method as used for plastic design of multi-storey frames in Clause 5.7 of the 1990 issue of BS 5950-1.

The Merchant-Rankine equation for predicting the load factor against failure, accounting for second-order effects, is:

$$\frac{1}{\lambda_{cr}} + \frac{1}{\lambda_p} = \frac{1}{\lambda_f}$$

where:

λ_{cr} is the elastic critical buckling factor $= V_{cr}/V_{ULS}$

λ_p is the plastic collapse factor $= V_{pl}/V_{ULS}$

λ_f is the load factor against failure, accounting for second-order stability effects $= V_f/V_{ULS}$

in which:

V_{cr} is the elastic critical buckling load of the lowest mode of the bare frame

V_{pl} is load to cause plastic collapse of the frame in the absence of second order stability effects

V_f is the failure load accounting for second-order stability effects

V_{ULS} is Ultimate Limit State load for the load case being considered.

Note that V_{pl} differs from V_{ULS} in that V_{pl} is the load that the frame can carry at plastic collapse (according to first order calculations) whereas V_{ULS} is the load that is applied at ULS.

The distribution of load in V_{cr}, V_{pl} and V_f should be the same as the distribution of load in V_{ULS}.

For $\lambda_f \geq 1.0$, the Merchant-Rankine equation reduces to:

$$\lambda_p \geq \frac{\lambda_{cr}}{\lambda_{cr} - 1}$$

Wood[8] recommended that the beneficial effects of cladding and strain hardening should be allowed for by the following modified version of the Merchant-Ranking criterion:

$$\text{For } 4 \leq \frac{\lambda_{cr}}{\lambda_p} \leq 10, \quad \lambda_f = \lambda_p (0.9 + \lambda_p / \lambda_{cr})$$

This equation can be re-written in the form of the Merchant-Ranking equation:

$$\frac{1}{\lambda_f} = \frac{0.9}{\lambda_p} + \frac{1}{\lambda_{cr}}$$

As explained in Kirby and Nethercot[3], this can be expressed for $\lambda_f \geq 1.0$ as the requirement that:

$$\text{For } 10 > \lambda_{cr} \geq 4.6, \quad \lambda_p \geq \frac{0.9\lambda_{cr}}{\lambda_{cr} - 1}$$

in which λ_{cr} is taken as the value for the lowest buckling mode of the bare frame.

This requirement is generalised in Clause 5.5.4.4 of BS 5950-1 as:

$$\lambda_r = 0.9\ \lambda_{cr} / (\lambda_{cr} - 1)$$

so that it can be applied to both plastic and elastic design.

4.3 Calculation of λ_{cr} for BS 5950-1

4.3.1 General

The value of λ_{cr} for use in Clause 5.5.4.4 must be the true value, not the approximate value derived from the formulae in Chapters 2 or 5 of BS5950-1. The elastic critical buckling load V_{cr} or the elastic critical buckling factor λ_{cr} for the first mode will be available in numerous software packages that perform elastic analysis. The value of λ_{cr} is calculated for the frame assuming it is entirely elastic and that no plastic hinges exist.

The value of λ_{cr} depends on the magnitude of the applied load. Therefore, unless the lowest value of λ_{cr} from all load combinations is used throughout, the value of λ_{cr} used in the equation for λ_r must be calculated for each load combination, giving different values of λ_r for each load combination.

4.3.2 Structural details that lower the value of λ_{cr}

The details of frames can affect the stability significantly. Where connections are not stiff moment resisting connections, or where the arrangement of a frame is irregular, the effects on stability should be carefully considered.

Portals are sometimes detailed with pin-ended props serving as the internal columns. A pin-ended prop tends to destabilise the whole structure because any lateral displacement causes the prop to induce an additional lateral load, instead of the restoring shear that would be induced by a continuous column, (see Davies[5,6]). Therefore, any pin-ended members must be correctly modelled as pin-ended.

Valley beams do not provide any stabilising effect to the whole structure, thus they should be modelled accordingly, e.g. as sliding supports. If a valley beam is free to twist at the frame it supports and if there is no lateral restraint to the top flange at this point, then the valley beam will act as a very short pin-ended prop and destabilise the frame, as discussed by Davies.

4.3.3 Computer solutions for λ_{cr}

Computer solutions will normally be preferable for design office use.

The most obvious differences between the various available computer solutions are the differences in the output. The principal differences are the number of buckling modes calculated and the output of the buckling mode shapes.

The buckling modes may be expressed as loads, V_{cr}, or as the ratio of (buckling load)/(applied load), λ_{cr} as explained in Section 2.3. Only the first buckling mode is required for the Amplified Moment method. However, it can be helpful to the designer to know the mode shape of higher modes if λ_{cr} is unpleasantly low. The mode shape helps to show how and where to stiffen the structure.

4.3.4 Solutions without a computer

Although a computer solution will normally be preferable, stability functions can be used to calculate the elastic critical buckling loads of frame structures. Unfortunately, a rigorous solution is long and complicated.

For hand calculations using stability functions, acceptable approximations may be introduced by making the following assumptions:

(i) The elastic critical buckling load is not affected by the distribution of transverse load along the members. Only the axial loads need be considered. This is an old and well-respected assumption.

(ii) The maximum axial load in each member is assumed to act along its full length. This is a conservative assumption.

(iii) The stiffening effect of haunches is ignored. This is a conservative assumption.

Axial loads should be calculated from an elastic analysis. They may be calculated from standard results, as illustrated in References 9, 10, 11, assuming fully pinned/fixed bases for the buckling analysis of frames with nominally pinned/fixed bases.

To reduce the calculation to a simple process suitable for design office use, Davies[5,6] used stability functions to produce simple formulae to calculate the approximate buckling load of portal frames with pinned or infinitely rigid bases. The original work by Davies on stability of portal frames was extended by King[12] to account for the partial fixity of nominally pinned bases and the slight flexibility of nominally rigid bases. The work of Davies and King is summarised in Section 4.4 below.

The formula in BS 5950-1, Clause 2.4.2.6, is not valid for single-storey portals as it ignores the compression in the rafters. That equation is intended for multi-storey buildings, not single-storey pitched-roof portals.

4.3.5 Base stiffness for calculation of λ_{cr}

BS 5950-1, Clause 5.1.3, gives guidance on the base stiffness that may be assumed in design. The provisions for ULS analysis may be used in the analysis model for λ_{cr}. It is important to note that the Amplified Moment method uses the stiffness of the frame at ULS, so only the ULS base stiffness values may be used, not the SLS values.

Therefore the following base stiffnesses may be used:

Base with a pin or rocker

The base stiffness should be taken as zero in the calculation of λ_{cr}.

Nominally pinned base

The base stiffness may be taken as 10% of the column stiffness for the calculation of λ_{cr}. For the same frame, the base moments transmitted to the foundation may be taken as zero, provided that the ULS analysis of the frame, from which the moments and forces around the frame are found, assumes that the bases are pinned.

Nominal semi-rigid base

A nominal base stiffness of up to 20% of the column stiffness may be assumed for the ULS calculations, provided that the foundation is designed for the moments and forces obtained in the analysis. Therefore, for the calculation of λ_{cr}, the base stiffness may be taken as 20% of the column stiffness but at the cost of designing the foundations to carry the moments from the ULS global analysis for every load case.

Nominally rigid base

The base stiffness should be taken as equal to the column stiffness and the foundation must be designed to resist moments from all load cases. Therefore, the base stiffness may be taken as equal to the column stiffness for the calculation of λ_{cr}. Note that the bases should not be assumed to be rigid for this check.

The above assumptions have been used in deriving the approximate formulae for λ_{cr} given in Section 4.4.

4.4 Simplified hand solutions for λ_{cr}

4.4.1 General

This section is from Davies[5,6,7] with the extensions of Davies' work to other base conditions by King[12].

In this method, the frame is considered as a series of sub-divisions (see Figure 4.1) including:

(i) Rafter pairs (see Section 4.4.2).

(ii) External column + rafter (see Section 4.4.3).

(iii) Internal column + rafter each side (see Section 4.4.4).

(iv) Equivalent frame for frames with props or valley beams (see Section 4.4.5).

For each ULS load combination analysed, λ_{cr} should be found for each of the above sub-divisions and the lowest λ_{cr} should be used throughout the structure for that particular load combination. (The very lowest λ_{cr} could be used for all load combinations, but it would result in a conservative design).

Column and rafter loads should be the values calculated by elastic analysis, which may be found by first-order computer analysis or by the formulae in reference books[9,10,11].

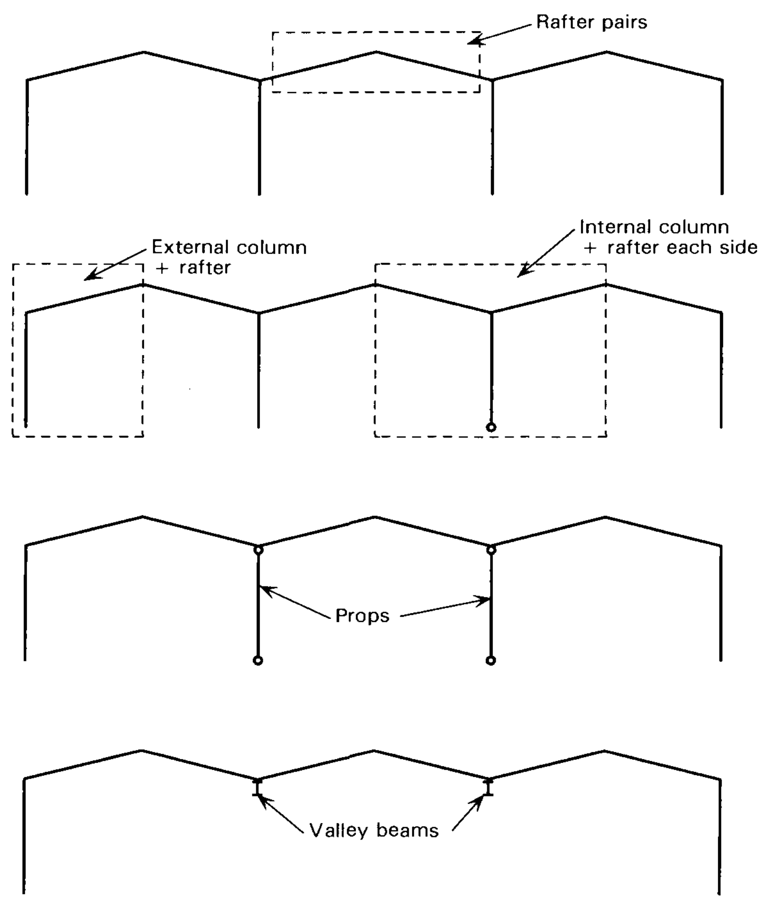


Figure 4.1 *Sub-divisions of frames for analysis without computer*

4.4.2 Rafter pairs

This Section checks the 'rafter pair' sub-divisions of the structure.

It checks that the 'arch' formed by each rafter pair does not collapse; see Figure 4.2.

The theory is due to Horne[13] and forms the basis of the rules of Clause 5.5.4.3 of BS 5950-1. It is re-expressed by Davies[7].

For roof slopes in the range $0 \leq \theta_r \leq 20^o$,

$$\lambda_{cr} \approx \left[\frac{D}{L_b}\right]\left[\frac{55(4 + L/h)}{\Omega - 1}\right]\left[1 + \frac{I_c}{I_r}\right]\left[\frac{275}{p_{yr}}\right]\tan 2\theta$$

where:

L_b is the effective span L–L_h

and other symbols are as defined in Section 3.4.2, except for θ which is defined in Section 3.5.

Figure 4.2 *Arching failure of rafters*

4.4.3 External column and rafter

This checks the 'external column and rafter' sub-divisions of the structure. The theory is due to Davies, but modified to include an explicit column base stiffness in (b) and (c) below.

(a) Truly pinned bases, or bases with rockers, as Clause 5.1.3.1 of BS 5950-1.

$$\lambda_{cr} = \frac{3EI_r}{s\left[0.3P_r s + \left(1 + \frac{1.2}{R}\right)P_c h\right]}$$

This may be expressed in terms of the rafter and column Euler buckling loads as:

$$\lambda_{cr} = \frac{1}{\left(\frac{P_r}{P_{r.crit}}\right) + (4 + 3.3R)\left(\frac{P_c}{P_{c.crit}}\right)}$$

(b) Nominally pinned bases, as Clause 5.1.3.3 of BS 5950-1.

$$\lambda_{cr} = \frac{(4.2 + 0.4R)EI_r}{s\left[0.42P_r s + \left(1.16 + \frac{1.2}{R}\right)P_c h\right]}$$

This may be expressed in terms of the rafter and column Euler buckling loads as:

$$\lambda_{cr} = \frac{(1 + 0.1R)}{\left(\frac{P_r}{P_{r.crit}}\right) + (2.9 + 2.7R)\left(\frac{P_c}{P_{c.crit}}\right)}$$

(c) Nominally rigid bases, as Clause 5.1.3.2 of BS 5950-1

$$\lambda_{cr} = \frac{5E(10 + 0.8R)}{\left(\frac{5P_r s^2}{I_r}\right) + (2.6R + 4)\left(\frac{P_c h^2}{I_c}\right)}$$

This may be expressed in terms of the rafter and column Euler buckling loads as:

$$\lambda_{cr} = \frac{(1 + 0.08R)}{\left(\frac{P_r}{P_{r.crit}}\right) + (0.8 + 0,52R)\left(\frac{P_c}{P_{c.crit}}\right)}$$

Where (for the above expressions):

E is the Young's modulus of steel = 205 kN/mm^2

I_r is the rafter inertia in the plane of the portal

I_c is the column inertia in the plane of the portal

s is the rafter length along the slope (eaves to apex)

h is the column height

$$R = \frac{\textit{column stiffness}}{\textit{rafter stiffness}} = \left(\frac{\frac{I_c}{h}}{\frac{I_r}{s}}\right) = \frac{I_c s}{I_r h}$$

P_c is the axial compression in column from elastic analysis

Note: *This differs from BS 5950-1 notation which defines P_c as the capacity of the compression member.*

P_r is the axial compression in rafter from elastic analysis

$$P_{c.crit} = \frac{\pi^2 EI_c}{h^2} = \text{Euler buckling load of the column}$$

$$P_{r.crit} = \frac{\pi^2 EI_r}{s^2} = \text{Euler buckling load of the rafter.}$$

4.4.4 Internal column and rafter each side

This checks the 'internal column and rafter' sub-divisions of the structure. The theory is as in Section 4.4.3 but modified for internal columns.

(a) Truly pinned bases, or bases with rockers, as Clause 5.1.3.1 of BS 5950-1

$$\lambda_{cr} = \frac{1}{\left(\frac{P_{r\ell}}{P_{r\ell.crit}}\right)R_\ell + \left(\frac{P_{rr}}{P_{rr.crit}}\right)R_r + \left(4_r + 3.3R_2\right)\left(\frac{P_c}{P_{c.crit}}\right)}$$

which, in the case of identical rafter forces, sections and lengths gives:

$$\lambda_{cr} = \frac{1}{\left(\frac{P_r}{P_{r.crit}}\right) + \left(4 + 3.3R_2\right)\left(\frac{P_c}{P_{c.crit}}\right)}$$

(b) Nominally pinned bases, as Clause 5.1.3.3 of BS 5950-1

$$\lambda_{cr} = \frac{(1 + 0.1R_2))}{\left(\frac{P_{r\ell}}{P_{r\ell.crit}}\right)R_\ell + \left(\frac{P_{rr}}{P_{rr.crit}}\right)R_r + \left(2.9 + 2.7R_2\right)\left(\frac{P_c}{P_{c.crit}}\right)}$$

which in the case of identical rafter forces, sections and lengths gives

$$\lambda_{cr} = \frac{(1 + 0.1R_2)}{\left(\frac{P_r}{P_{r.crit}}\right) + \left(2.9 + 2.7R_2\right)\left(\frac{P_c}{P_{c.crit}}\right)}$$

(c) Nominally rigid bases, as Clause 5.1.3.2 of BS 5950-1

$$\lambda_{cr} = \frac{(1 + 0.08R_2)}{\left(\frac{P_{r\ell}}{P_{r\ell.crit}}\right)R_\ell + \left(\frac{P_{rr}}{P_{rr.crit}}\right)R_r + \left(0.8 + 0.52R_2\right)\left(\frac{P_c}{P_{c.crit}}\right)}$$

which, in the case of identical rafter forces, sections and lengths gives

$$\lambda_{cr} = \frac{(1 + 0.8R_2)}{\left(\frac{P_r}{P_{r.crit}}\right) + \left(0.8 + 0.52R_2\right)\left(\frac{P_c}{P_{c.crit}}\right)}$$

The symbols for the above expressions are the same as in Section 4.4.3, except:

$P_{r\ell}$ is the axial compression in left-hand rafter from elastic analysis

P_{rr} is the axial compression in right-hand rafter from elastic analysis

$P_{r\ell.crit}$ is the Euler buckling load of left hand rafter $= \pi^2 EI_{rl}/s_l^2$

$P_{rr.crit}$ is the Euler buckling load of left hand rafter $= \pi^2 EI_{rr}/s_r^2$

$$R_\ell = \frac{\textit{left hand rafter stiffness}}{\textit{total rafter stiffness}} = \frac{EI_{r\ell}/s_\ell}{\left(EI_{r\ell}/s_\ell + EI_{rr}/s_r\right)}$$

$$R_r = \frac{\textit{right hand rafter stiffness}}{\textit{total rafter stiffness}} = \frac{EI_{rr}/sr}{\left(EI_{r\ell}/s_\ell + EI_{rr}/s_r\right)}$$

$$R_2 = \frac{\textit{column stiffness}}{\textit{total rafter stiffness}} = \frac{EI_c/h}{\left(EI_{r\ell}/s_\ell + EI_{rr}/s_r\right)}$$

$I_{r\ell}$ is the left hand rafter inertia in the plane of the portal

I_{rr} is the right hand rafter inertia in the plane of the portal

s_ℓ is the left hand rafter length along the slope (valley to apex)

s_r is the right hand rafter length along the slope (valley to apex).

4.4.5 Portal frame with props or valley beams

The theory is due to Davies[5,6,7], but modified to include an explicit column base stiffness in (b) and (c) below. It assumes that all the valleys are supported either by props or by valley beams.

A simple equivalent frame with one column pinned top and bottom is used, representing an end bay. This is loaded by a share (normally 50%) of the total of all the prop loads in the frame on the pin-ended column. Assuming that the internal column load is twice the external column load, the equivalent frame prop load is NP_c,

where:

P_c is the axial compression in the external column from elastic analysis

N is the total number of props in the frame

The rafter beyond the first bay contributes little to the sway stability, so is ignored.

Valley beams do not add appreciably to the stability of the portal and do not destabilise it when well detailed with a rigid connection to the eaves. Therefore, rigidly connected valley beams make no contribution to N. A portal with valley beams but no props has $N = 0$.

(a) Truly pinned bases, or bases with rockers as Clause 5.1.3.1 of BS 5950-1

$$\lambda_{cr} = \frac{3EI_r}{s_2\left[0.3P_r s_2 + \left(\frac{1.2}{R_p} + 1\right)(N+1)P_c h\right]}$$

which may be expressed in terms of the Euler loads as

$$\lambda_{cr} = \frac{1}{\left(\frac{P_r}{P_{2r.crit}}\right) + (4 + 3.3R_p)(N+1)\left(\frac{P_c}{P_{c.crit}}\right)}\lambda$$

(b) Nominally pinned bases, as Clause 5.1.3.3 of BS 5950-1

$$\lambda_{cr} = \frac{(1 + 0.1R_p)}{\left(\frac{P_r}{P_{2r.crit}}\right) + (2.9 + 2.7R_p)(N+1)\left(\frac{P_c}{P_{c.crit}}\right)}$$

(c) Nominally rigid bases, as Clause 5.1.3.2 of BS 5950-1

$$\lambda_{cr} = \frac{(1 + 0.08R_p)}{\left(\frac{P_r}{P_{2r.crit}}\right) + (0.8 + 0.52R_p)(N+1)\left(\frac{P_c}{P_{c.crit}}\right)}$$

where, for rafters of equal cross-section and equal length

I_2 is the rafter inertia in the plane of the frame

s_2 is the length of rafter pair (i.e. eaves to apex to valley) but for asymmetrical arrangements of rafters, I_2/s_2 is the value that gives the true ratio of column stiffness to stiffness of the pair of rafters (length = sum of rafter lengths, i.e. eaves to apex to valley) for rotation about the eaves.

$P_{2r.crit}$ is the Euler critical buckling load of the pair of rafters adjacent to the external column

$= \frac{\pi^2 EI}{(s_2)^2}$ for a symmetrical pair of rafters.

$$R_p = \frac{\textit{stiffness of column}}{\textit{stiffness of rafter pair}} = (EI_c/h)(EI_2/s_2)$$

5 SECOND-ORDER ANALYSIS

5.1 Introduction

Second-order analysis is the term used to describe analysis methods in which the effects of increasing deflection under increasing load are considered explicitly in the solution method, so that the results include the $P.\Delta$ (P-big delta) and $P.\delta$ (P-little delta) effects described in Section 2.4. The results will differ from the results of first-order analysis by an amount dependent on the magnitude of the $P.\Delta$ and $P.\delta$ effects.

Second order analysis will normally be more accurate than first-order analysis with magnification factors.

The method and limitations of a second order analysis should be clearly understood before it is used and before resistance checks are applied to the results.

5.2 Design steps

This Section gives the steps required to satisfy Clauses 5.5.2 or 5.5.3 of BS 5950-1 using second-order analysis, as Clause 5.5.4.5 of BS 5950-1. For second-order analysis, λ_r is taken as 1.0.

Out-of-plane stability member must also be checked as required by BS 5950-1 Chapters 4 and 5, but is outside the scope of this document.

Design steps for plastic design

1. Carry out a plastic analysis of the frame.

   Apply the loads to the frame. For load combinations other than Load combination 1 (see Clause 2.4.1.2 of BS 5950-1), notional horizontal forces (NHF) need not be applied (see Clause 2.4.2.4 of BS 5950-1). Where NHF are applied to asymmetric frames or symmetric frames with asymmetric loading, it will generally be necessary to apply two load cases, one with the NHF in one direction and the other with the NHF in the other direction to ensure that the most unfavourable load case is applied.

2. Check the strength of the frame

   (a) Calculate λ_p and check that $\lambda_p \geq 1.0$. Note that second-order analysis should not give higher values of λ_p than first-order analysis

   (b) Check the member strength and out-of-plane stability at 1.0 x ULS loads.

Design steps for elastic design

1. Carry out an elastic analysis of the frame. Loading should be the same as given in Step 1 for plastic design.
2. Calculate the moments and forces around the frame. Check the cross-sectional resistances using BS 5950-1 Clause 4.8 using the output of the second-order analysis at 1.0 x ULS loads.

5.3 Structure model

5.3.1 Division of members into elements for P.δ effects

In second-order analysis of portal frames, all the members in a frame may need to be divided into several elements along their lengths for an accurate analysis. The reason is that $P.\delta$ (P-little delta) effects described in Section 2.4.2 have a significant influence on the behaviour of practical portal frames, but most analysis methods do not allow for these effects within the model element. The $P.\delta$ (P-little delta) effects are not the same as the $P.\Delta$ (P-big delta) effects described in Section 2.4.3. Many software packages include modules of 'P-delta' analysis, but these are usually designed to consider the relative displacements of the ends of the element only. This is the same as the $P.\Delta$ (P-big delta) effects on the element. When a member is divided into a number of elements, then the $P.\Delta$ effects of all the elements will approximate to the $P.\delta$ effects of the entire member. As the number of elements is increased, the approximation is improved. It is recommended, as a simple guide, that members should be divided into 10 elements for analysis of portal frames, because portal members are commonly slender and $P.\delta$ effects are more important on slender members.

There are analysis methods in which the $P.\delta$ effects are modelled within the length of each element. In these methods, the coefficients of each of the bending terms in the stiffness matrix are modified by factors similar to 'stability functions' such as those published by Livesiey and Chandler[14]. Where this method is used, the member length need not be divided into numerous elements to allow for $P.\delta$ effects. Division of a member into elements may still be required to provide nodes at which loads can be applied if loads are applied within the length of a member. It is important that the designer is sure that this 'stability function' type method is incorporated in an analysis method before deciding not to sub-divide members into 10 or more elements. An example of a 'stability function' type of solution is provided by the worked examples of the 'hand' method. In these examples, the stiffness of the members is reduced by a factor $(1 - P_{ULS}/P_{cr})$, where P_{ULS} is the axial compression at the Ultimate Limit State and P_{cr} is the elastic critical buckling load of the member. Whilst this is an approximation, it is an example of modification of the member stiffness by a factor to allow for $P.\delta$ effects. An alternative factor that may be used is $(1 - V_{ULS}/V_{cr}) = (1/\lambda_{cr})$, where V_{ULS}, V_{cr}, and λ_{cr} are as defined in Section 2.3.

5.3.2 Initial imperfections of frames

All frames must be designed to allow for initial imperfections. BS 5950-1: 2000 requires that the effects of these imperfections are included by application of the notional horizontal forces. These are taken at 0.5% of the factored loads applied in Load combination 1 (gravity loads) in cases without significant horizontal loads, as BS 5950-1 Section 2.4.2.4. Notional horizontal forces are not applied in load combinations including horizontal loads.

The notional horizontal forces are assumed to act in any one direction, thus two analyses are required, except for symmetrical frames with symmetrical loading. In one analysis, the notional horizontal forces applied in one direction in the plane of the portal frame, and in the other analysis, the forces are applied in the opposite direction in the plane of the portal frame.

5.3.3 Initial imperfections of members

In-plane buckling checks of members is covered in detail in Section 6. This section only covers principles that affect the analysis model.

Where the buckling resistance of a member is to be checked by a method other than direct application of the buckling checks in BS 5950-1, it is important that the effect of residual stresses is included. This may be achieved by using initial imperfections equivalent to those used in BS 5950-1, Annex C to define the buckling curves. Alternatively, the moment due to strut action can be calculated from BS 5950-1 Annex C.3.

If initial imperfections of members are allowed for in the analysis model of a complete frame, it should be remembered that these imperfections might be either destabilising or stabilising, depending on the direction of the deflections induced by each load combination. Numerous analyses for each load case may be required to ensure that the worst case has been considered. Because of this, it is recommended that the frame is analysed assuming initially perfect members and that the initial imperfection effects are then added in the most unfavourable direction to each individual member, in addition to the moments and forces from the frame analysis.

5.3.4 Base stiffness

BS 5950-1 Clause 5.1.3 gives guidance on the base stiffness that may be assumed in design. The provisions for ULS analysis may be used in the analysis model. It is important to note that the second-order analysis uses the stiffness of the frame at ULS, hence only the ULS base stiffness values may be used, not the SLS values.

The following base stiffnesses may be used:

Base with a pin or rocker

The base stiffness should be taken as zero.

Nominally pinned base

If a column is nominally pin-connected to a foundation assuming that the base moment is zero, the base should be assumed to be pinned in the global analysis. Therefore, where the moment applied to the foundation is required to be zero, the 10% column stiffness value cannot be applied in the global analysis. However, the 10% column stiffness can be used in the calculations of stability functions that allow for $P.\delta$ (P–little delta) effects. This is the reason for the reduction of the effective length of columns when calculating P_{cr} in Appendix A.3.2, which is also referred to in Appendix B.3.2. The reduction of effective length is also applicable in the internal column in-plane checks recommended in Section 6.

Nominally semi-rigid base

A nominal base stiffness of up to 20% of the column stiffness may be assumed for the ULS calculations, provided that the foundation is designed for the moments and forces obtained in the analysis.

Nominally rigid base

The base stiffness should be taken as equal to the column stiffness and the foundation must be designed to resist moments from all load cases.

5.3.5 Tied portals

Tied portals are especially sensitive to second-order effects because of the high axial forces in the rafters. In addition, tie portals with low pitch rafters are very sensitive to the vertical deflection of the apex. This is because the apex acts as a central support to the rafters, which act as a two span beam supported by the columns and the apex. The axial forces in the rafters are determined by the vector component of the reaction from the rafters acting as a two-span beam. Therefore, the axial forces are inversely proportional to the actual slope of the rafters in their loaded position. As the apex deflects, the axial force in the rafters must increase to provide the same vertical reaction at the apex. This phenomenon is illustrated in B.5.3. It is a non-linear effect that must be accounted for.

An additional consideration is that the rafters in a tied portal may be so shallow, relative to their span from eaves to apex, that the deflections are significant. Where this occurs, the curvature shortens the end to end distance of the rafter. This effect adds to the apex drop. This curvature shortening effect is illustrated in B.5.3. It is another non-linear effect that must be accounted for.

Because of the above two non-linear effects, tied portals must be analysed using routines that can model these non-linear effects. It is essential that designers must check the functionality of their software with the suppliers before designing tie portals. It is unlikely that any software that uses the original geometry of the rafters throughout the analysis can be reliable for low pitch rafters, unless an iterative modification of the rafter geometry is used at some point in the analysis.

Some software packages use a system of modification to the stiffness matrix while maintaining the original geometry of the model. This is often referred to as 'P-delta' analysis. This type of analysis routine is not appropriate for analysis of tied portals with low pitch rafters, because of the non-linearity, unless a routine is added to account for this. Equally, routines that use stability functions but retain the original geometry throughout the analysis cannot model the non-linearity of the apex deflection and are not appropriate for portals with low pitch rafters, a procedure is added to account for this.

5.4 Analysis methods

5.4.1 General

Second order analysis may be carried out by numerous methods, including:

- Closed solutions using a geometrical or algebraic function.
- Matrix methods.
- Energy methods.

It is important that the method chosen is suitable for the particular application.

One of the most common cases of allowing for second-order effects is a strut buckling curve. The resistance of the strut is reduced below its squash load by the bending moments caused by the axial load. This is a case of second-order analysis that can be performed using a geometrical function to produce a closed solution. The classic solution is the Perry-Robertson solution, which uses a sine curve to model the strut deflected form. However, it is not normally practical to produce an accurate analysis of a frame using geometrical functions, although an approximate analysis can be made using the first mode of frame buckling as the geometrical function. An example of this method is the amplification of sway effects in Clause 2.4.2.7 of BS 5950-1.

The more universally applicable method is by iterative application of matrix analysis described in Section 5.4.2.

In the case of single-storey plastic portal frames, the energy method provides one of the simplest solution techniques. Using the energy method, it is possible to perform a second-order analysis without computer software, although the solution is laborious and more conservative than a computer solution. Examples of this method are given at the end of this document.

In iterative solutions, the number of iterations should be sufficient to ensure that the stiffness of the frame is not overestimated as the loading approaches the collapse load. The solution method should include an equilibrium check to ensure that the applied loads are in equilibrium with the frame resistance within a satisfactory tolerance.

5.4.2 Matrix methods

There are at least two matrix methods of second order analysis available. One modifies the geometry after each load increment, and then recalculates the stiffness matrix using the new geometry and unmodified member properties. The other, often called P-delta analysis, uses the initial geometry throughout but modifies the terms of the stiffness matrix according to the displacements and axial load in each member but always referring to the original member geometry. There are special requirements for modelling tied portals, which are given in Section 5.3.5. It is unlikely that any routine using the initial geometry can be reliable for tied portals with low pitches unless a separate iterative procedure is used in addition.

Also, the analysis may be either elastic/perfectly-plastic or elasto-plastic (in which the modulus of elasticity reduces to model the stress-strain behaviour in a real member).

It is very important that a matrix method addresses all the issues raised in Section 5.1. It is equally important to recognise that the overall frame behaviour (the $P.\Delta$ effects) cannot be correctly calculated unless the member effects (the $P.\delta$ effects) are correctly included. For example, if the column of a portal is modelled as one element, it will give unsafe answers unless the stiffness is correctly modified to allow for the appropriate column buckling mode.

5.4.3 Energy methods

The energy method is a long-established method of structural analysis. The method uses the principle of conservation of energy, equating the strain energy in the structure under load with the potential energy given up by the load as the structure deflects.

It is very important that an energy method addresses all the issues raised in Section 5.1. It is equally important to recognise that the overall frame behaviour (the $P.\Delta$ effects) cannot be correctly calculated unless the member effects (the $P.\delta$ effects) are correctly included. For example, if the column of a portal is modelled as one element, it will give unsafe answers unless the stiffness is correctly modified to allow for the appropriate column buckling mode.

The strain energy in the structure is given by the area under the load-deflection diagram. This is illustrated for a typical single bay portal frame in Figure 5.1. The load factors at the formation of the first and second hinges are denoted by λ_1 and λ_p.

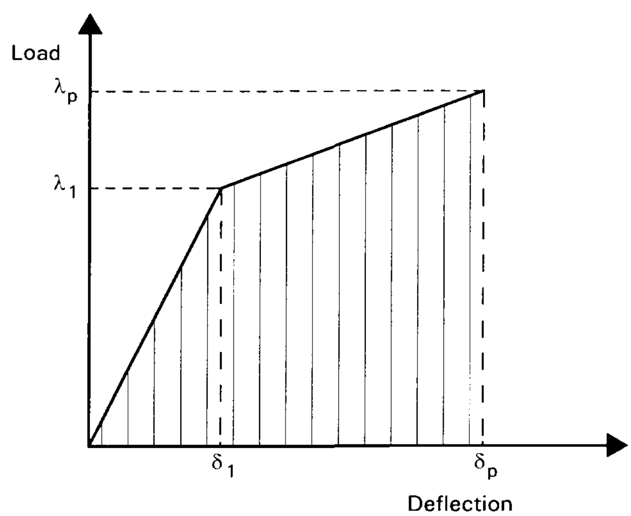


Figure 5.1 *Typical load-deflection diagram*

The second-order effects reduce the resistance of the frame to externally applied loads. This is simply illustrated by the column of a portal frame. The rotation of a typical exterior column is shown in Figure 5.2. In the deflected state, the top of the column has moved down by δ. This is mostly due to the rotation ϕ of the column top relative to the column base. In addition, the deflection δ is partly due to the curvature of the column, which reduces the distance between the top and the bottom of the column, causing the column top to move down. This column top deflection moves the rafter shear force, V, downwards, releasing potential energy that is not calculated in first-order analysis.

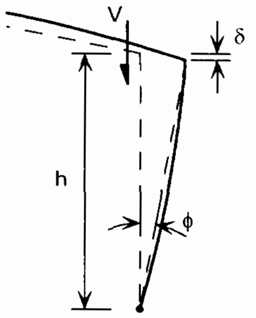


Figure 5.2 *Column top deflections*

By the principal of conservation of energy, the potential energy released by a given range of deflection is equal to the strain energy absorbed in the structure over that same range of deflection. This may be written:

$E_p = E_s$

where:

E_p is the potential energy released

E_s is the strain energy absorbed in the structure.

For a given range of deflection, some potential energy is released by second-order deflections, which is defined here as E_{p2}. The remaining potential energy is released by first-order deflections and is defined here as E_{p1}. Therefore, we can write:

$E_s = E_p = E_{p1} + E_{p2}$

These energies are illustrated in Figure 5.3.

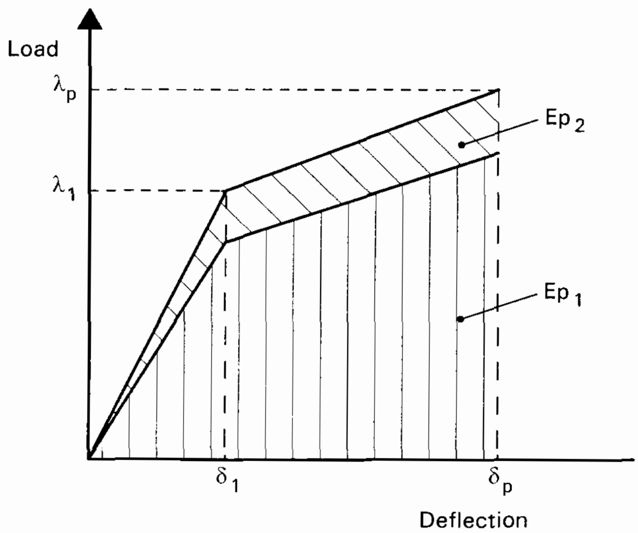


Figure 5.3 *Energy*

At any point on the load-deflection curve, the resistance to externally applied loads can be found from the conservation of energy over an infinitesimal increment of deflection, as shown in Figure 5.4.

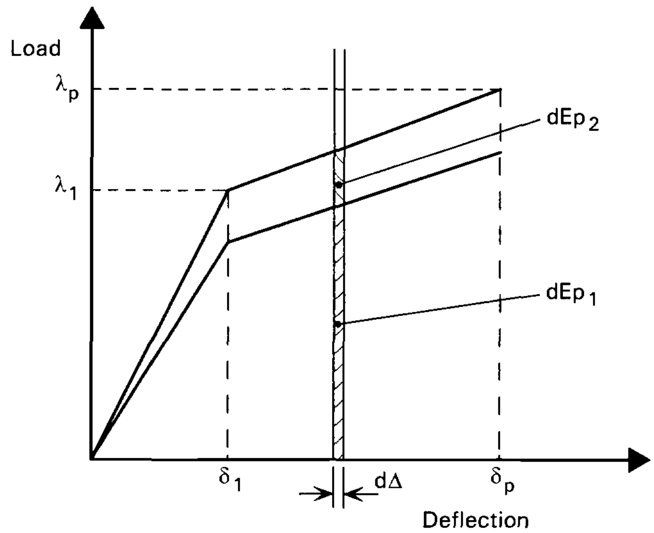


Figure 5.4 *Energy over an increment of deflection*

The energy equation is:

$dE_s = dE_{p1} + dE_{p2}$

where, at the particular level of load being considered:

$dE_s = \int(Mdk) + \Sigma M_{pr} d\theta$

$dE_{p1} = \lambda_i \Sigma w d\Delta$

$dE_{p2} = \Sigma P_2 s \phi d\phi + \Sigma\int(P_2 ds)$

λ_i is the first-order load factor at the load level considered

w is the applied loads

$d\Delta$ is the increments of displacements at λ_i

P_2 is the axial loads in the member including the effect of the drop of the apex

s is the straight line distance between the ends of members (or parts of members between hinges)

ds is the increment of shortening of the distance between the ends of the members due to change of curvature

M_{pr} is the plastic moment reduced by axial force at the hinges

$d\theta$ is the increments of rotation of the hinges

ϕ is the rotation of the member between the 'frame unloaded' position and the position at λ_i

$d\phi$ is the increment of rotation of the members

M is the first-order moments throughout the frame

dk is the increments of curvature.

The second-order load factor at the particular level of load being considered is:

$$\lambda_M = \lambda_p \left[1 - \frac{dE_{p2}}{dE_s} \right]$$

The load factor at failure in each load case is taken as the maximum value of λ_M found for that load case.

The energy method is convenient for 'hand' calculations, as shown in Appendices A and B. This application of the Energy Method is similar to the method given by Horne and Morris[4], but it is made more rigorous by including the effects of $P.\delta$ (P-little delta) in the members and by more rigorous calculation of the deflections. The energy equations used in the hand method are given in Appendix A.2.2.

The energy method has been calibrated with rigorous iterative second-order elastic-plastic matrix analysis methods on frames in which the sway-check deflections, δ (calculated as in Section 3.3.2), do not exceed $h/500$. This implies that λ_{cr} is approximately 2.0. It is recommended that the method is not applied for frames with greater flexibility without due caution. Frames that are more flexible are generally inefficient and it is advisable either to stiffen the frame or choose a stiffer structural concept.

6 MEMBER CHECKS

6.1 General

This section describes the member checks that should be performed and how to calculate the bending moment diagram used for these checks.

Section 6.2 provides a general introduction to the differences between the first-order moments in initially straight members and the actual moments occurring, including second-order effects. Section 6.3 describes the cases in which the members may need in-plane buckling checks, even when the frame has been proved to be stable in-plane.

6.2 Additional bending moments from strut action

The effects of both geometry and residual stresses must be included in member design. BS 5950-1 uses the concept of an equivalent geometrical imperfection.

In the elastic domain, the following relationship can be proved by a closed solution for a pin-ended member supporting both an axial compressive force and a distributed lateral load in the form of a half sine curve, as shown in Figure 6.1

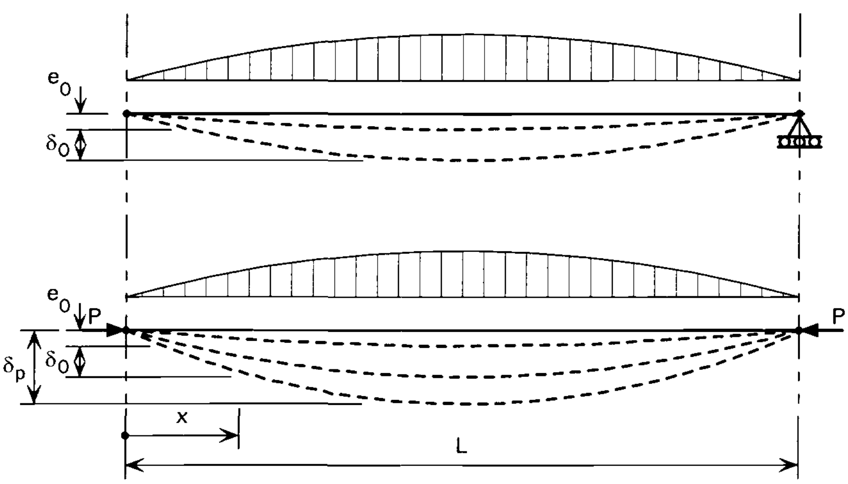


Figure 6.1 *Additional deflections from strut action*

The maximum moment is given by:

$$M_{\max} = P \times \delta_{P} + M_{1} = P \times (e_{0} + \delta_{0}) \left(\frac{1}{1 - \frac{P}{P_{cr}}} \right) + M_{1}$$

where:

P is the axial compressive force

e_0 is the initial imperfection

M_1 is the first-order bending moment

δ_0 is the deflection from the first-order moments

δ_p is the second-order deflection

P_{cr} is the Elastic Critical Buckling load.

Assuming that the above relationship applies to other patterns of loads and moments, the equation can be used to study how the code value of strut imperfection should be applied to a second-order analysis, that used initially straight members.

The French NAD[15] to ENV1993-1-1[16] gives a method of analysing members with axial compression and bending that relies on this equation as an alternative to ENV 1993-1-1 Clause 5.5.4.

Second-order analysis assuming initially straight members will calculate the maximum moment as:

$$M_{max} = P \times (\delta_0) \left(\frac{1}{1 - \dfrac{P}{P_{cr}}} \right) + M_1$$

Therefore the only difference between second order with the strut imperfection and second-order without the strut imperfection is:

$$\delta M = P \times (e_0) \left(\frac{1}{1 - \dfrac{P}{P_{cr}}} \right) \mathrm{Sin} \left(\frac{\pi x}{L} \right)$$

Therefore, all that is needed to include the effect of the code strut imperfection is to add the above moments to the second-order bending moments calculated for initially straight members. In external columns of portals, this will usually be demonstrably small. In internal columns with no applied moments, this is the strut moment from the code imperfection. In rafters, this will add a very small amount to the moments that could be considered to be covered by the factor that allows for strain hardening and the increased reliability arising from the redundancy of a plastic design.

6.3 In-plane member checks

6.3.1 General

For most structures, all the members resisting axial compression must be checked to ensure adequate resistance to buckling about both the major and minor axes. However, in portal frames checked for in-plane stability by the methods defined in this publication, in-plane buckling of the members is not the critical design case for many members. This Section gives guidance for the majority of portal frames, that is the frames, in which the bending moments around the frame, are predominantly from loads distributed along the rafters so as to cause relatively large bending moments. In members with both:

(i) relatively low axial compression, and

(ii) relatively large bending moments which occur away from the maximum strut action moments at mid-length of members,

the strut action is so low relative to the maximum moments that buckling is not the critical failure criterion. Simplified guidance is presented for common portals in Section 6.3.2 and for tied portals in Section 6.3.3. The guidance is very different for the rafters of these two types of frame.

6.3.2 Common portals

Common portals are portals without ties at or near rafter level. In these frames, guidance on in-plane buckling checks may be simplified as follows:

Rafters and columns that resist the full haunch moment of the adjoining rafter.

In these members, the bending moments at the ends of the members are very large, but at mid-length the bending moment is much less. Examples are shown in Figure 6.2.

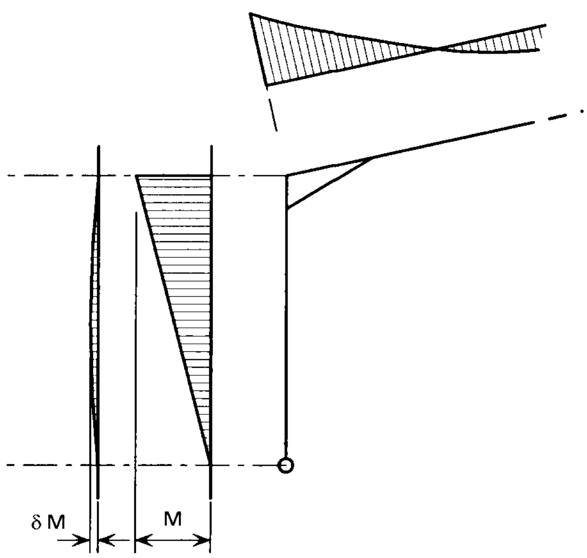


Figure 6.2 *Members with relatively low moments at mid-length compared with the ends*

In these cases the strut action moment is at a maximum where the first-order bending moments are approximately half the maximum. In addition, the strut action moment is relatively small compared with the moment of resistance of the section. Therefore, these members need not be checked for in-plane buckling.

Columns that do not resist the full haunch moment of the adjoining rafter.

In members that do not resist the full haunch moment of the adjoining rafter, it is possible that the strut action moment is relatively large compared with the applied bending moment. The principal example of this is the internal columns of multi-span frames as shown in Figure 6.3. These members should be checked for in-plane buckling. These members may be checked by BS 5950-1 Clause 4.8.3.3.2(a)(i) using a segment length, L, defined in BS 5950-1

Clause 4.7.1.1, of the height from the base to the eaves in the absence of intermediate in-plane restraint. The effective length, L_E, may be taken as:

L_E	=	$1.0\,L$	for truly pinned base columns
L_E	=	$0.85\,L$	for nominally pinned base columns
L_E	=	$0.7\,L$	for nominally fixed base columns.

These effective lengths are not from Annex E of BS 5950-1 because Annex E assumes the adjoining beams remain generally elastic, which is not common in portal rafter design.

The exceptions are columns that have sufficient bending capacity to resist the full haunch moment. One example of this exception is where an extension of a frame is constructed, so that the original external column has become the internal column of the extended frame. Another example is where a column supports rafters at levels so different that the column section is sufficient to resist the full haunch bending moment and this section is continued to the foundation, as shown in Figure 6.4.

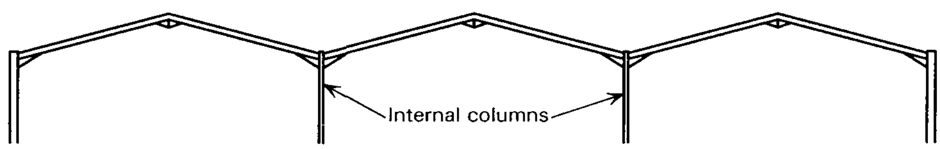


Figure 6.3 *Internal columns in a typical multi-span frame*

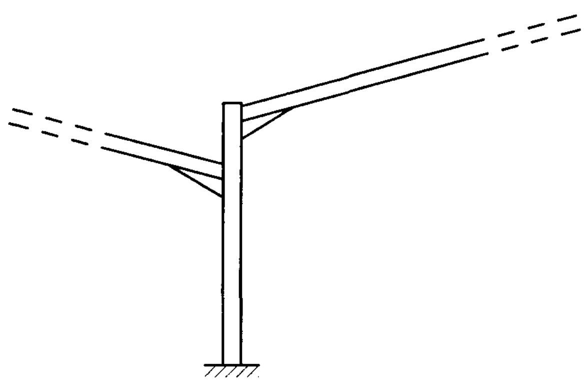


Figure 6.4 *Internal column supporting rafters at different levels*

6.3.3 Tied portals

Tied portals are portal frames in which there is a tie at or near rafter level. The tie at this level causes very high axial loads in the rafters and reduces the bending moments in the rafters as shown in Figure 6.5.

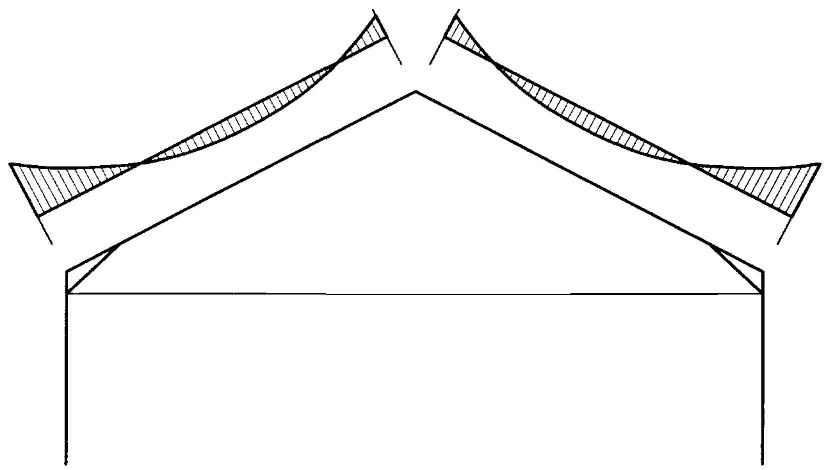


Figure 6.5 *Tied portal bending moments in the rafters*

The bending moment near the mid-length of the rafter approaches the moment of resistance of the rafter. This large bending moment occurs near the point of maximum strut action, so the in-plane buckling of the rafter must be checked with the coexistent bending moment diagram. This may be checked by BS 5950-1 Clause 4.8.3 taking the effective length as the length along the rafter from apex to eaves/valley. The member check may be made using BS 5950-1 Clause 4.8.3.3.2(a)(i) taking the rafter section as constant throughout the effective length (i.e. ignoring the increase in section properties of the haunch), using the bending moment diagram of the full length from apex to eaves/valley to define the values of moment used in BS 5950-1, Table 26 and calculate m_x.

6.4 Bending moments for frames using plastic design

6.4.1 General

Where the in-plane stability of individual members of a frame should be checked (see Section 6.3), the bending moments can be found directly from elastic-plastic analysis.

Where elastic-plastic analysis has not been used, the moments may be calculated approximately. This can be done by modifying the moments and forces from a plastic analysis by multiplying by λ_r/λ_p. This is a method allowed in BS 5950-1 Clause 5.3.1 for calculating the bending moments for out-of-plane buckling. Where this method is used, it must be remembered that the plastic hinges will occur unless they can be proved not to occur. This can only be done by elastic unloading calculations unless the reduction by λ_r/λ_p is clearly very large. Unless the plastic hinges are demonstrated not to occur, the stability of the members must be demonstrated assuming plastic hinges occur at the same points as in the bending moment diagram at collapse.

6.4.2 Sway-check and amplified moments methods

The bending moments around the frame at ULS, excluding the moments from strut action, may be calculated as given below.

Where the analysis is by elastic-plastic analysis, the bending moment diagram may be calculated by interpretation between the bending moments at the load steps above and below ULS.

For other analysis techniques, the moments may be calculated approximately (see Section 6.4.1) as:.

$$M = M_1 \times \frac{\lambda_r}{\lambda_p}$$

where:

- M is the design ULS moment at any point
- M_1 is the bending moment at that point from the first-order plastic analysis
- λ_r is the required load factor (see below)
- λ_p is the collapse factor from first-order plastic analysis.

The values of λ_r are given by:

(a) Sway-check method: Gravity loads

BS 5950-1 Section 5.5.4.2.2 gives:

$\lambda_r = 1.0$ (see Section 3.3.2 and Section 3.4.2)

(b) Sway-check method: Horizontal loads

BS 5950-1 Section 5.5.4.2.3 gives:

$$\lambda_r = \lambda_{sc}/(\lambda_{sc} - 1)$$

where:

λ_{sc} may be recalculated for each load case, see Section 3.3.3 and Section 3.4.3 .

(c) Amplified moments method

BS 5950-1 Section 5.5.4.4 gives:

if $\lambda_{cr} \geq 10$: $\lambda_r = 1.0$

if $10 > \lambda_{cr} \geq 4.6$: $\lambda_r = \dfrac{0.9\lambda_{cr}}{\lambda_{cr} - 1}$

where:

λ_{cr} may be recalculated for each load case, see Section 4.1.

6.4.3 Second-order analysis by iterative methods

The bending moment diagram may be calculated by interpolation between the moments calculated at the load steps above and below ULS.

6.4.4 Second-order analysis 'by hand'

The bending moments for frames analysed by the 'hand' methods of Section 5 and Section 6 may be calculated as:

$$M = M_l / \lambda_M$$

where:

M is the design bending moment at ULS at any point excluding the effects of strut action

M_l is the bending moment at that point in the plastic collapse mechanism

λ_M is defined and the value of λ_M calculated in Appendix A and Appendix B. Note that λ_p/λ_M is equivalent to λ_r in Section 6.3.2.

6.5 Bending moments for frames using elastic design

BS 5950-1 Clause 5.5.2 required that the bending moments should be taken as the values from linear elastic analysis multiplied by the required load factor λ_r.

Where the linear elastic analysis is first-order analysis, the values of λ_r may be determined either by the sway-check method or the amplified moments method.

Where the linear elastic analysis is second-order analysis, the value of λ_r may be taken as 1.0

6.6 Other member checks

Portal frames must satisfy all the relevant requirements of BS 5950-1, including out-of-plane buckling checks. However, the purpose of this publication is to give guidance on the in-plane stability of portal frames, so detailed guidance on other checks is not included.

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APPENDIX A Second-order analysis of common portals 'by hand'

A.1 Range of application and design steps

This Appendix shows how second-order analysis may be performed without second-order software for common forms of portal frames designed by plastic design methods. These portals do not have ties at or near the rafter level. Portals with ties at or near the rafter level should be checked using Appendix B. It is recommended that this method is not used for frames for which the deflection from a sway-check exceeds $h/500$, as explained in Section 5.4.3.

Design steps

The design steps are as follows:

1. Identify from the first order analysis:
   - the plastic collapse mechanism
   - the hinge incremental rotations
   - the axial forces in the members
   - the load factor at the formation of the first hinge, λ_1 (excluding hinges at bases where the moment of resistance of the base is less than the moment of resistance of the columns)
   - the plastic collapse factor, λ_p
   - the deflections of the frame at λ_1.
2. Calculate the second-order deflections of the 'elastic' frame.
   (a) Calculate the stiffness reduction factors to allow for $P.\delta$ (P-little delta) effects
   (b) Calculate the second-order magnification factors for the 'elastic' frame
   (c) Calculate the deflections of the 'elastic' frame. These are the values of the deflections at λ_1 magnified to account for second-order effects.
3. Calculate the second-order deflections of the 'plastic' frame
   (a) Calculate the stiffness reduction factors to allow for $P.\delta$ (P-little delta) effects
   (b) Calculate the second-order magnification factor for the 'plastic' frame.
   (c) Calculate the deflections of the 'plastic' frame from gravity loads.
   (d) Calculate the deflections of the 'plastic' frame from horizontal loads.
4. Calculate the increased axial loads in the rafters to account for second-order effects.
5. Sum the energies.
6. Calculate the reserve factor, λ_M.
7. Check that $\lambda_M \geq 1.0$

A.2 Basis of method

A.2.1 General

In this method, the load factor of the frame is calculated by the energy method, allowing for any strength reduction caused by second-order stability effects. It is derived from the plastic collapse load factor λ_p calculated by first-order analysis.

This load factor may be called the 'reserve factor at ULS', λ_M, defined as:

$$\lambda_M = \frac{V_{2,P(ULS)}}{V_{ULS}}$$

where:

$V_{2,P(ULS)}$ is the load predicted to cause failure of the frame including the second-order effects, but calculated using the axial forces in the members at the ULS level of load.

V_{ULS} is the ULS load on the frame.

The factor, λ_M, allows for the de-stabilising effects on the frame of second order effects, including both $P.\delta$ effects and $P.\Delta$ effects described in Sections 2.4.2 and 2.4.3 respectively. The $P.\delta$ and $P.\Delta$ effects are calculated using the axial forces that occur in the frame at ULS, including the second-order effect of any drop of the apex of the frame.

This reserve factor, λ_M, must be equal to or greater than 1.0 to demonstrate that the frame is stable at ULS.

The reserve factor λ_M is not exactly the same as the failure factor, commonly referred to as λ_f, except in the case where $\lambda_M = 1.0$. This is because λ_f is defined as:

$$\lambda_f = \frac{V_f}{V_{ULS}}$$

where:

V_f is the failure load including second-order stability effects

V_{ULS} is the ULS load on the frame.

The failure load V_f is calculated using the axial forces at failure and, therefore, the $P.\delta$ effects arising from the axial forces at failure. This contrasts with λ_M which is calculated using the $P.\delta$ effects from the axial forces at ULS. Where the loading at ULS is equal to the loading at failure, λ_M is equal to λ_f.

The method uses the plastic collapse mechanism found by first-order analysis. This is sufficiently accurate because there is very little difference between the bending moment diagrams from first-order and second-order analysis. The plastic hinges limit the bending moment diagram and prevent any significant change of curvature along the members between the plastic hinges. Therefore, the energy calculation can be made using only the deflections arising from the rigid-body motions of the elements between the hinges. The energy calculation is made using the deflected form of the plastic collapse mechanism to calculate the second-order effects. The stiffness reduction to allow for $P.\delta$ (P-little delta) effects, as described in Section 2.4.2 is made using the axial forces at ULS calculated from first-order analysis.

This method is called a 'hand' method because it is possible to perform the second-order analysis by this method without a computer. However, it will be much easier if computer software is used for various steps, such as calculating the deflections of the two different structure models used in the method. Hand calculations will still be required to prepare the input data and combine the output data.

A.2.2 Application of the energy method

The basis of the energy calculation is shown in Figure A.1. An increment of deflection is considered at the formation of the collapse mechanism. The strain energy dE_s absorbed over this increment of deflection can be calculated from the rotation of the hinges:

$$dE_s = \Sigma M_{pr} d\theta$$

The potential energy released by second-order effects can be calculated from the rigid body rotations as shown in Figure A.2.

$$dE_{p2} = \Sigma P_2 (\phi s d\phi)$$

$$dE_{p1} + E_{p2} = dE_s$$

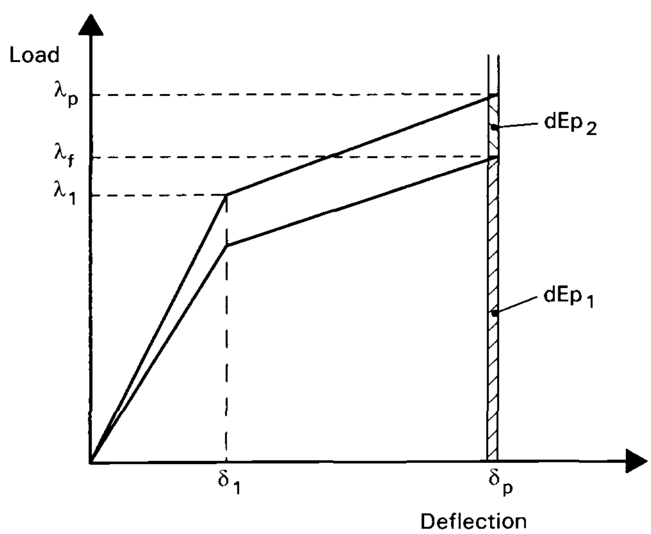


Figure A.1 *Energy over an increment of deflection at collapse*

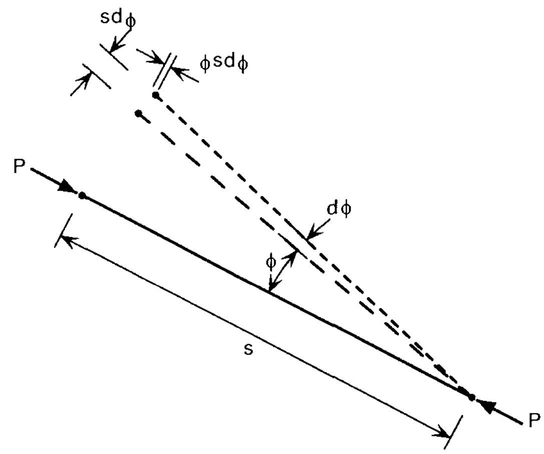


Figure A.2 *Potential energy release from second-order effects*

The potential energy released by the loads at failure can be calculated from:

$dE_{p1} = \lambda_M \Sigma w \Delta$

Therefore the conservation of energy that is:

$dE_{p1} + dE_{p2} = dE_s$ can be written as:

$\lambda_M \Sigma w \Delta + \Sigma P_2\ \phi\ s d\phi = \Sigma M_{pr}\, d\theta$

where:

λ_M	is the reserve factor on the collapse mechanism at ULS (see Section 5.2.1)
w	is the set of applied loads at ULS
Δ	is the set of incremental displacements of the applied loads in the collapse mechanism including the $P.\delta$ (P-little delta) effects.
P_2	is the set of axial forces in the members at ULS including second order effects.
s	is the set of member lengths
ϕ	is the set of member rotations at the onset of the collapse mechanism allowing for the reduced stiffness of the members due to $P.\delta$ (P–little delta) effects
$d\phi$	is the set of incremental member rotations in the collapse mechanism
M_{pr}	is the set of plastic moments of resistance reduced by co-existent axial force from first-order analysis
$d\theta$	is the set of incremental hinge rotations in the collapse mechanism.

Noting that in the first-order collapse analysis:

$\lambda_p \Sigma w \Delta = \Sigma M_{pr} d\theta$

giving:

$$\Sigma w \Delta = \frac{\Sigma M_{pr}\, d\theta}{\lambda_p}$$

the requirement can be expressed as:

$$\lambda_{\mathrm{M}}\left(\frac{\Sigma M_{\mathrm{pr}}d\theta}{\lambda_{\mathrm{p}}}\right)+\Sigma P_2\phi s d\phi \quad = \quad \Sigma M_{\mathrm{pr}}d\theta$$

or

$$\frac{\lambda_{\mathrm{M}}}{\lambda_{\mathrm{p}}}\left(\Sigma M_{\mathrm{pr}}d\theta\right) \quad = \quad \Sigma M_{\mathrm{pr}}d\theta-\Sigma P_2\phi s d\phi$$

Thus

$$\lambda_{\mathrm{M}} \quad = \quad \lambda_{\mathrm{p}}\left[1-\frac{\Sigma P_2\phi s d\phi}{\Sigma M_{\mathrm{pr}}d\theta}\right]$$

The minimum acceptable load factor at failure, λ_{M}, is 1.0 for any load combination.

This method of calculating λ_{M} is used in the worked examples.

A.2.3 Deflection calculations

The accuracy of the calculation depends on the angles ϕ shown in Figure A.2. These deflections must allow for second-order effects, so wherever first-order analysis is used, either the member stiffnesses must be reduced or the results must be amplified.

This section shows how the deflections can be found with a combination of:

(i) 'Elastic frame' deflections

(ii) 'Plastic frame' deflections.

The 'elastic frame' deflections are the deflections of the frame at the load factors, λ_1, at which the first hinge is about to form. The frame behaviour up to this point is elastic.

The 'plastic frame' deflections are the deflections of the frame at load factors above λ_1. This means that the first hinge has formed and the frame is partially plastic.

To make the calculation as simple as possible, it is most convenient to assume that all the plastic hinges (except the final hinge that forms in any span to create the collapse mechanism) occur at load factor λ_1. The deflected form of a typical 2-bay frame is shown in Figure A.3.

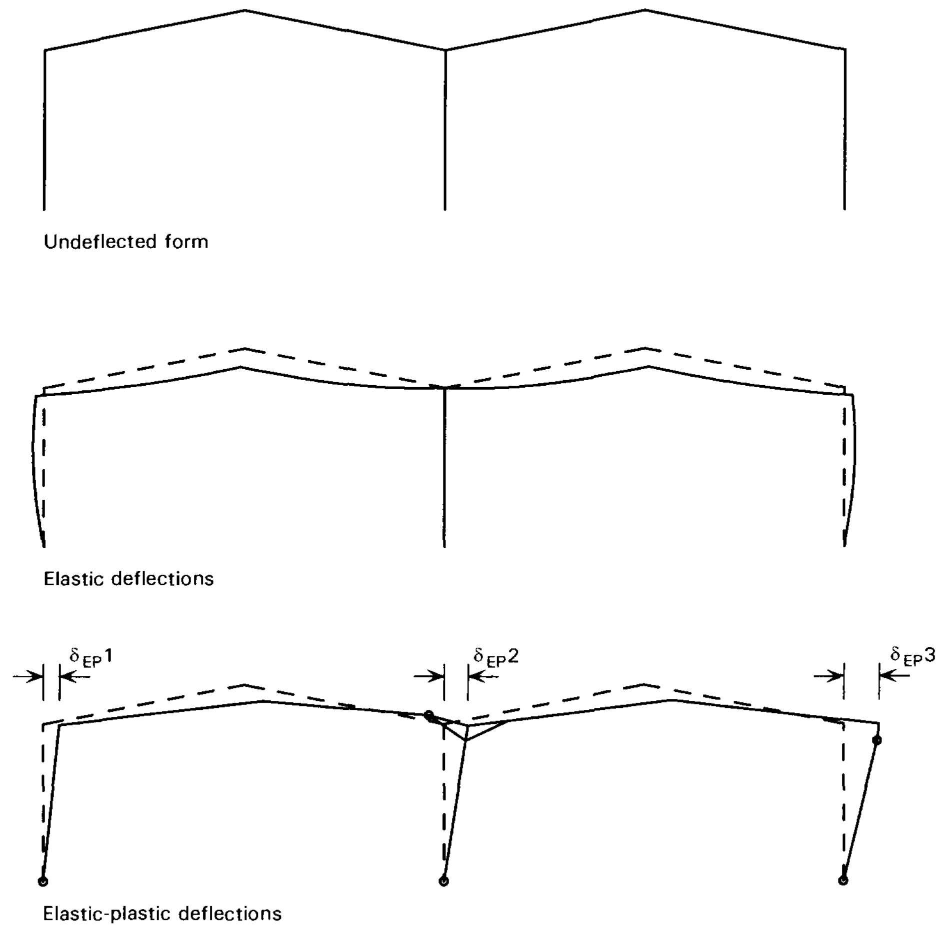


Figure A.3 *Deflections of typical frame*

In Figure A.7, $\delta_{EP}1$, $\delta_{EP}2$ and $\delta_{EP}3$ are the sums of the elastic deflections and the deflections after the plastic hinges have formed. The sum of the deflections comprises:

(i) The 'elastic frame' deflections at load factor λ_l.

(ii) The 'plastic frame' deflections from load factor λ_l to load factor λ_p, using an elastic structure model with pins at the position of the plastic hinges.

The deflections are calculated by elastic analyses, as described in Appendices A.3 and A.4.

A.2.4 Base stiffness

BS 5950-1 Clause 5.1.3 gives guidance on the base stiffness that may be assumed in design. The provisions for ULS analysis may be used in the analysis model. It is important to note that the second-order analysis uses the stiffness of the frame at ULS, so only the ULS base stiffness values may be used, not the SLS values.

Therefore the following base stiffnesses may be used:

Base with a pin or rocker

The base stiffness should be taken as zero.

Nominally pinned base

If a column is nominally pin-connected to a foundation assuming that the base moment is zero, the base should be assumed to be pinned in the global analysis. Therefore, where the moment applied to the foundation is required to be zero, the 10% column stiffness value cannot be applied in the global analysis. However, the 10% column stiffness can be used in the calculations of stability functions that allow for $P.\delta$ (P–little delta) effects. This is the reason for the reduction of the effective length of columns when calculating P_{cr} in A.3.2, which is also referred to in B.3.2. The reduction of effective length is also applicable in the internal column in-plane checks recommended in Section 6.

Nominally semi-rigid base

A nominal base stiffness of up to 20% of the column stiffness may be assumed for the ULS calculations, provided that the foundation is designed for the moments and forces obtained in the analysis.

Nominally rigid base

The base stiffness should be taken as equal to the column stiffness and the foundation must be designed to resist moments from all load cases.

A.3 Deflections of the 'elastic' frame

A.3.1 General

The deflections of the frame will be elastic until the first hinge forms at a load factor λ_1. The deflections are referred to as the 'elastic frame' deflections.

The value of λ_1 may be given by elastic-plastic analysis software or it may be calculated from an elastic analysis of the frame. The load factor λ_1 is the lowest load factor at which the applied bending moment at any section of the frame reaches the plastic moment of resistance, as Clause 4.8.2.3 of BS 5950-1.

The second-order deflections are calculated using the equation from Section 2.4.3:

$$\delta_v = \delta_1 \left(\frac{\lambda_{cr1}}{\lambda_{cr1} - 1} \right) + \delta_2 \left(\frac{\lambda_{cr2}}{\lambda_{cr2} - 1} \right)$$

Vertical deflections and eaves spread deflections are generally similar to the symmetrical mode of buckling of the frame, so vertical and spread deflections are treated as δ_2 above. Sway deflections are generally similar to the sway mode of buckling of the frame, so sway deflections are treated as δ_1 above.

A.3.2 Reduction factor to allow for $P.\delta$ (P–little delta) effects

The second-order effects within the member lengths reduce the effective stiffness of the members, and this effect must be included in the analysis from which the deflections are found. This is done by calculating a reduction factor and then applying it to the gross inertia to give an effective inertia value for the members, I_{eff}.

Rafters:

Reduction factor $= (1 - P_{ULS}/P_{cr}$

where:

P_{ULS} is the axial compressive force in the member from ULS loads calculated by first-order analysis. P_{ULS} may be calculated by elastic analysis, plastic analysis or elastic-plastic analysis. A consistent set of forces from any of these analyses should be used throughout the frame. Where columns support intermediate loads, for example from cranes or mezzanine floors, these intermediate loads must be included in P_{ULS}.

P_{cr} is taken as $\pi^2 EI/L^2$

where:

L = L_r, the developed length of the rafters in the relevant span, see Figure 3.8 for both the elastic frame and the 'plastic frame in multi-span frames and the 'plastic' frame for single-span frames.

L = $0.5L_r$ for single span 'elastic' frames only.

Hence, $I_{eff} = I\,(1 - P_{ULS}/P_{cr})$

Columns:

Reduction factor $= (1 - P_{ULS}/P_{cr}$

where:

P_{ULS} is as defined for rafters

P_{cr} is taken as $\pi^2 EI/(\alpha h)^2$

where:

h is the height from the base to the neutral axis of the rafter

α = 2.0 for columns with truly pinned bases or rockers

α = 1.7 for columns with nominally pinned bases

α = 1.4 for columns with nominally fixed bases.

Hence, $I_{eff} = I\,(1 - P_{ULS}/P_{cr})$

A typical column supporting rafters at different levels is shown in Figure A.8. The appropriate values of P_{ULS} and P_{cr} are those in the greater of

$$\left(\frac{P_{ULS}}{P_{cr}}\right)_A \quad \text{or} \quad \left(\frac{P_{ULS}}{P_{cr}}\right)_B$$

where:

$$\left(\frac{P_{ULS}}{P_{cr}}\right)_A = \frac{P_A}{\left[\dfrac{\pi^2 EI}{(\alpha h_A)}\right]}$$

$$\left(\frac{P_{ULS}}{P_{cr}}\right)_B = \frac{(P_A + P_B)}{\left[\frac{\pi^2 EI}{(\alpha h_B)}\right]}$$

P_A is the axial compression in the column between the higher and the lower rafters

$P_A + P_B$ is the axial compression in the column between the lower rafter and the base

h_A is the column height from the base to the higher rafter

h_B is the column height from the base to the lower rafter.

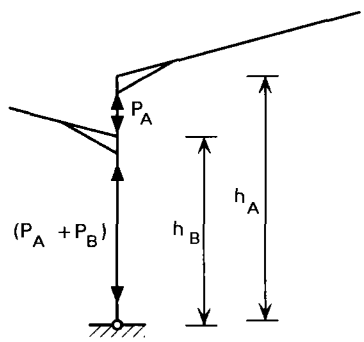


Figure A.4 *Column supporting rafters at different levels*

For columns supporting intermediate vertical loads such as crane loads and mezzanine floors without continuous connection into the column, the maximum column force should be used.

A.3.3 Second-order magnification factors, $\lambda_{cr}/(\lambda_{cr} - 1)$

Vertical deflections and eaves spread deflections

The second-order deflections are greater than the first-order deflections, δ_I, by the factor $\lambda_{cr2}/(\lambda_{cr2} - 1)$.

λ_{cr2} is taken as the minimum of either:

the minimum value of P_{cr}/P_{ULS} for any pair of rafters, where P_{cr} and P_{ULS} are taken as for rafters in A.3.2., or

$\Sigma P_{cr}/\Sigma P_{ULS}$ for the columns

where:

ΣP_{cr} is the sum of the values of P_{cr} for all the columns in the frame, where P_{cr} is taken as for columns in A.3.2

ΣP_{ULS} is the sum of the axial forces in all the columns in the frame.

Sway deflections

The second-order deflections are greater than the first-order deflections, δ_I, by the factor $\lambda_{cr1}/(\lambda_{cr1} - 1)$

λ_{cr1} is taken as the minimum value of $h/200\ \delta_{n2}$

where:

h is the height of any column, as A.3.2.

δ_{n2} is the sway deflection of the frame with member inertias I_{eff} calculated as in A.3.2, loaded only with notional horizontal forces, as defined in Section 1.6 and applied as in Section 3.3. δ_{n2} can be calculated from the simplified 'elastic' frame method in Appendix D using the values of I_{eff}.

A.3.4 Calculation of deflections

The first-order deflections are calculated using the gross I values.

Where elastic-plastic analysis is used, the deflections at the load factor of the first hinge, λ_1, is given directly by the software. The structural model for the elastic analysis must use the same member and base stiffnesses appropriate to ULS analysis, not to SLS analysis. Base stiffness values are given in A.2.4.

The loads applied to the elastic frame are λ_1 x (ULS loads), which is a set of loads in the same proportions as the ULS loads. It is recommended that software be used for the deflection calculations of the 'elastic frame' because hand calculations for portals is a long process.

The second-order deflections, (δX_2, δY_2), are calculated from the first-order deflections, (δX_1, δY_1), as follows:

$$\delta X_2 = (\delta X_1 - \delta X_{1s}) \{\lambda_{cr2}/(\lambda_{cr2} - 1)\} + \delta X_{1s} \{\lambda_{cr1}/(\lambda_{cr1} - 1)\}$$

$$\delta Y_2 = \delta Y_1 \{\lambda_{cr2}/(\lambda_{cr2} - 1)\}$$

where:

λ_{cr1} and λ_{cr2} are calculated as in A.3.3

δX_{1s} are the sway deflections from the horizontal component of externally applied loads resisted by the 'elastic' frame, $\lambda_1 H_{ULS}$,

where:

λ_1 is the load factor at the formation of the first plastic hinge

H_{ULS} is the nett horizontal component of the ULS loads. This includes the notional horizontal forces where they are applied in a load case.

A.4 Deflections of the 'plastic' frame

A.4.1 General

To simplify the calculations, this method assumes that all the spans develop plastic hinges at one end at the load factor λ_1 at which the first hinge forms in the frame. Then the analysis model becomes an elastic frame with a hinge at (or near) the end of each span as shown in Figure A.3. The pins are used at the plastic hinges because at a pure plastic hinge there is no increase of bending moment.

This mode of deflection is a sway mode, even for gravity loads, so all the second-order deflections are calculated from the magnification arising from the sway mode of buckling. Therefore, the equation in Section 2.4.3 reduces to:

$$\delta_v = \delta_l \{\lambda_{cr}/(\lambda_{cr}-1)\}$$

A.4.2 Reduction factor to allow for P.δ (P–little delta) effects

The second-order effects within the member lengths reduce the effective stiffness of the members. This effect must be included in the analysis. This is done by calculating effective inertia values for the members, I_{eff}, as for the 'elastic' frame' in A.3.2 above.

A.4.3 Second-order magnification factor, $\lambda_{crp}/(\lambda_{crp}-1)$

The second-order deflections are greater than the first-order deflections, δ_l, by the factor $\lambda_{crp}/(\lambda_{crp}-1)$.

λ_{crp} is taken as the minimum value of $h/200\ \delta_{np}$ for the 'plastic' frame

where:

h is the height of any column, as A.3.2.

δ_{np} is as defined for δ_{n2} A.3.3 but for the 'plastic' frame. This can be calculated from the simplified 'plastic' frame method in Appendix D.

A.4.4 Calculation of deflections

The loads applied to the 'plastic frame' must be the difference between the collapse loads, λ_p × (ULS loads), and the loads resisted by the 'elastic frame'. Therefore the loads to be applied to the 'plastic frame' are $(\lambda_P - \lambda_l)$ × (ULS loads).

Typical deflections due to gravity loads alone are shown in Figure A.5. Note that the frame sways under gravity loading plus notional horizontal forces. This is partly due to the direct effect of the notional horizontal forces and partly because the notional horizontal forces causes the hinge to appear on one side only, creating an asymmetric frame that results in sway.

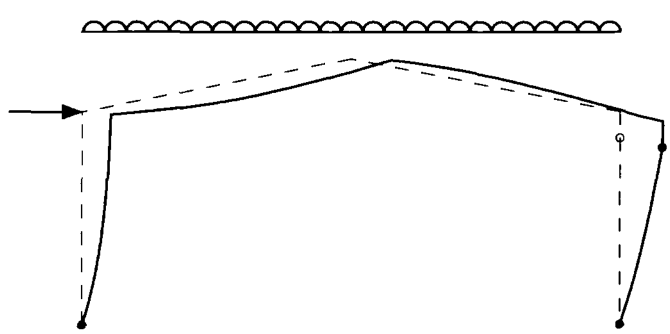


Figure A.5 *Typical deflection of a 'plastic frame'*

In the absence of a more detailed analysis, the hinges in the 'plastic frame' model should be assumed to be asymmetric, as in Figure A.6, to avoid unconservative deflections at collapse load.

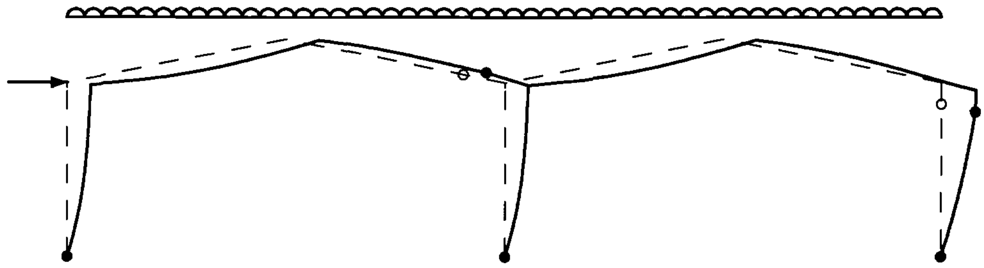


Figure A.6 *Typical design hinge pattern for a 'plastic frame'*

A.4.5 The 'plastic frame' resisting vertical loads

The deflections from λ_l to λ_p are calculated by simple beam theory based on simply supported beams because the plastic hinges mean that there can be no increase in end moments of the rafters. To simplify the calculations, this 'hand' method assumes that all rafters will behave as simply supported beams from load factor λ_l.

The horizontal deflections $\delta_{EP}1$ due to gravity load result from the end rotation of the rafters. For simplicity, this method takes the maximum sway deflection arising from any span. If a frame analysis is performed with a pin at the hinge positions, a less conservative result may be obtained.

The load on the span at ULS is w_{ULS} and the load applied to the span on the elastic frame is w_E. Therefore, the load to be applied to the 'plastic frame' $w_P = \lambda_p \, w_{ULS} - w_E$.

The column top displacement for the external column/rafter that remains elastic is governed by the rafter end. The rafter end rotation is approximately the same as for a simply supported beam of the same developed length, *S*, as shown in Figure A.7.

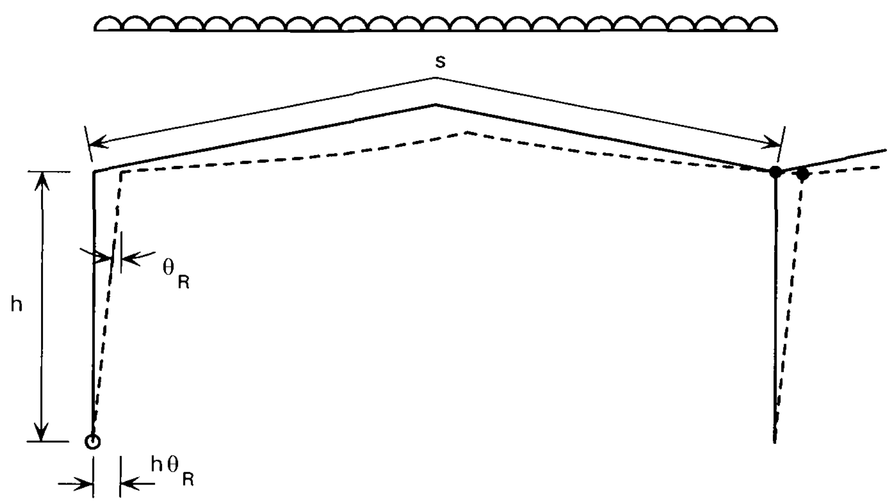


Figure A.7 *Column top displacement*

For simplicity, the calculations are based on the deflections of a symmetrical pitched roof, but the method may be used for other geometries.

The second-order rafter end slope is given by:

$$\theta_{R2} = \frac{w_p S^3}{24 EI_R}\left(\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right)$$

where:

- S is the developed length of the rafter
- w_P is the load on the span of the 'plastic frame'
- w_{ULS} is the Ultimate Limit State load on the span
- w_E is the load applied to the 'elastic frame'
- λ_p is the plastic collapse factor
- I_R is the gross value of the major axis inertia of the rafter.
- E is Young's modulus
- $\lambda_{crp}/(\lambda_{crp}-1)$ is the deflection magnification factor from A.4.3.

Note that w_p, w_{ULS} and w_l here denote the component of load acting transverse to the rafter measured as a distributed load along the developed length of the rafter.

For vertical loads measured on plan, $w_{v.plan}$, the component transverse to the rafter is given by:

$$w = w_{v.plan} \text{Cos}^2\alpha$$

For vertical loads measured along the slope, $w_{v.slope}$, the component transverse to the rafter is given by:

$$w = w_{v.slope} \text{Cos}\alpha$$

where:

- w is the set of loads w_p, w_{ULS} or w_l
- α is the slope of the rafter.

The second-order change of the column top deflections $= h\ \theta_{R2}$

The spread of each span is calculated from the deflection of a simply supported beam as shown in Figure A.8. The second-order midspan deflection of a straight simply supported beam of length S carrying a distributed load w_p is given by:

$$\delta_{B2} = \frac{5 w_p S^4}{384 EI_R}\left(\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right)$$

The spread of the rafter ends results from the deflection at the crank in the rafter. The spread is given by:

$$\text{Spread} = \delta_{B2}\ (\text{Sin}\alpha_1 + \text{Sin}\alpha_2)$$

where;

- α_1 is the slope of one rafter in the span

α_2 is the slope of the other rafter in the span.

Note that there is no spread if the rafter is straight from column to column.

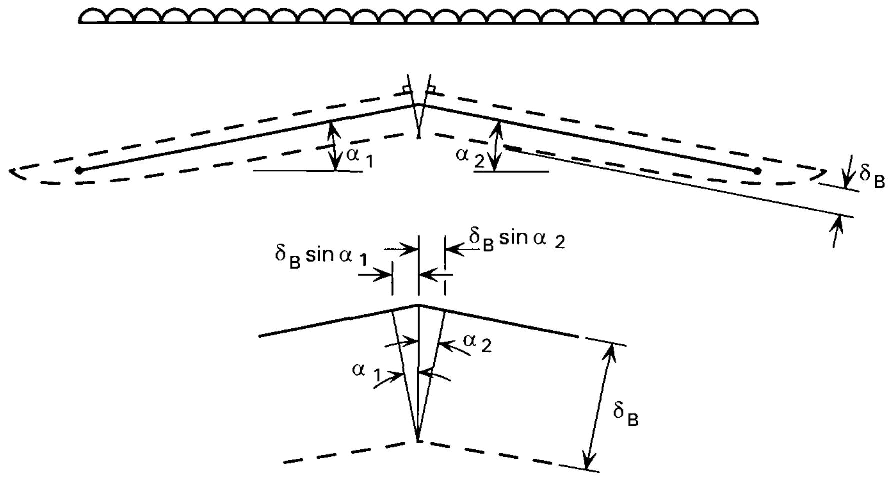


Figure A.8 *Rafter spread*

A.4.6 The 'plastic frame' resisting horizontal loads

The sway deflections due to the notional horizontal forces or externally applied horizontal loads are calculated assuming the reduced stiffness of the frame following the formation of plastic hinges as shown in Figure A.9. The formulae are derived in Appendix D.

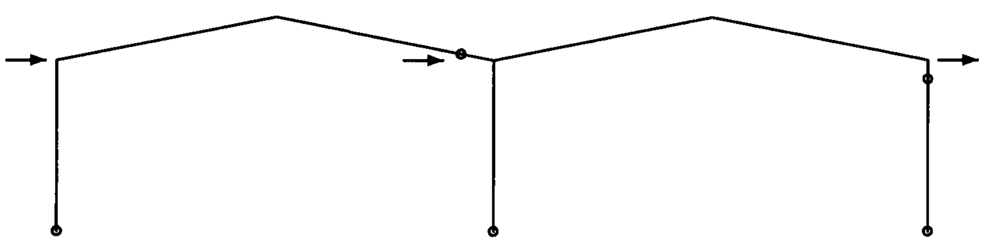


Figure A.9 *Sway deflection model*

The second-order sway stiffness of each bay is given by:

$$K_s = \frac{1}{\delta_{s2}} = \frac{1}{\left(\dfrac{Sh^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} \times \frac{1}{\left(\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}\right)}$$

where:

K_s is the sway stiffness of the span including second-order effects

δ_{s2} is the sway deflection of the top of the column which does not have a hinge in it or in the adjacent length of rafter including second-order effects

S is taken as L_r, the developed length of the rafter from support to support where the support may be either a column or valley beam

h is the height of the column from the base to the neutral axis of the rafter

I_R and I_c are the gross values of the major axis inertias of the rafter and the column

$\lambda_{crp}/(\lambda_{crp}-1)$ is the second-order magnification factor from A.4.3.

The second-order least sway deflection of the top of the column in a frame may be taken as:

$$\Delta_2 = \frac{(\lambda_p - \lambda_1)\Sigma H}{\Sigma K_s}$$

where:

ΣH is the sum of the equivalent horizontal loads H resisted by the frame. At each column, H is given by $H = \Sigma(H_i h_i)/h$, as shown in Figure A.9

ΣK_s is the sum of the sway stiffnesses allowing for second-order effects of all the spans in the frame.

In addition to this sway deflection, there is an additional spread arising from the angle in the rafter at the apex of the span. The second-order sagging deflection of a straight rafter would be:

$$\delta_{sm2} = \frac{ML_r^2}{16EI_R}\left(\frac{\lambda_{crp}}{\lambda_{crp}-1}\right)$$

where:

M is the moment caused by the horizontal loads resisted by the span and is given by $M = K_s \Delta_2 h$

L_r is the developed length of the rafter from column top to column top as shown in Figure 2.5

$\lambda_{crp}/(\lambda_{crp}-1)$ is the second-order magnification factor from A.4.3.

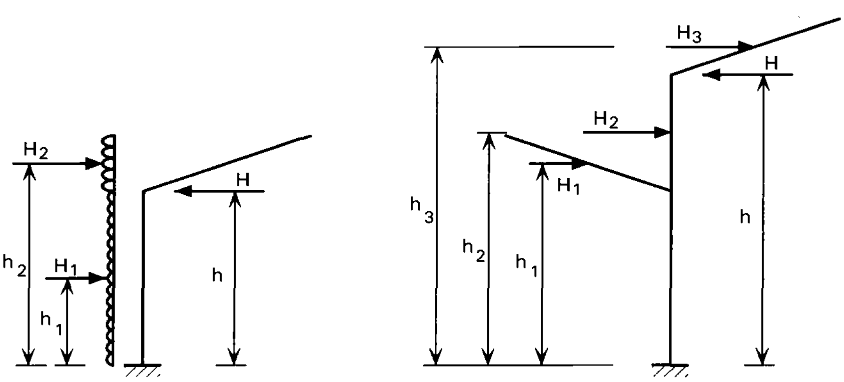


Figure A.10 *Horizontal forces and moments from horizontal forces*

Therefore the additional spread in each span is given by:

Spread $= \delta_{sm2}$ (Sin α_1 + Sinα_2)

Where:

α_1 is the slope of one rafter in the span

α_2 is the slope of the other rafter in the span.

A.5 Axial forces

Some of the axial compression forces differ from the first-order analysis values because the shape of the frame differs from the undeformed shape. The axial forces in the deformed structure must be used in the energy summations.

There can be no change in the total axial forces in the columns. However the drop of the apex will change the axial compression in the rafters. Therefore, in the absence of a detailed analysis the axial loads at collapse should be taken as:

Columns $P_2 = P_1$

Rafters $$P_2 = \frac{P_{1a}}{\left(1 - \frac{\delta_a}{h_a}\right)} + P_{1b}$$

where:

P_2 is the set of axial forces in the members at ULS, including the second-order effects

P_1 is the set of axial forces at ULS in the first-order analysis

P_{1a} is the axial compression force in the rafter at ULS at the hinge nearest mid-span calculated by first-order analysis. The value at mid-span may be used

P_{1b} is the additional axial compression force in the rafter such that $P_1 = P_{1a} + P_{1b}$

A.6 Reserve factor at Ultimate Limit State

The energy released by second-order effects is calculated from the expression $\Sigma P_2 \phi s d\phi$, which is defined in A.2.2. This summation is shown in the worked examples.

The energy absorbed by the plastic mechanism is calculated from the expression $\Sigma M_{pr} d\theta$ which is defined in A.2.2. This summation is shown in the worked examples.

The reserve factor on moment, λ_M, is calculated from the first-order plastic collapse factor, λ_p, as follows:

$$\lambda_M = \lambda_p \left[1 - \frac{\Sigma P_2 \phi s d\phi}{\Sigma M_{pr} d\theta}\right]$$

The load factor of the frame at failure is taken as λ_M.

APPENDIX B Second order analysis of tied portals 'by hand'

B.1 Range of application and design steps

This Appendix shows how second-order analysis may be performed without second-order software for tied portal frames, designed by plastic design methods. These portals have ties at or near the rafter level. The method is only intended for frames in which:

(i) the tie connects either to the column/rafter haunch or directly to the rafter

(ii) no hinges occur below ULS when analysed by first-order analysis

(iii) the tie does not yield below ULS.

It should be recognised that tied portals with low rafter slopes are potentially susceptible to snap-through of the rafters. The worked example shows a significant increase in axial force and consequent apex drop for a frame with an 8° slope. It is recommended that slopes less than 6° should not be used without additional consideration of the stiffnesses of members and components.

Portals in which there is a tie at an internal floor level, but where there is no tie at or near the rafter level, should be designed using the method for common portals, given in Appendix A.

It is recommended that the ties in tied portals are designed so that they do not yield below Ultimate Limit State, because yielding of the tie may cause a major change in both the geometry of the structure and the structural behaviour. It is also recommended that this method is not used for frames for which the deflections from a sway-check exceed $h/500$, as explained in Section 5.4.3.

Design steps

The design steps are as follows:

1. Identify from the first order analysis:
   - the plastic collapse mechanism
   - the hinge incremental rotations
   - the axial forces in the members
   - the load factor at the formation of the first plastic hinge, λ_1 (excluding hinges at bases where the moment of resistance of the base is less than the moment of resistance of the columns)
   - the plastic collapse factor, λ_p
   - the deflections of the frame at λ_1.

2. Calculate the second-order deflections of the 'elastic' frame.

   (a) Calculate the stiffness reduction factors to allow for $P.\delta$ (P-little delta) effects

(b) Calculate the second-order magnification factors for the 'elastic' frame.

(c) Calculate the deflections of the 'elastic' frame. These are the values of the deflections at λ_1 plus additional deflections calculated to allow for the second-order effects.

3. Calculate the second-order deflections of the 'plastic' frame.

   (a) Calculate the stiffness reduction factors to allow for $P.\delta$ (P-little delta) effects

   (b) Calculate the second-order magnification factor of the 'plastic' frame.

   (c) Calculate the deflections of the 'plastic' frame from gravity loads.

   (d) Calculate the deflections of the 'plastic' frame from horizontal loads.

4. Calculate the apex drop using an iterative method.
5. Calculate the increased axial loads in the rafters to account for second-order effects.
6. Sum the energies.
7. Calculate the reserve factor, λ_M.
8. Check $\lambda_M \geq 1.0$

B.2 Basis of method

B.2.1 General

The basis of the method is the same as described in A.2.1 for common portals, which are portals without tie at or near the rafter level.

B.2.2 Application of the energy method

The energy calculations are as described in A.2.2 for common portals.

B.2.3 Deflection calculations

The deflection calculations are not identical to the calculations for common portals in A.2.3. This is because, where the tie in a tied portal is at or near the rafter level, the rafters and tie act more like a truss than like the rafters in a common portal. Therefore, the vertical deflection of the apex and the spread of the column tops is dominated by the axial deformations of the rafters and the tie instead of the bending deformations of the rafters and the columns.

The deflections governed by bending stiffness are calculated using the same principles as in A.2.3 for common portals.

The following deflections are governed by bending stiffness:

- sway of the frame, for both 'elastic' and 'plastic' frames
- mid-rafter deflection from sway and transverse loads on the rafter.

The deflection of the apex is calculated from the following components:

- The elastic deflection calculated at ULS from the first-order frame analysis.

- A drop from curvature shortening of the rafters.
- Further deflection arising from the increase in axial force in the rafters to maintain the same vertical component of force as in first-order analysis because the rafter slope has decreased due to the drop of the apex.

B.2.4 Base stiffness

The base stiffness may be taken as in A.2.4.

B.3 Bending deflections of the 'elastic' frame

B.3.1 General

The methods are the same as described in A.3 except for the differences noted below. Only the differences from A.3 are given below, to clarify the comparison of the methods for common and tied portals.

B.3.2 Reduction factor to allow for P.δ (P–little delta) effects

For rafters of tied portals with ties that have not yielded at ULS, the method is similar to the method in A.3.2 but the calculation of P_{cr} differs from A.3.2. The length L is taken as the length along the slope from apex to eaves/valley. The axial forces in the rafter should be taken as the forces occurring in the length resisting the tie force because these are the highest forces occurring within the length of the rafter.

For columns of tied portals, the method is identical to that in A.3.2.

B.3.3 Second-order magnification factors, $\lambda_{cr}/(\lambda_{cr}-1)$

The methods are the same as in A.3.3, except that in the calculation of δ_n, the rafter span is taken as the length from apex to eaves/column.

B.3.4 Calculation of deflections

The principles are as in A.3.4. However, the second-order vertical deflections are governed by truss action, which is calculated according to B.5. Therefore, only the sway deflections are magnified by $\lambda_{cr}/(\lambda_{cr}-1)$.

The second-order deflections on the elastic frame, excluding the deflections within the truss system, are calculated as follows:

$$\delta X_2 = (\delta X_1 - \delta X_{1s}) + \delta X_{1s}\,\{\lambda_{cr1}/(\lambda_{cr1} - 1)\}$$

δY_2 are calculated for the truss system as in B.5.

B.4 Bending deflections of the 'plastic' frame

B.4.1 General

To simplify the calculations, this method assumes that all the spans develop plastic hinges at one end at the load factor λ_1 at which the first hinge forms in the frame. Then the analysis model becomes an elastic frame with a hinge at (or near) the end of each rafter as shown in Figure B.1. The pins are used at the plastic hinges because at a pure plastic hinge there is no increase of bending moment.

This mode of deflection is a sway mode even for gravity loads, so all the second-order deflections are calculated from the magnification arising from the sway mode of buckling. Therefore, the second-order deflections are given by:

$$\delta = \delta_1 \{\lambda_{cr}/(\lambda_{cr}-1)\}$$

B.4.2 Reduction factor to allow for $P.\delta$ (P–little delta) effects

The second-order effects within the member lengths reduce the effective stiffness of the members. This effect must be included in the analysis. This is done by calculating an effective inertia value for the members, I_{eff}, as for the 'elastic' frame in Section B.3.2 above.

B.4.3 Second-order magnification factor, $\lambda_{crp}/(\lambda_{crp}-1)$

The principles that apply are as in A.4.3, except that in the calculation of δ_{np}, the rafter span is taken as the length from apex to eaves/column.

B.4.4 Calculation of deflections

The loads applied to the 'plastic frame' must be the difference between the collapse loads, λ_p × (ULS loads), and the loads resisted by the 'elastic frame'. Therefore the loads to be applied to the 'plastic frame' are $(\lambda_p - \lambda_1)$ × (ULS loads). Typical deflections are shown in Figure B.1, which shows that the frame sways under gravity loading plus notional horizontal forces. This sway is partly due to the direct effect of the notional horizontal forces and partly because the notional horizontal forces cause the hinge to appear on one side only, creating an asymmetric frame and an asymmetric response.

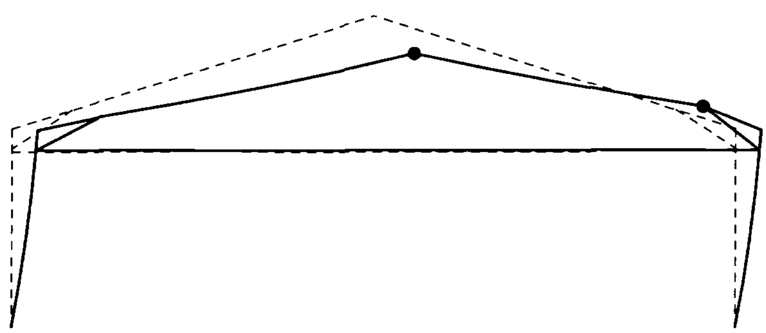


Figure B.1 *Typical deflection of a 'plastic frame'*

In the absence of a more detailed analysis, the hinges in the 'plastic frame' model should be assumed to be asymmetric, as in Figure B.2, to avoid unconservative deflections at collapse load.

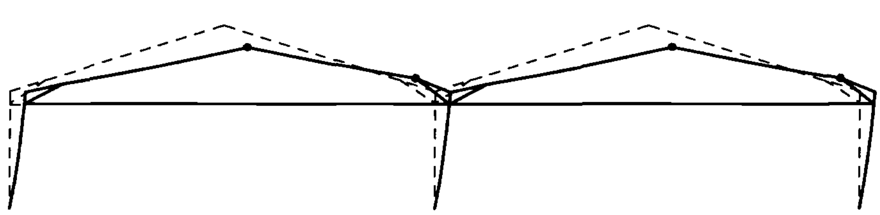


Figure B.2 *Typical design hinge pattern for a 'plastic frame'*

B.4.5 The 'plastic frame' resisting vertical loads

The deflections from λ_1 to λ_p are calculated by simple beam theory based on simply supported beams because the plastic hinges mean that there can be no increase in end moments of the rafters. To simplify the calculations, this 'hand' method assumes that all rafters will behave as simply supported beams from load factor λ_1.

The horizontal deflections due to gravity load results from the end rotation of the rafters of one of the spans. Where the analysis model is a series of individual spans instead of the entire frame, the greatest end rotation should be used.

The load on the span at ULS is w_{ULS} and the load applied to the span on the elastic frame is w_E. Therefore, the load to be applied to the 'plastic frame' $w_P = \lambda_p w_{ULS} - w_E$.

The column top displacement for the external column/rafter that remains elastic is governed by the rafter end. The rafter end rotation is approximately the same as for a simply supported beam of the same developed length, S, as shown in Figure B.3.

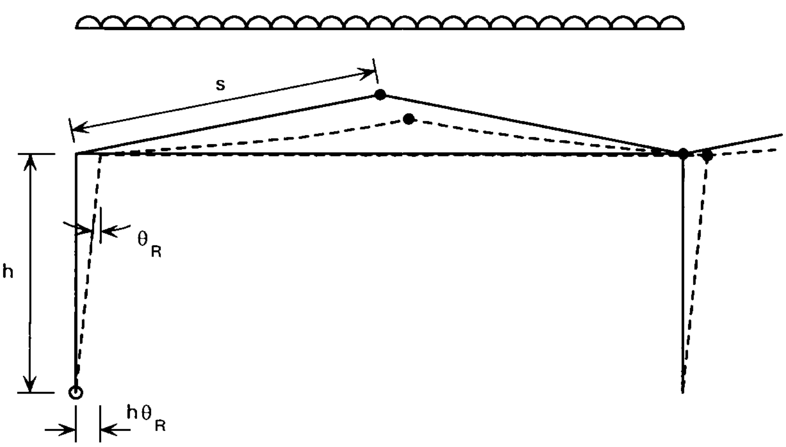


Figure B.3 *Column top displacement*

The second-order rafter end slope is given by:

$$\theta_{R2} = \frac{w_p S^3}{24 EI_R}\left(\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right)$$

giving a sway deflection of the column top $= h\theta_R$.

The transverse deflection of the rafter at mid-rafter (where there is a plastic hinge) is given by:

$$\delta_{R2} = \frac{5 w_p S^4}{384 EI_R}\left(\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right)$$

where:

S is the rafter length from apex to eaves/valley

w_P is the load on the span of the 'plastic frame'

w_{ULS} is the Ultimate Limit State load on the span

w_E is the load applied to the 'elastic frame'

λ_p is the plastic collapse factor

I_R is the gross value of the major axis inertia of the rafter

E is Young's modulus

$\lambda_{crp}/(\lambda_{crp}-1)$ is the deflection magnification factor from B.4.3.

Note that w_p, w_{ULS} and w_l here denote the component of load acting transverse to the rafter measured as a distributed load along the developed length of the rafter.

For vertical loads measured on plan, $w_{v.plan}$, the component transverse to the rafter is given by:

$$w = w_{v.plan} \, \text{Cos}^2\alpha$$

For vertical loads measured along the slope, $w_{v.slope}$, the component transverse to the rafter is given by:

$$w = w_{v.slope} \, \text{Cos}\alpha$$

where:

w is the set of loads w_p, w_{ULS} or w_l

α is the slope of the rafter.

B.4.6 The 'plastic frame' resisting horizontal loads

The sway deflections due to the notional horizontal forces or externally applied horizontal loads are calculated assuming the reduced stiffness of the frame following the formation of plastic hinges as shown in Figure B.4. The formulae are derived in Appendix C.

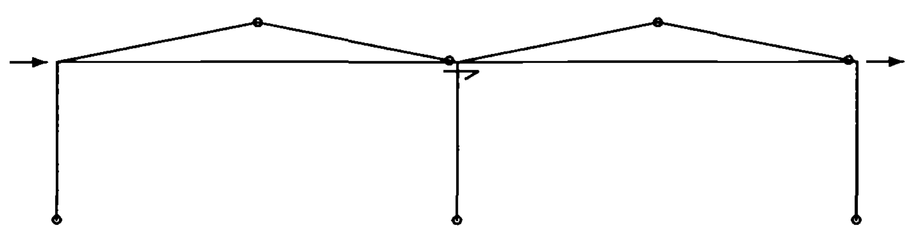


Figure B.4 *Sway deflection model*

The second-order sway stiffness of each bay is given by:

$$K_s = \frac{1}{\delta_{s2}} = \frac{1}{\left(\frac{Sh^2}{3EI_R} + \frac{h^3}{3EI_c}\right)} \times \frac{1}{\left(\frac{\lambda_{crp}}{\lambda_{crp}-1}\right)}$$

where:

K_s is the sway stiffness of the span including second-order effects

δ_{s2} is the sway deflection of the top of the column which does not have a hinge in it or in the adjacent length of rafter, including second-order effects

S is taken as the length of the rafter from apex to eaves/valley

h is the height of the column from the base to the neutral axis of the rafter

I_R and I_c are the nominal values of the inertias of the rafter and the column

$\lambda_{crp}/(\lambda_{crp}-1)$ is the second-order magnification factor from B.4.3.

The second-order least sway deflection of the top of the column in a frame may be taken as:

$$\Delta_2 = \frac{(\lambda_p - \lambda_1)\Sigma H}{\Sigma K_s}$$

where:

ΣH is the sum of the equivalent horizontal loads H resisted by the frame. At each column, H is given by $H = \Sigma(H_i h_i)/h$, as shown in Figure B.5.

ΣK_s is the sum of the sway stiffnesses, allowing for second-order effects of all the spans in the frame.

The sagging deflection of the rafter at mid-length between the apex and the eaves/valley, caused by horizontal loads, is given by

$$\delta_{sm2} = \frac{MS^2}{16EI_R}\frac{\lambda_{cr1}}{(\lambda_{cr1}-1)}$$

where:

S is the length along the rafter slope from apex to eaves/valley

I_R is the gross I_x value of the rafter

M is the moment caused by the horizontal loads resisted by the span and is given by $M = K_s \Delta_2 h$.

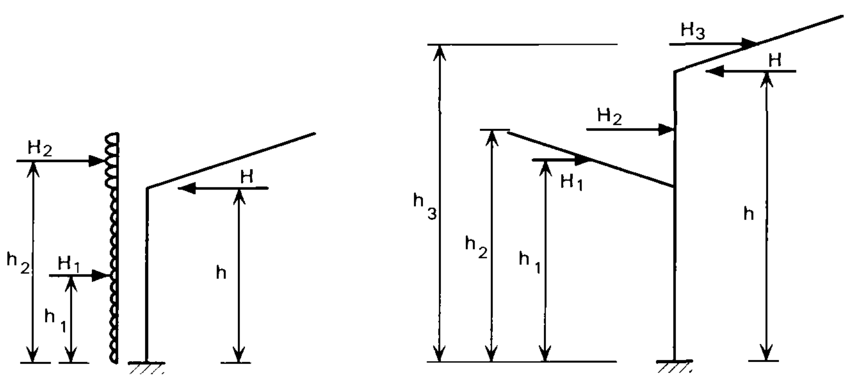


Figure B.5 *Horizontal forces and moments from horizontal forces*

B.5 Deflections of the rafters/tie 'truss' system

B.5.1 General

The apex deflection is calculated from

(i) First-order elastic deflections of the frame at ULS.

(ii) Apex drop from rafter curvature.

(iii) Increased rafter forces so that the vertical component of the rafter force at the apex remains equal to the first-order values despite the reduced slope of the rafters due to apex drop.

Step (iii) may require iteration until the solution is acceptably close to equilibrium. This is measured in terms of increase of the drop of the apex. The solution may be taken as acceptable when the increase in apex drop, from one iteration, is less than 3% of the total apex drop calculated before the iteration.

B.5.2 Apex drop from first-order elastic deflections

The apex drop at ULS from first-order behaviour may be calculated from

$$\delta_{ULS} = \delta_l / \lambda_l$$

where:

λ_l is the load factor at the formation of the first hinge

δ_l is the apex drop at λ_l calculated from the first-order behaviour.

B.5.3 Apex drop from curvature shortening δ_c

It is unusual to consider the shortening of the end to end length of members caused by curvature. However, where tied portals have low roof slopes, for example around 8° or lower, the apex drop is very sensitive to member shortening. Tied portals tend to have slender rafters, so curvature shortening should be considered. The shortening is calculated from the deflection of the length of the rafter between the 'sharp' end of the column/rafter haunch and the apex of the roof, S_r, shown in Figure B.6.

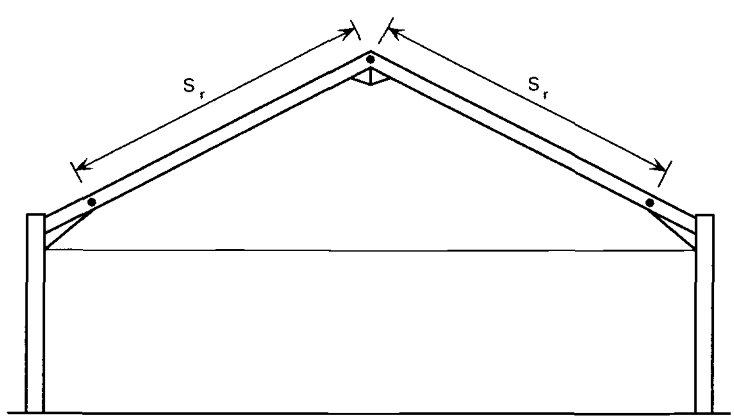


Figure B.6 *Length of rafter in curvature shortening calculations*

The deflection within S_r is calculated assuming that the bending moment diagram is as shown in Figure B.7. The hogging moments at the ends of the length S_r are equal to the sagging moment at mid-length due to plastic redistribution of moment, so that

$$M_R = \frac{w_{ULS} S_r^2}{16}$$

where:

w_{ULS} is the transverse load along the rafter at ULS.

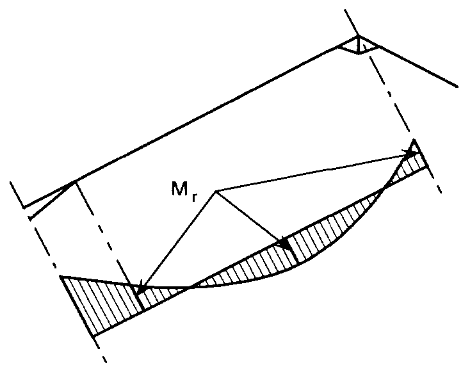


Figure B.7 *Bending moment diagram for curvature shortening calculations*

The second-order transverse deflection, $\delta_{cs2} = \frac{2}{384} \frac{w_{ULS} S_r^2}{EI_{eff.R}}$

where:

$I_{eff.R}$ is calculated according to B.3.2.

The shortening is then approximated as $\Delta = \frac{\pi^2 (\delta_{cs2})^2}{4 S_r}$

The apex drop from shortening is calculated as shown in Figure B.8 from:

$$\text{Apex drop} = \frac{\Delta}{\text{Sin}\alpha}$$

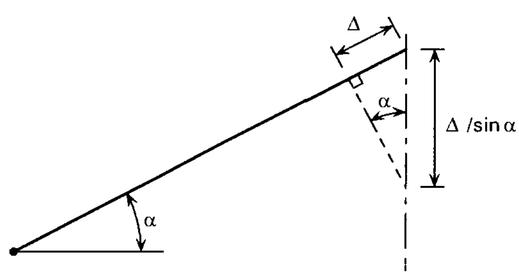


Figure B.8 *Apex drop from rafter shortening*

B.5.4 Apex drop from increased rafter axial force

The apex drop reduces the slope of the rafters, so the force must increase to provide the same vertical component of rafter force. The increase in axial force causes additional apex drop, which is calculated by an iterative process as follows:

(i) Calculate the vertical component of the rafter axial force at the apex according to first-order analysis.

$$\therefore \quad V_1 = P_{R1} \text{ Sin } \alpha_1$$

(ii) Calculate the reduced slope of the rafter from first order deflections from Section B.5.2 and from curvature shortening from Section B.5.3. The deflection is:

$$d_1 = \delta_1 + \frac{\Delta}{\text{Sin}\alpha_1}$$

The unstressed rise of the rafter above the hinge at the bottom of the rafter, $h_1 = S_r \text{ Sin}\alpha$, as shown in Figure B.9.

$$\therefore \quad \text{Reduced rise, } h_2 = h_1 - d_1$$

$$\therefore \quad \text{Reduced slope, } \alpha_2 = \text{Sin}^{-1}\left(\frac{h_1 - d_1}{S_r}\right)$$

(iii) Calculate the reduced vertical component of the rafter

$$V_2 = P_{R1} \text{ Sin}\alpha_2.$$

(iv) Calculate the required increase in rafter axial force:

$$\delta P_R = \frac{V_2 - V_1}{\text{Sin}\alpha_2}$$

(v) Calculate the resultant increase in horizontal reaction at the column top:

$$\delta H_c = \delta P_R \text{ Cos}\alpha_2.$$

(vi) Calculate the resulting increase in tie force:

$$\delta T = \delta H_c \left(\frac{c + h_T}{h_T}\right)$$

where, e and h_T are defined in Figure B.9 and c is defined in Figure B.10

(vii) Calculate the horizontal movement of the hinge Z due to tie stretching:

$$\text{Lateral displacement of tie end, } \delta X_T = \frac{\delta T \times \text{halfspan}}{A_T E}$$

where A_T is the cross-sectional area of the tie

$$\text{Lateral displacement of } Z,\ \delta X_{ZT} = \delta X_T \left(\frac{e + h_T}{h_T}\right)$$

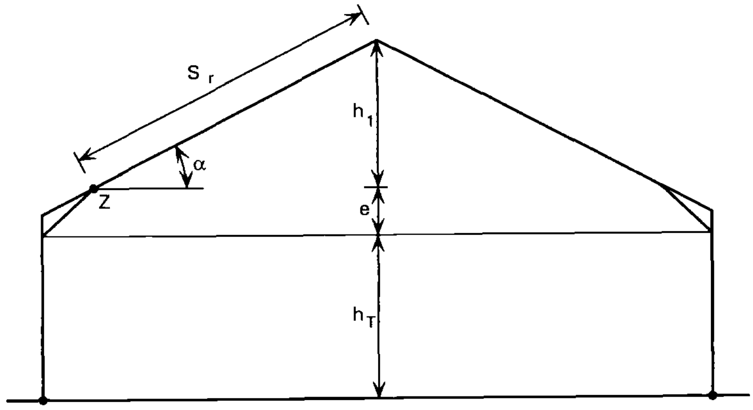


Figure B.9 *Tied portal geometry*

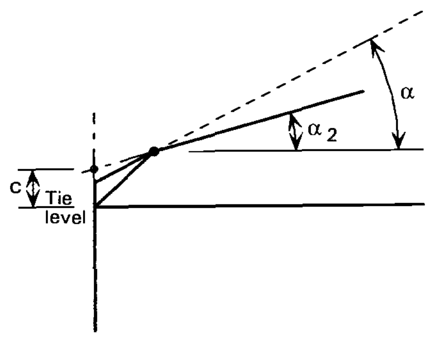


Figure B.10 *Effective cantilever above tie*

(viii) Calculate the horizontal movement of hinge Z due to column bending:

$$\delta X_{ZC} = \frac{\delta H_c c^2 (h_T + c)}{3EI_{\text{eff.c}}}$$

where $I_{\text{eff.c}}$ is calculated according to B.3.2.

(ix) Calculate the rafter shortening from increased axial strain

$$\delta S_r = \frac{\delta P_R S_r}{A_R E}$$

where A_R is the cross-sectional area of the rafter.

(x) Calculate the increased rafter drop from increased rafter axial as Figure B.11.

$$d_A = \frac{\delta X_z}{\text{Tan}\alpha_2} + \frac{\delta S_r}{\text{Sin}\alpha_2}$$

where $\delta X_Z = \delta X_{ZT} + \delta X_{ZC}$

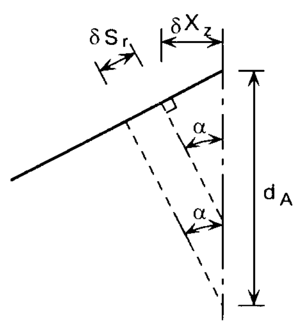


Figure B.11 *Apex drop from increased rafter axial force*

(xi) Re-calculate the rafter slope. The definition is now given by:

$$d_2 = \delta_1 + \frac{\Delta}{\mathrm{Sin}\alpha_2} + d_A$$

$$\text{Revised slope } \alpha_2 = \mathrm{Sin}^{-1}\left(\frac{h_1 - d_2}{S_r}\right)$$

(xii) Calculate d_2/d_1

If $d_2/d_1 \leq 1.03$, take $\alpha = \alpha_2$ and $P_R = P_{R1} + \delta P_R$.

If $d_2/d_1 > 1.03$, repeat steps (iii) to (xi).

If (revised d_2)/(previous d_2) ≤ 1.03, take $\alpha = \alpha_2$ and $P_R = P_{R1} + \delta P_R$; otherwise, repeat again.

B.6 Axial forces

The energy calculation uses the axial forces at ULS including the second-order effects.

The axial forces in the columns may be taken as the values from first-order analysis because the total of the axial forces must remain the same to preserve vertical equilibrium whatever analysis is used.

The axial forces in the rafters are greater than calculated by first-order analysis. The axial force increases as the slope decreases so that the vertical component of the force remains the same as from the first-order analysis.

Therefore, the forces used in the energy calculation may be calculated as follows:

Columns $P_2 = P_1$

Rafter $P_2 = P_1 + \delta P_R$

Where:

P_2 is the axial force used in the energy calculation

P_2 is the axial force at ULS in the first-order analysis

α_1 is the slope of the rafter in the unstressed condition

α_2 is the slope of the rafter allowing for second-order effects.

B.7 Reserve factor at Ultimate Limit State

The reserve factor on moments, λ_M, is calculated from the first order collapse factor, λ_p, as follows:

$$\lambda_M = \lambda_p \left[1 - \frac{\Sigma P_2 \phi s d\phi}{\Sigma M_{pr} d\theta} \right]$$

The summations are shown in the worked examples.

The load factor of the frame at failure is taken as λ_M.

APPENDIX C Effective stiffness of members

The horizontal thrust on a span of a common portal (i.e. not a tied portal) is limited if a plastic hinge develops at one end of the span, either at the column top or in the rafter. For a column with a pinned base, the horizontal thrust, H, is limited to:

$$H = \frac{M_{pr}}{h}$$

where:

M_{pr} is the plastic moment of the hinge

h is the height of the hinge above the base.

A simple closed solution for the effective stiffness of a uniform rafter is possible if the geometry of the rafters is idealised as a half sine-curve and the loading is idealised as a varying distributed load of a half sine-curve intensity as shown in Figure C.1. The deflections from a sinusoidal load on a uniform member are sinusoidal, allowing a simple solution.

For in increment of loading above the load that forms the first hinge, the deflection of the roof is entirely determined by the bending deflection. This is because the horizontal thrust cannot increase above the limiting value determined by the plastic hinge. For a load increment $w\text{Sin}\frac{\pi x}{L}$, the deflected form calculated by first-order analysis is given by:

$$y = a\text{Sin}\frac{\pi x}{L} = \int\int\left(\frac{d^2y}{dx^2}\right)dx = \int\int\left(-\frac{M}{EI}\right)dx = \int\int\left(-\frac{1}{EI}\right)\int\int(\text{loading})dx$$

$$= \int\int -\left(\frac{1}{EI}\right)\int\int w\,\text{Sin}\frac{\pi x}{L}$$

$$= \frac{wL^4}{\pi^4 EI}\text{Sin}\frac{\pi x}{L}$$

$$\therefore\ w\text{Sin}\frac{\pi x}{L} = EIa\frac{\pi^4}{L^4}\text{Sin}\frac{\pi x}{L}$$

Considering the effects of deflection, the vertical reaction from the horizontal thrust decreases if the roof member deflects downwards, reducing the rise of the arch-shaped member. This loss of vertical reaction must be compensated for by increased reaction from bending to maintain vertical equilibrium.

Taking the undeflected shape of the roof member as $A\,\text{Sin}\dfrac{\pi x}{L}$ and the actual deflection as $b\ \text{Sin}\dfrac{\pi x}{L}$, then the loss of vertical reaction from H is:

$$w_H = H\frac{d^2 y_1}{dx^2} - H\frac{d^2 y_2}{dx^2}$$

$$= HA\frac{\pi^2}{L^2}\text{Sin}\frac{\pi x}{L} - H(A-b)\frac{\pi^2}{L^2}\text{Sin}\frac{\pi x}{L}$$

$$= Hb\frac{\pi^2}{L^2}\text{Sin}\frac{\pi x}{L}$$

The vertical reaction from bending is:

$$w_B = \frac{d^2 M}{dx^2} = \frac{d^2}{dx^2}\left(-EI\frac{d^2 y}{dx^2}\right)$$

∴ for a uniform member

$$w_B = -EI\frac{d^4}{dx^4}\left(b\text{Sin}\frac{\pi x}{L}\right) = -EIb\frac{\pi^4}{L^4}\text{Sin}\frac{\pi x}{L}$$

For vertical equilibrium

$$w + w_H + w_B = 0$$

$$\therefore\quad w = -w_B - w_H$$

$$\therefore\quad w\text{Sin}\frac{\pi x}{L} = EIb\frac{\pi^4}{L^4}\text{Sin}\frac{\pi x}{L} - Hb\frac{\pi^2}{L^2}\text{Sin}\frac{\pi x}{L}$$

But $w\text{Sin}\dfrac{\pi x}{L} = EIa\dfrac{\pi^4}{L^4}\text{Sin}\dfrac{\pi x}{L}$

$$\therefore\quad EIb\frac{\pi^4}{L^4}\text{Sin}\frac{\pi x}{L} - Hb\frac{\pi^2}{L^2}\text{Sin}\frac{\pi x}{L} = EIa\frac{\pi^4}{L^4}\text{Sin}\frac{\pi x}{L}$$

$$\therefore\quad bEI\frac{\pi^2}{L^2} - bH = aEI\frac{\pi^2}{L^2}$$

Writing $EI\dfrac{\pi^2}{L^2} = P_{cr}$

Then $bP_{cr} - bH = aP_{cr}$

$$\therefore\quad b(P_{cr} - H) = aP_{cr}$$

$$\therefore \qquad b = a\frac{P_{cr}}{P_{cr} - H} = a\frac{1}{\left(1 - \dfrac{H}{P_{cr}}\right)}$$

The axial thrust P in the rafter is always greater than the horizontal thrust H, so conservatively

$$b = a\frac{1}{1 - \dfrac{P}{P_{cr}}}$$

Thus the deflection including second-order effects is greater than the deflection from first order calculations by the factor $1/(1 - P/P_{cr})$. Therefore, the second-order effects may be included by using an effective rafter stiffness $I_{eff.R} = I_R \times (1 - P/P_{cr})$.

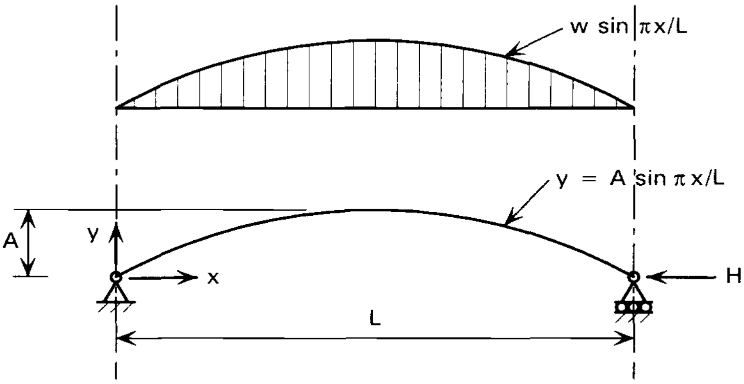


Figure C.1 *Idealised rafter after formation of the first hinge in the span*

APPENDIX D Deflections from horizontal loads for 'hand' second-order calculations

D.1 General

This Appendix describes 'hand' methods of calculating the sway deflections arising from horizontal loads for frames. These methods are intended for use in Appendix A and Appendix B.

Methods are given for 'elastic' frames in D.2 and for 'plastic' frames in D.3.

An alternative and more accurate method of calculating the deflections would be to use software to analyse the frame. For 'plastic' frames, this can be done by inserting pins at the positions of all the plastic hinges assumed in the method described in this Appendix.

D.2 'Elastic' frame sway deflection

D.2.1 General

This Section describes methods of calculating notional sway deflections for the frame when it is entirely elastic.

D.2.2 Simplifying assumptions

The majority of multi-span portal frames have slender internal columns. When a horizontal load is applied to these frames, there is only a small bending moment induced in these slender internal columns, because the external columns are much stiffer. A typical bending moment diagram is shown in Figure D.1.

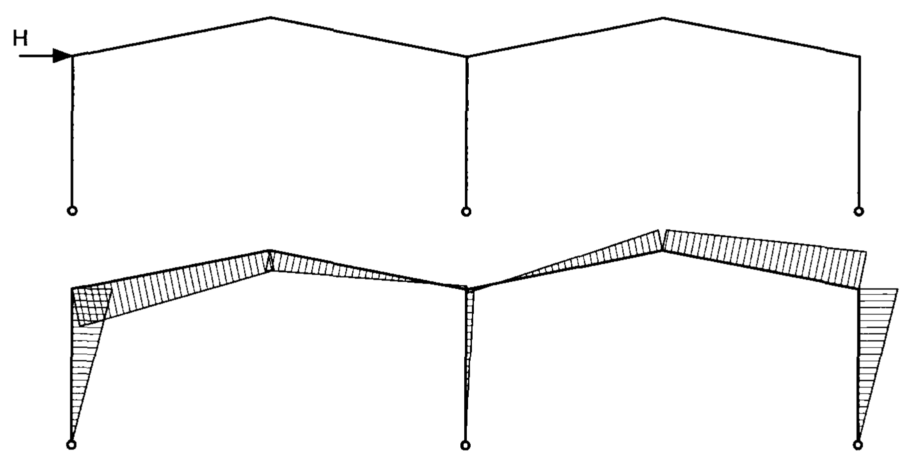


Figure D.1 *Bending moments in a typical two-span frame under horizontal loading*

This can be considered as two sub-frames, each comprising an external column and a rafter pair, as shown in Figure D.2. For multi-span frames in general, the two external sub-frames provide the majority of the stiffness, so the same model of a pair of sub-frames could be used for hand calculations. Where the stiffness of the internal columns is to be included, it is preferable to use software for the analysis of the entire frame.

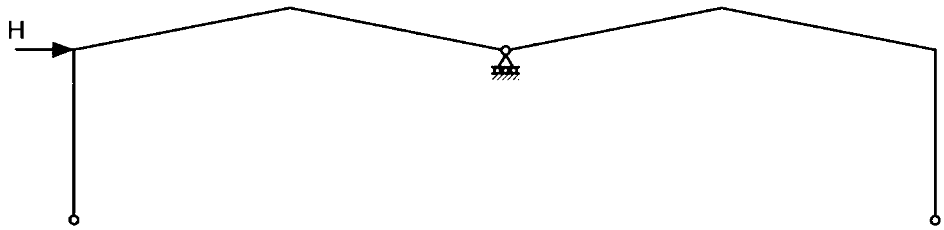


Figure D.2 *Sub-frames for a typical two-span frame*

Where the internal columns provide significant stiffness, it is uneconomic to ignore them and a detailed analysis of the entire frame by software would be preferable to the simple methods given here.

Single-span portals and tied portals are better modelled with the rafter length taken as eaves to mid-span, as shown in Figure D.3.

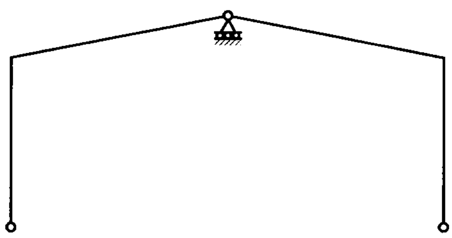


Figure D.3 *Sub-frames for a typical single-span frame.*

Given the above assumptions, the calculation of the sway deflections caused by horizontal loads becomes a reasonable task.

D.2.3 Method for first-order sway deflections

The frame is considered to be a pair of sub-frames as shown in Figure D.2 for multi-span frames or Figure D.3 for single-span frames. A typical sub-frame is shown in Figure D.4.

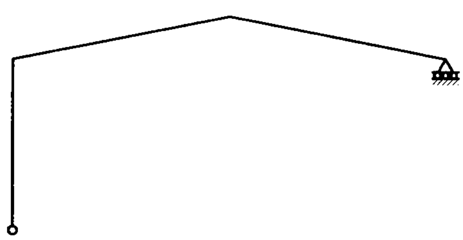


Figure D.4 *Idealised span*

When a horizontal force H is applied to the structure in Figure D.4, the resulting bending moment diagram is a shown in Figure D.5. The bending moment diagram has been drawn on the compression side for clarity.

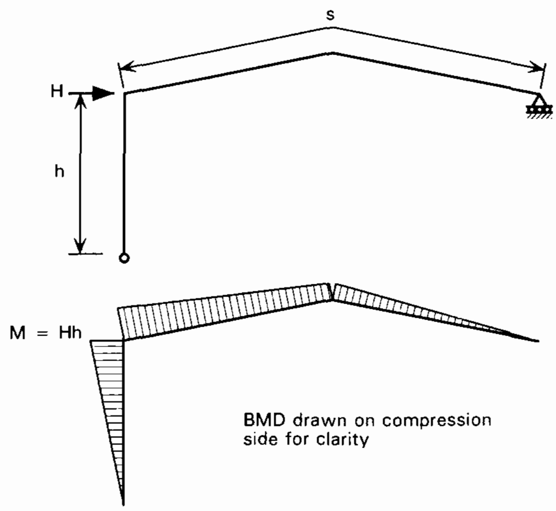


Figure D.5 *Bending moment diagram*

The resulting deflections are shown in Figure D.6.

The rafter end slope is given by:

$$\theta = \frac{MS}{3EI_R}$$

Therefore the column top deflection due to θ is given by:

$$\delta_R \quad = h\theta \quad = h\left(\frac{MS}{3EI_R}\right) \quad = h\left(\frac{(Hh)S}{3EI_R}\right) \quad = \frac{HSh^2}{3EI_R}$$

The column top deflection due to column flexure is given by:

$$\delta_c \quad = \frac{Mh^2}{3EI_c} \quad = \frac{(Hh)h^2}{3EI_c} \quad = \frac{Hh^3}{3EI_c}$$

The total column top deflection δ is given by:

$$\delta \quad = \delta_R + \delta_c \quad = H\left(\frac{Sh^2}{3EI_R} + \frac{h^3}{3EI_c}\right)$$

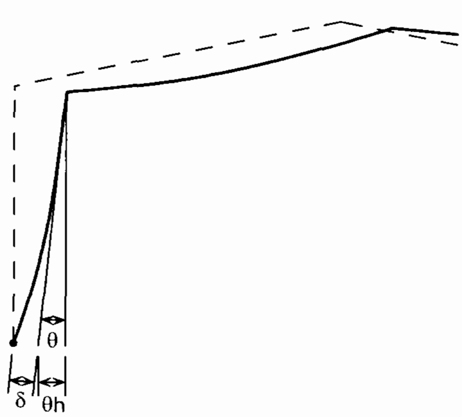


Figure D.6 *Column top deflection*

Therefore the first-order column top stiffness K is given by:

$$K = \frac{H}{\delta} = \frac{1}{\left(\dfrac{Sh^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)}$$

In the frame shown above, which has been idealised as two sub-frames, the total column top stiffness of the whole frame is the sum of the stiffnesses of the two sub-frames, ΣK.

The first-order sway deflection, δX, is given by:

$$\delta X = \frac{H}{\Sigma K}$$

D.2.4 First-order rafter spread and apex drop

In addition to the column top deflection, the span will spread because of the sagging deflection in the rafter coupled with the angle at the apex. By simple hand methods, it is only possible to produce approximate allowance for this effect in multi-bay frames. This is done by assuming the sub-frames illustrated in Figure D.4 are independent, calculating the spread of each span and then calculating the total accumulation of spread across each frame.

The deflection at mid-span of a symmetrical rafter of length S can be calculated by the moment area method as shown in Figure D.7. This method can be adapted for any other apex position. The deflection at the mid-span δ is given by:

$$\delta = \frac{8ML^2}{48EI} - \frac{5ML^2}{48EI} = \frac{3ML^2}{48EI} = \frac{ML^2}{16EI}$$

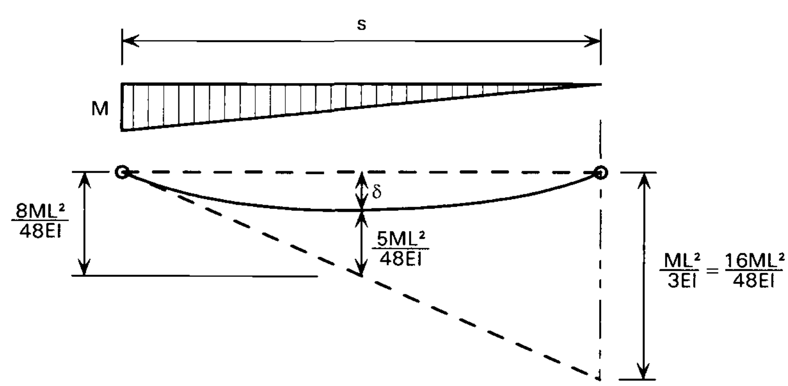


Figure D.7 *Deflection of a straight rafter*

The spread of the pitched-roof rafter is calculated from the deflection of a straight rafter as shown in Figure D.8. The spread is given by:

Spread = $\delta(\text{Sin}\ \alpha_1 + \text{Sin}\alpha_2)$

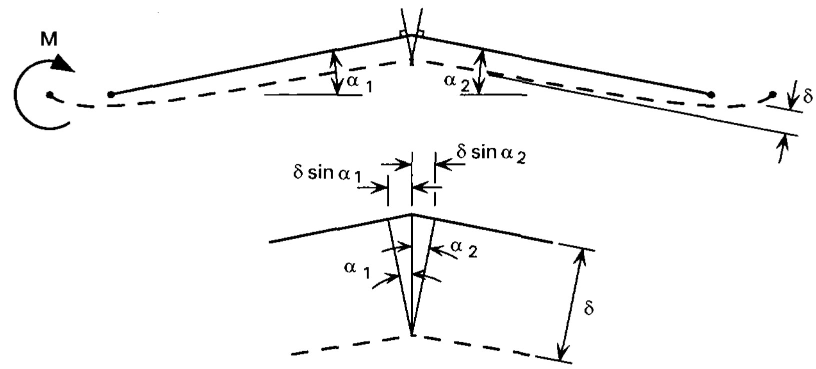


Figure D.8 *Spread of pitched-roof rafter*

D.2.5 Method for second-order notional sway deflection

The second-order notional sway deflections, δ_{n2}, is used to calculate the critical buckling ratio for the sway mode, λ_{cr1}, for the frame when it is entirely elastic.

The method is similar to the method for calculating first-order sway deflections in D.2.3. The difference is that $P\delta$ effects are included by using the effective inertias, I_{eff}, of the members as calculated in A.3.2. These are used in the method in D.2.3 in place of the gross inertias, I. In addition, the effect of base stiffness may be added as explained in D.4.

The applied horizontal load, H, is the notional horizontal forces which must be taken as the full 0.5% of the Ultimate Limit State (ULS) loads, see Section 1.6, because the formula for calculating the critical buckling ratio,

$$\lambda_{cr} = h/200\delta$$

assumes that 0.5% of the ULS loads has been applied to calculate δ.

The total column top deflection δ is given by:

$$\delta = \delta_R + \delta_c = H\left(\frac{Sh^2}{3EI_{eff.R}} + \frac{h^3}{3EI_{eff.c}}\right)$$

Therefore the second-order column top stiffness K_2 is given by:

$$K_2 = \frac{H}{\delta} = \frac{1}{\left(\dfrac{Sh^2}{3EI_{eff.R}} + \dfrac{h^3}{3EI_{eff.c}}\right)}$$

In the frame shown above, which has been idealised as two sub-frames, the total column top stiffness of the whole frame is the sum of the stiffnesses of the two sub-frames, ΣK_2.

The second-order notional sway deflection, δ_{n2} , is given by:

$$\delta_{n2} = \frac{H}{\Sigma K_2 + \Sigma K_b}$$

where K_b is calculated in D.4.

The critical buckling ratio for the first mode of frame buckling (which is the sway mode) for the 'elastic' frame is given by:

$$\lambda_{cr1} = h/200\delta_{n2}$$

D.3 'Plastic' frame sway deflection

D.3.1 General

This Section describes methods of calculating notional sway deflections for the frame when the stiffness of the frame has been reduced by the formation of plastic hinges.

D.3.2 Simplifying assumptions

The load factor λ_1 is defined as the load factor at which the first plastic hinge forms. To simplify hand calculations, it is assumed that a plastic hinge occurs at one end of every span at λ_1. This is a conservative assumption. It is also assumed that all the spans become mechanisms as the same load factor, which is λ_p.

Given the above assumptions, the calculation of the sway deflections caused by horizontal loads becomes a reasonable task.

D.3.3 Method

A typical two span frame is shown in Figure D.9 with the Ultimate Limit State (ULS) loads. The load factor at the formation of the first hinge is defined as λ_1 and it is assumed that a plastic hinge forms in all spans so that the structure can be idealised as shown in Figure D.10. This ideal structure behaves as a series of beam plus rafter pairs as shown in Figure D.11, which is the same concept as in D.2.3.

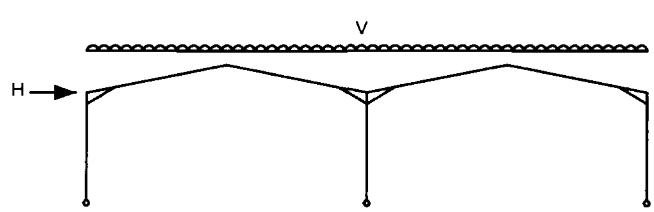


Figure D.9 *Typical multi-span frame*

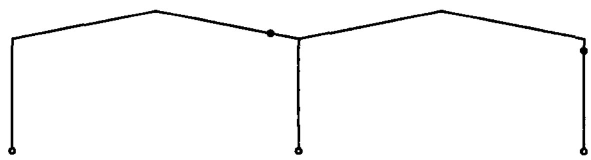


Figure D.10 *Multi-span frame with first hinge in each span*

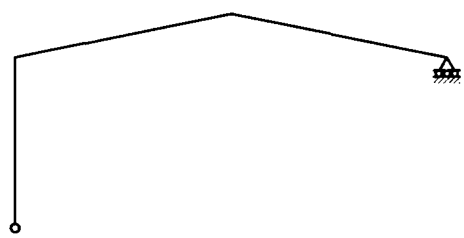


Figure D.11 *Idealised span*

The horizontal load applied to the 'plastic' frame is the difference between the load applied to the fully elastic model of the frame and the load applied at plastic collapse, i.e. at load factor λ_p. The elastic deflections will include the effects of λ_1 × (ULS loads). Therefore, the deflections of the 'plastic' frame should be calculated for applied horizontal forces equal to $(\lambda_p - \lambda_1)$ × (ULS loads).

The horizontal force is applied to sub-frames comprising a column and a rafter pair as shown in Figure D.11, which is similar in concept to the sub-structures used for the analysis of the 'elastic' frames in D.2.

The difference between the 'plastic' frame and the 'elastic' frame is that only one of the sub-frames in the plastic frame includes an external column. The other external column is separated by a plastic hinge.

The deflections are calculated using the methods for the 'elastic' frame but with the different sub-frames resulting from the plastic hinges.

Therefore the column top stiffness K is given by:

$$K = \frac{H}{\delta} = \frac{1}{\left(\frac{Sh^2}{3EI_R} + \frac{h^3}{3EI_c} \right)}$$

Where the frame is multi-span, the total column top stiffness is the sum of the stiffnesses of all the sub-frames, ΣK.

The first-order sway deflection, δX_{1s}, is given by:

$$\delta X_{1s} = \frac{H}{\Sigma K}$$

Tied portals are best modelled by taking S equal to the length of the rafter from the apex to the eaves/valley. This is because the trusss behaviour of the rafter-tie system provides some positional restraint to the apex in the sway mode of the frame.

D.3.4 First-order rafter spread and apex drop

In addition to the column top deflection, the span will spread because of the sagging deflection in the rafter coupled with the angle at the apex in the same way as in D.2.4.

D.3.5 Method for second-order notional sway deflection

The method of calculating the second-order notional sway deflections, δ_{np}, used to calculate the critical buckling ratio for the sway mode, λ_{crp} , for the frame when the stiffness of the frame has been reduced by the formation of plastic hinges is similar to the method in D.2.5.

The method accounts for $P\delta$ effects by using the effective inertias, I_{eff} , of the members as calculated in A.2.3. These are used in the method described in D.2.3 in place of the gross inertias, I. The effect of base stiffness may be added as explained in D.4.

The applied horizontal load, H, is the notional horizontal force, which must be taken as the full 0.5% of the Ultimate Limit State (ULS) loads, see Section 1.6, for the reasons given in D.2.5.

The total column top deflection δ is given by:

$$\delta = \delta_R + \delta_c = H\left(\frac{Sh^2}{3EI_{eff.R}} + \frac{h^3}{3EI_{eff.c}}\right)$$

Therefore the second-order column top stiffness K_2 is given by:

$$K_2 = \frac{H}{\delta} = \frac{1}{\left(\frac{Sh^2}{3EI_{eff.R}} + \frac{h^3}{3EI_{eff.c}}\right)}$$

In the frame shown above, which has been idealised as more than one sub-frame, the total column top stiffness of the whole frame is the sum of the stiffnesses of the two sub-frames, ΣK_2 .

The second-order notional sway deflection, δ_{np} , is given by:

$$\delta_{np} = \frac{H}{\Sigma K_2 + \Sigma K_b}$$

where, K_b is as calculated in D.4.

The critical buckling ratio for the first mode of frame buckling (which is the sway mode) for the 'plastic' frame is given by:

$$\lambda_{crp} = h/200\delta_{np}$$

Tied portals are best modelled by taking S equal to the length of the rafter from the apex to the eaves/valley for the reason given in D.3.3

D.4 Nominally pinned bases

BS 5950-1, Clause 5.1.3.3 says that the stiffness of the nominally pinned base may be assumed to have a rotational stiffness equal to 10% of the column stiffness, which may be used when checking frame stability, even where the strength calculations assume no moment is applied to the foundation. This stiffness is very useful in portal design, particularly for more flexible frames such as multi-bay portals and tied portals. This stiffness is used for calculating δ_n, which is used to find λ_{cr} for the frame stability. The base stiffness of all the columns with base fixity may be added to the sway stiffness of the frame.

An individual column loaded by a horizontal force H at the top of the column is shown in Figure D.12.

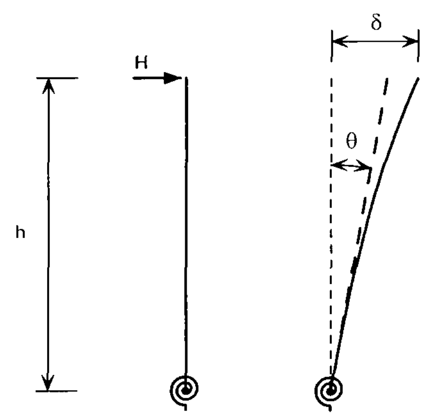


Figure D.12 *Sway stiffness from base fixity*

Base stiffness, $K = 0.4\dfrac{EI}{h}$

Base moment, $M = Hh$

$$\therefore \text{ Base rotation, } \theta = \frac{M}{K} = \frac{Hh}{\left(\dfrac{0.4EI}{h}\right)} = \frac{5Hh^2}{2EI}$$

$$\therefore \text{ Deflection of column top due to } \theta = \frac{5Hh^3}{2EI}$$

$$\text{Deflection of column top due to flexure} = \frac{Hh^3}{3EI_{\text{eff}}}$$

$$\therefore \text{ Total column top deflection, } \delta = \frac{5Hh^3}{2EI} + \frac{Hh^3}{3EI_{\text{eff}}}$$

$$\therefore \text{ Sway stiffness due to base stiffness, } K_b = \frac{H}{\delta} = \frac{1}{\left(\dfrac{5h^3}{2EI} + \dfrac{h^3}{3EI_{\text{eff}}}\right)}$$

This stiffness is additive to the frame stiffness for calculating δ_{n2} and δ_{np} in D.2.5 and D.3.5.

APPENDIX E Hinge deflections by interpolation

E.1 Vertical deflections

This appendix gives an approximate method that may be used in the energy method of second-order analysis (see Section 5.6 and Appendices A and B) to calculate the deflections of plastic hinges in rafters that do not occur at the points for which deflection output is available. For example, software commonly gives the deflections at the apex of a span. The hand calculation methods in Appendix A and Appendix D give deflections at mid-span. Where there are two hinges in any one span, it is safe to assume that the deflections of one hinge are the deflections of the apex (or mid-span) and that the deflections of the other hinge are the deflections of the nearest eaves (or valley). This assumption does not normally affect the economy of the method significantly. However, where there is only one hinge in the span, the deflections of the hinge should be taken as the deflections of the apex (or mid-span), unless they are calculated more accurately. Where the hinge is not far from the eaves (or valley), more accurate values of deflection may improve the economy significantly. For these mechanisms, the deflection can be found by interpolation. This may be done by assuming the deflection at the hinge is related to the known deflection according to the deflected form of a simply supported beam.

The deflection, y, of a simply supported beam of span L supporting a uniformly distributed load, w, is derived from:

$$EIy = \left(\frac{w}{24}\right)x^4 - \left(\frac{wL}{12}\right)x^3 + \left(\frac{wL^3}{24}\right)x$$

$$= \left[16\left(\frac{x}{L}\right)^4 - 32\left(\frac{x}{L}\right)^3 + 16\left(\frac{x}{L}\right)\right]\left(\frac{wL^4}{384}\right)$$

Therefore, the ratio of the deflection at point αL to the deflection at point βL (see Figure E.1), is given by:

$$\frac{y_\alpha}{y_\beta} = \frac{\alpha^4 - 2\alpha^3 + \alpha}{\beta^4 - 2\beta^3 + \beta}$$

where βL is the mid-span, this reduces to:

$$\frac{y_\alpha}{y_\beta} = 3.2\left(\alpha^4 - 2\alpha^3 + \alpha\right)$$

Taking β for the point in the span for which the deflections are known and α for the point at which the hinge occurs, the deflection at the hinge can be calculated.

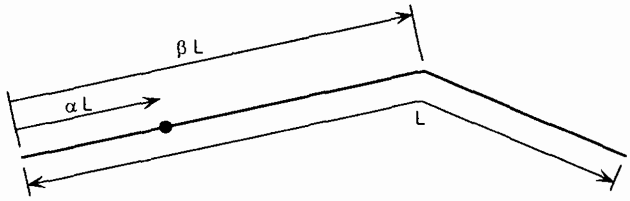


Figure E.1 *Distances αL and βL for interpolation*

E.2 Horizontal deflection

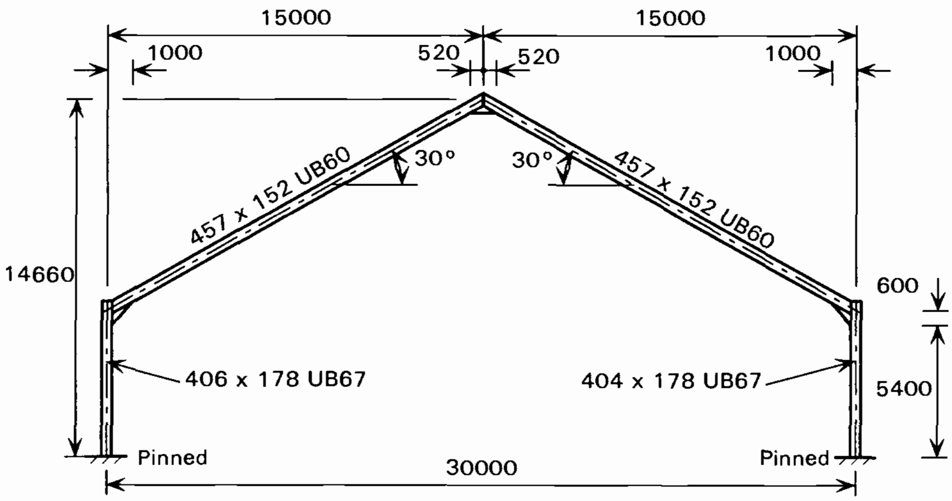
The horizontal deflection may be calculated by linear interpolation between the deflection at the apex (or mid-span) and the deflection of the nearest columns (or valley) on the other side of the hinge.

WORKED EXAMPLES

INPUT FROM FIRST-ORDER ANALYSIS

1.1 General Arrangement

The calculations have been carried out using by spreadsheet software. The numerical values presented below are the values from the spreadsheet rounded to a suitable number of significant figures.



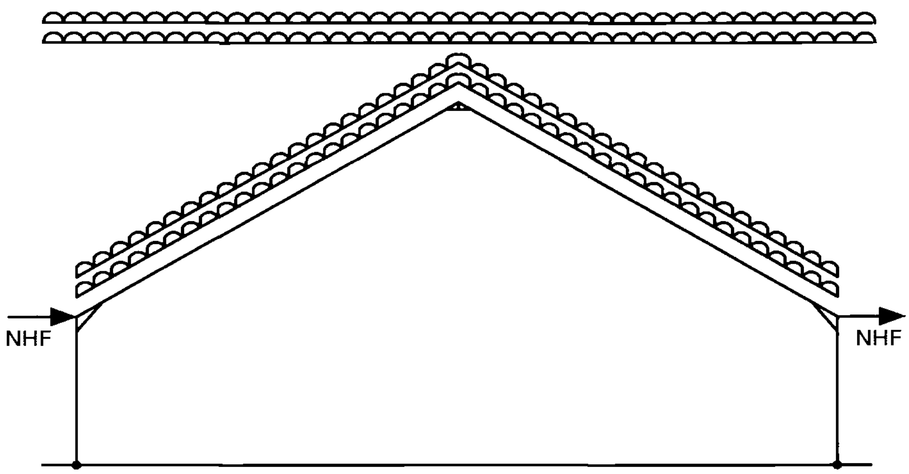
Angle of rafters: $\alpha_1 = \alpha_2 = 30°$

Span = 30 m

Developed length of rafter $= \dfrac{30}{\cos 30°} = 34.64$ m

Height of column from base to Neutral Axis of Rafter = 6.0 m

1.2 Loading



Frame spacing = 6 m

Dead	$= 0.100 \times 6.000 \times 1.4$	$= 0.840$	along slope
Service	$= 0.150 \times 6.000 \times 1.4$	$= 1.260$	on plan
Imposed	$= 0.600 \times 6.000 \times 1.6$	$= 5.760$	on plan
Self Weight	$= 80 \times 10^{-2} \times 1.000 \times 1.4$	$= 1.120$	along slope

ULS loads on plan $= 1.26 + 5.76 = 7.02$ kN/m

ULS loads on slope $= 0.84 + 1.12 = 1.96$ kN/m

ULS load transverse to slope $= 7.02 \mathrm{Cos}^2 30° + 1.96 \mathrm{Cos}\, 30° = 6.96$ kN/m

Taking notional horizontal forces (NHF) as 0.5% of the column base reactions — Clause 2.4.2.4

Total vertical load $= 30(7.02 = 1.96/\mathrm{Cos}\, 30) = 279$ kN

Required NHF $= 0.005 \times 279 = 1.39$ kN

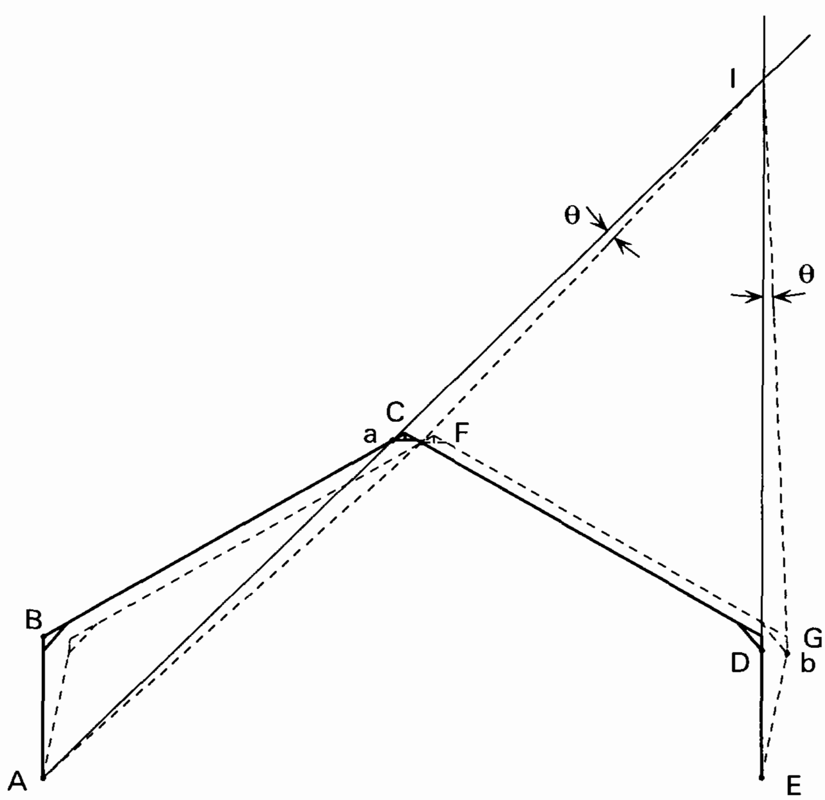
1.3 Hinge Incremental Rotations

The values of incremental rotation of the hinges are taken from the first-order collapse mechanism. These are the incremental rotations as used to calculate the collapse factor of the frame using the classic Rigid-Plastic (Virtual Work) method.

The second-order analysis uses the relative magnitude of the instantaneous rotations, so the absolute magnitude of each rotation does not affect the calculations.

Where the analysis has been performed by methods other than the Rigid-Plastic method (e.g. by the Semi-Graphical method), the incremental rotations can be deduced from the geometry of the frame and the position of the hinges. It is not necessary to repeat the calculation of the collapse factor by the Rigid-Plastic method.

Failure Mechanism



Find Node Locations

Pt A → (0, 0) = (0, 0)

Pt B → (0, 6) = (0, 6)

Pt C → (15, 6 + 15 tan 30) = (15, 14.660)

Pt D → (30, 6) = (30, 6)

Pt E → (30, 0) = (30, 0)

Pt a → (12.698 Cos 30, 6 + 12.698 sin 30) = (10.997, 12.349)

Pt b → (30, 5.400) = (30, 5.400)

Find Centre of Rotation I

$$Y_I = 30 \times \frac{Y_a - Y_A}{X_a - X_A} = 30 \times \frac{12.349-0.0}{10.997-0.0} = 33.689 \text{ m}$$

$$X_I = 30$$

Pt I → (30, 33.689)

Hinge Rotations

Taking the instantaneous rotation at A = θ

$$\text{Rotation at I, } \theta_I = \theta \times \frac{12.345}{33.689-12.345} = 0.579\,\theta$$

$$\text{Rotation at E, } \theta_E = \theta_I \times \frac{33.689 - 5.400}{5.400} = 0.579\,\theta \times 5.239 = 3.032\,\theta$$

1.4 Axial forces at ULS from first-order analysis

LH column:	at base	= 142 kN,	at haunch	= 136 kN
LH rafter:	at column	= 126 kN,	at apex	= 58 kN
RH rafter:	at column	= 126 kN,	at apex	= 59 kN
LH column:	at base	= 142 kN,	at haunch	= 136 kN

1.5 Reduced Plastic Moments at ULS from first-order analysis

Use the reduced moment capacity for the sections to account for the co-existent axial force, calculated in accordance with BS5950-1:2000 Annex I.2. These may be taken from section tables.

$M_{rx} = p_y S_{rx}$

For the axial forces in this frame under this load case

M_{pr} rafter = 403 kNm

M_{pc} column = 452 kNm

1.6 Load factor at formation of the first hinge, λ_1

From the frame analysis output, the load factor at the formation of the first hinge is:

$\lambda_1 = 1.232$

1.7 Plastic collapse factor, λ_p

From the frame analysis, the plastic collapse factor calculated by first-order analysis is:

$\lambda_p = 1.503$

1.8 Member inertias, I_x

LH column: 457×191×74 UB : $I_x = 33320$ cm^4

LH rafter: 457×191×67 UB: $I_x = 29380$ cm^4

RH rafter: as LH rafter

RH column: as LH column

1.9 Deflections of frame at λ_1 (formation of the first hinge)

$\delta_{xB} = -79.4$ mm $\delta_{yB} = 0.5$ mm

$\delta_{xC} = 3.4$ mm $\delta_{yC} = 146.1$ mm

$\delta_{xD} = 86.3$ mm $\delta_{yD} = 0.5$ mm

2. SECOND-ORDER ANALYSIS

2.1 Axial force in members

The axial force in the members is calculated from the first-order analysis output. The values are taken as the mean of the axial force at the ends of the member where there is no major step in the axial force. Where there is a major step in the axial force, e.g. at the underside of a crane bracket on a column, the value should be taken as the mean of the ends of the most highly loaded segment.

LH col:	take mid-height P_{ULS}	= (142 + 136)/2	= 139 kN
LH rafter:	take mid-length P_{ULS}	= (126 + 58)/2	= 92 kN
RH rafter:	take mid-length P_{ULS}	= (126 + 59)/2	= 93 kN
RH col:	take mid-height P_{ULS}	= (142 + 136)/2	= 139 kN

2.2 Bending deflections of the "elastic" frame — A.3

2.2.1 Stiffness reduction factors allowing for P.δ effects — A.3.2

LH column:

$I_x = 33320$ cm^4, $h = 6000$ mm, $\alpha = 2.0$ for truly pinned bases,

$P_{cr} = \pi^2 EI/(\alpha h)^2 = \pi^2 \times 205000 \times 33320 \times 10^4/(2.0 \times 6000)^2 = 4682$ kN

$P_{ULS} = 139$ kN

Stiffness reduction factor $(1 - P_{ULS}/P_{cr}) = 1 - 139/4682 = 0.970$

RH column:

P_{cr} = as LH column

P_{ULS} = 139 kN

Stiffness reduction factor $(1-P_{ULS}/P_{cr}) = 1 - 139/4682 = 0.970$

LH rafter:

$I_x = 29380$ cm^4, $L = 34640$ mm, $\alpha = 0.5$ for single span "elastic" frame

$P_{cr} = \pi^2 EI/(\alpha L)^2 = \pi^2 \times 205000 \times 29380 \times 10^4/(0.5 \times 34640)^2 = 1982$ kN

P_{ULS} = 92 kN

Stiffness reduction factor $(1-P_{ULS}/P_{cr}) = 1 - 92/1982 = 0.953$

RH rafter:

P_{cr} = as LH rafter

P_{ULS} = 93 kN

Stiffness reduction factor $(1-P_{ULS}/P_{cr}) = 1 - 93/1982 = 0.953$

2.2.2 Second-order magnification factors

A.3.3

Sway mode magnification factor $\lambda_{cr1}/(\lambda_{cr1} - 1)$

The notional sway deflection is calculated from the sum of the stiffnesses K for each of the column and rafter pairs:

Sway stiffness from column and rafter stiffness:

D.2.5

$$K_2 = \frac{1}{\left(\dfrac{S\,h^2}{3EI_{eff.R}} + \dfrac{h^3}{3EI_{eff.c}}\right)}$$

This frame is taken as truly pinned, so there is no contribution to sway stiffness from base stiffness.

LH rafter and column:

Rafter : $I_{eff.R} = I_x(1-P_{ULS}/P_{cr}) = 29380 \times 0.953 = 28012$ cm^4

Column : $I_{eff.c} = I_x(1 - P_{ULS}/P_{cr}) = 33320 \times 0.970 = 32334\ cm^4$

$$K_2 = \frac{1}{\left(\dfrac{17321\ (6000)^2}{3 \times 205000 \times 28012 \times 10^4} + \dfrac{(6000)^3}{3 \times 205000 \times 32334 \times 10^4}\right)}$$

$= 1/(0.00362 + 0.00109) = 1/0.00471 = 212.5$ N/mm

RH rafter and column:

Rafter : $I_{eff.R} = I_x(1 - P_{ULS}/P_{cr}) = 29380 \times 0.953 = 28008\ cm^4$

Column : $I_{eff.c} = I_x(1 - P_{ULS}/P_{cr}) = 33320 \times 0.970 = 32329\ cm^4$

$$K_2 = \frac{1}{\left(\dfrac{17321\ (6000)^2}{3 \times 205000 \times 28008 \times 10^4} + \dfrac{(6000)^3}{3 \times 205000 \times 32329 \times 10^4}\right)}$$

$= 1/(0.00362 + 0.00109) = 1/0.00471 = 212.5$ N/mm

Total frame:

$\Sigma K = 212.5 + 212.5 = 425$ N/mm

$H = 0.005 \times$ [Sum of the column reactions]

$= 0.005 \times (278) = 1.39$ kN

$\delta_{n2} = \Sigma H / \Sigma K = 1390/425 = 3.27$ mm

$\lambda_{cr1} = h/200\delta_{n2} = 6000/(200 \times 3.27) = 9.2$

Sway mode magnification factor $\lambda_{cr1}/(\lambda_{cr1} - 1) = 9.2/(9.2 - 1) = 1.122$

Symmetrical mode magnification

The symmetrical mode magnification is taken as the magnification arising from using the effective inertia $I_{eff} = I(1 - P_{ULS}/P_{cr})$ of the members.

Sway mode magnification factor $\lambda_{cr2}/(\lambda_{cr2} - 1) = 1/[\text{minimum } (1 - P/P_{cr})]$

$= 1/0.953 = 1.049$

2.2.3 Deflection calculations

A.3.4

Sway deflections

D.2.3

The first-order sway deflection δX_{1s} is calculated from the sum of the stiffnesses K for each of the column and rafter pairs. (The base stiffness of nominally pinned bases is not included because this is not a stability calculation like the calculation of λ_{cr})

$$K = \frac{1}{\left(\frac{S\,h^2}{3EI_R} + \frac{h^3}{3EI_c}\right)}$$

LH rafter and column:

Rafter : I_R = 29380 cm^4

Column : I_c = 33320 cm^4

$$K = \frac{1}{\left(\frac{17321\ (6000)^2}{3 \times 205000 \times 29380 \times 10^4} + \frac{(6000)^3}{3 \times 205000 \times 33320 \times 10^4}\right)}$$

$= 1/(0.00345 + 0.00105) \quad = 1/0.00451 \quad = 222.0$ N/mm

RH rafter and column:

As LH rafter and column

Total frame:

First-order sway deflections:

$\Sigma K \quad = 222 + 222 \quad = 444$ N/mm

$H \quad = \lambda_1 \times H_{ULS} \quad = 1.232 \times 1.39 \quad = 1.71$ kN

$\Delta \quad = \Sigma H / \Sigma K \quad = 1710/444 \quad = 3.85$ mm

$\delta X_{1s} \quad = \Delta \quad = 3.85$ mm

Second-order deflections

$\delta X_2 = (\delta X_1 - \delta X_{1s})\{\lambda_{cr2}/(\lambda_{cr2} - 1)\} + \delta X_{1s}\{\lambda_{cr1}/(\lambda_{cr1} - 1)\}$ A.3.4

$\delta Y_2 = \delta Y_1\{\lambda_{cr2}/(\lambda_{cr2} - 1)\}$

$\delta X_B = (-79.4 - 3.85)\{1.049\} + 3.85\{1.122\} = -83.0$ mm

$\delta Y_B = 0.5\{1.049\} = 0.5$ mm

$\delta X_C = (3.4 - 3.85)\{1.049\} + 3.85\{1.122\} = 3.8$ mm

$\delta Y_C = 146.1\{1.049\} = 153.2$ mm

$\delta X_D = (86.3 - 3.85)\{1.049\} + 3.85\{1.122\} = 90.8$ mm

$\delta Y_D = 0.5\{1.049\} = 0.5$ mm

The hinges at "a" and "b" are so close to points C and E that the deflections at the hinges can be assumed to be at points C and E. The effect on the Energy summation is insignificant.

2.3 Bending deflections of the "plastic" frame

A.4

2.3.1 Stiffness reduction factors allowing for P.δ effects

A.4.2

LH column:

As "elastic" frame, stiffness reduction factor $(1-P_{ULS}/P_{cr}) = 0.970$

RH column:

As "elastic" frame, stiffness reduction factor $(1-P_{ULS}/P_{cr}) = 0.970$

LH rafter:

$I_x = 29380$ cm^4, $L = 34640$ mm, $\alpha = 1.0$ for "plastic" frame

$P_{cr} = \pi^2 EI/(\alpha L)^2 = \pi^2 \times 205000 \times 29380 \times 10^4/(1.0 \times 34640)^2 = 495$ kN

$P_{ULS} = 92$ kN

Stiffness reduction factor $(1-P_{ULS}/P_{cr}) = 1 - 92/495 = 0.814$

RH rafter:

P_{cr} = as LH rafter

$P_{ULS} = 93$ kN

Stiffness reduction factor $(1 - P_{ULS}/P_{cr}) = 1 - 93/495 = 0.813$

2.3.2 Second-order magnification factors A.4.3

Sway mode magnification factor $\lambda_{crp}/(\lambda_{crp} - 1)$

The notional sway deflection is calculated from the stiffness K_2 of the LH column and rafter pair (the RH column is hinged at the top and pinned at the bottom, reducing the stiffness to zero):

Sway stiffness from column and rafter stiffness: D.2.5

$$K_2 = \frac{1}{\left(\dfrac{S\,h^2}{3EI_{eff.R}} + \dfrac{h^3}{3EI_{eff.c}}\right)}$$

This frame is taken as truly pinned, so there is no contribution to the sway stiffness from base stiffness.

LH column and rafters:

Rafters : $I_{eff.R} = I_x(1 - P_{ULS}/P_{cr}) = 29380 \times 0.813 = 23894$ cm^4

Column : $I_{eff.c} = I_x(1 - P_{ULS}/P_{cr}) = 33320 \times 0.970 = 32334$ cm^4

$$K_2 = \frac{1}{\left(\dfrac{34641\ (6000)^2}{3 \times 205000 \times 23894 \times 10^4} + \dfrac{(6000)^3}{3 \times 205000 \times 32334 \times 10^4}\right)}$$

$= 1/(0.00849 + 0.00109) = 1/0.00957 = 105$ N/mm

Total frame:

$\Sigma K = 105$ N/mm

$H = 0.005 \times$ [Sum of the column reactions]

$= 0.005 \times (279) = 1.39$ kN

$\Delta = \Sigma H/\Sigma K = 1390/105 = 13.3$ mm

$\lambda_{crp} = h/200\delta_{np} = 6000/(200 \times 13.3) = 2.26$

Sway mode magnification factor $\lambda_{crp}/(\lambda_{crp} - 1) = 2.26/(2.26 - 1) = 1.80$

2.3.3 Deflections from vertical loads

A.4.5

The loads applied to the "plastic" frame $= (\lambda_p - \lambda_1)$(loads at ULS)

$(\lambda_p - \lambda_1) = (1.503 - 1.232) = 0.271$

$wP = 0.271(7.02\text{Cos}^2 30° + 1.96\text{Cos}30°) = 1.89$ kN/m

Sway

This arises due to the rotation of the column without an adjacent hinge caused by the end rotation of the rafter spanning from column to column.

Second-order end slope of the rafter, $\theta_{R2} = \dfrac{w_P S^3}{24\, EI_R} \dfrac{\lambda_{crp}}{\lambda_{crp} - 1}$

$$\theta_{R2} = \frac{1.89 \times 34640^3}{24 \times 205000 \times 29380 \times 10^4} \times 1.80 = 0.0975 \text{ radians}$$

Horizontal deflection of B, C, D $= h\theta_R$

$= 6000 \times 0.0975 = 585$ mm

Mid-span drop

Mid-span deflection of the rafter, $\delta_{B2} = \dfrac{5}{384} \dfrac{w_P S^4}{E\, I_R} \dfrac{\lambda_{crp}}{\lambda_{crp} - 1}$

$$\delta_{B2} = \frac{5}{384} \frac{1.89 \times 34640^4}{205000 \times 29380 \times 10^4} \times 1.80 = 1055 \text{ mm}$$

$\delta_{apex} = \delta_{B2} / \text{Cos}\alpha = 1055/\text{Cos } 30° = 1218$ mm

Spread

This is caused by the drop of the angle in the rafter which is at the apex in this frame. Because this frame is a symmetrical pitched roof portal, the drop of the angle is the mid-span drop calculated above.

$\delta_{spread,D} \quad = \delta_{B2}\,(\mathrm{Sin}\alpha_1 + \mathrm{Sin}\alpha_2) \;=\; \delta_B\,(2\mathrm{Sin}\;30°)$

$= 1055(2 \times 0.500) \;=\; 1055$ mm

$\delta_{spread,C} \quad = \delta_{B2}\;\mathrm{Sin}\;\alpha_1 \;=\; 1055 \times 0.500 \;=\; 528$ mm

Column hinge horizontal displacement

The hinge occurs at the underside of the haunch, which is at a distance from the neutral axis of the rafter, causing an additional horizontal displacement.

Second-order end slope of the rafter, θ_{R2} = 0.0975 radians

Off-set of the hinge below the rafter, e = 6000 - 5400 = 600 mm

Horizontal displacement of the hinge at G = $e\theta_R$ = 600 × 0.0975

= 59 mm

2.3.4 Deflections from horizontal loads A.4.6

The loads applied to the "plastic" frame = $(\lambda_p - \lambda_1)$(loads at ULS)

$(\lambda_p - \lambda_1) \;=\; (1.503 - 1.232) \;=\; 0.271$

The ULS horizontal load in this load case is the notional horizontal force

$H \;=(\lambda_p - \lambda_1) \times \mathrm{NHF} \;=\; 0.271 \times 1.39 \;=\; 0.38$ kN

The sway deflection is calculated from the sum of the stiffnesses K of the LH column and rafter pair (the RH column is hinged at the top and pinned at the bottom, reducing the stiffness to zero).

$$K_s = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)}\;\frac{1}{\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}}$$

The first-order stiffness of the LH rafter and column pair is the same as for the "elastic" frame:

$$\frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} = 125.7$$

$K_s \;=\; 125.7 \times (1/1.80) \;=\; 70.0$ N/mm

Sway deflection = 380/70 = 5.4 mm

Mid-span drop

The sway deflection induces an additional deflection at the apex

$$\delta_{sm2} = \frac{M\,L_r^2}{16\,EI_R}\;\frac{\lambda_{crp}}{\lambda_{crp}-1}$$

where $M = \Sigma H_i\,h_i = 380 \times 6000 = 2.26$ kNm

$$\therefore \quad \delta_{sm2} = \frac{2.26 \times 10^6 \times 34641^2}{16 \times 205000 \times 29380 \times 10^4} \times 1.80 = 5.1 \text{ mm}$$

Spread

This is caused by the drop of the angle in the rafter which is at the apex in this frame. Because this frame is a symmetrical pitched roof portal, the drop of the angle is the mid-span drop calculated above.

$\delta_{spread,D} = \delta_{sm2}\,(\mathrm{Sin}\alpha_1 + \mathrm{Sin}\alpha_2) = \delta_B\,(2\mathrm{Sin}\,30°)$

$= 5.1(2 \times 0.500) = 5.1$ mm

$\delta_{spread,C} = \delta_{sm2}\,\mathrm{Sin}\,\alpha_1 = 5.1 \times 0.500 = 2.5$ mm

2.4 Axial forces for the energy calculation A.5

The total of the axial loads in the columns is not affected by second-order effects, so $P_2 = P_1$ which is taken as the mid-height value calculated in 2.1 above.

LH column: $P_2 = 139$ kN

RH column: $P_2 = 139$ kN

The rafter axial forces are affected by the drop of the rafters at mid-span. Calculate P_{1a} and P_{1b} from the values in Section 2.2 of the worked example.

Mid-span drop = drop from "elastic" + drop from "plastic"

= from 2.2.3 + from 2.3.3 + from 2.3.4

= 153.2 + 1218 + 5.1 = 1376 mm

Increase in P_2 = $1/\{[1-(\delta_a/h_a)] - 1\}$ = $1/\{[1-(1376/14660)] - 1\}$ = 0.104

LH rafter:

Mid-span axial = 58.3, giving $P\Delta$ increase = 0.104×58.3 = 6.0 kN

P_2 = 92.3 + 6.0 = 98.3 kN

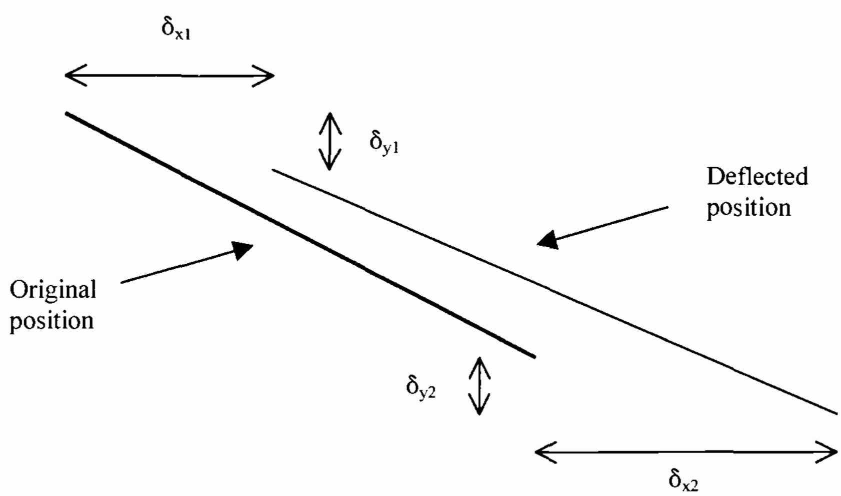
RH rafter:

Mid-span axial = 58.6, giving $P\Delta$ increase = 0.104×58.6 = 6.1 kN

P_2 = 92.5 + 6.1 = 98.6 kN

2.5 Second-order Energy Summation

A.2.2



The energy summation is required to calculate λ_M following the methods in A.2.2

Element for evaluation of Pc*phi*s*d(phi)	AB	BC	CD	bD	Eb		
X-AXIS DEFLECTIONS							
Deflections from the "elastic" frame							
dxa	0.0	-83.0	3.8	0.0	90.8		
dxb	-83.0	3.8	90.8	90.8	90.8		
(dxb - dxa)	**-83.0**	**86.8**	**87.0**	**90.8**	**0.0**		
Deflections from the "plastic" frame							
From gravity loads							
Sway of top of elastic column							
dxa	0.0	584.7	584.7	584.7	0.0		
dxb	584.7	584.7	584.7	584.7	584.7		
(dxb - dxa)	**584.7**	**0.0**	**0.0**	**0.0**	**584.7**		
Spread							
dxa	0.0	0.0	527.5	1055.0	0.0		
dxb	0.0	527.5	1055.0	1055.0	1055.0		
(dxb - dxa)	**0.0**	**527.5**	**527.5**	**0.0**	**1055.0**		
Column hinge horizontal displacement							
dxa	0.0	0.0	0.0	58.5	0.0		
dxb	0.0	0.0	0.0	0.0	58.5		
(dxb - dxa)	**0.0**	**0.0**	**0.0**	**-58.5**	**58.5**		
From horizontal loads							
Sway							
dxa	0.0	5.4	5.4	5.4	0.0		
dxb	5.4	5.4	5.4	5.4	5.4		
(dxb - dxa)	**5.4**	**0.0**	**0.0**	**0.0**	**5.4**		
Spread							
dxa	0.0	0.0	2.5	5.1	0.0		
dxb	0.0	2.5	5.1	5.1	5.1		
(dxb - dxa)	**0.0**	**2.5**	**2.5**	**0.0**	**5.1**		
Totals of (dxb-dxa) at collapse	**507.1**	**616.9**	**617.0**	**32.3**	**1708.6**		
Y-AXIS DEFLECTIONS							
Deflections from the "elastic" frame							
dya	0.0	0.5	153.2	0.5	0.0		
dyb	0.5	153.2	0.5	0.5	0.5		
(dyb - dya)	**0.5**	**152.7**	**-152.7**	**0.0**	**0.5**		
Deflections from the "plastic" frame							
Mid-span drop from gravity loads							
dya	0.0	0.0	1218.2	0.0	0.0		
dyb	0.0	1218.2	0.0	0.0	0.0		
(dyb - dya)	**0.0**	**1218.2**	**-1218.2**	**0.0**	**0.0**		
Deflections from the "plastic" frame							
Mid-span drop from horizontal loads							
dya	0.0	0.0	5.1	0.0	0.0		
dyb	0.0	5.1	0.0	0.0	0.0		
(dyb - dya)	**0.0**	**5.1**	**-5.1**	**0.0**	**0.0**		
Total of (dyb - dya) at collapse	**0.5**	**1375.9**	**-1375.9**	**0.0**	**0.5**		
psi (angle from X axis)	90.0	30.0	-30.0	90.0	90.0		
[(dxb - dxa) at collapse]*Sin(psi)	507.1	308.4	-308.5	32.3	1708.6		
[(dyb - dya) at collapse]*Cos(psi)	0.0	1191.6	-1191.6	0.0	0.0		
phi * s at collapse	507.1	1500.0	-1500.1	32.3	1708.6		
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.579	0.579	3.032		
Shortening = phi*s*d(phi) (modulus)	**507.1**	**1500.0**	**868.1**	**18.7**	**5179.7**		
AXIAL FORCES							
Pc for columns and rafters at ULS	138.6	92.3	92.5	139.2	139.2		
Total midspan drop		1376.5	1376.5				
Midspan height		14660	14660				
Increase rafter midspan axial by {1/(1-drop/height) - 1}		0.104	0.104				
Midspan axial		58.3	58.6				
Increase in rafter axial		6.0	6.1				
Design axial	138.6	98.3	98.6	139.2	139.2		
Incremental energy = Pc*phi*s*d(phi)	**70.3**	**147.4**	**85.6**	**2.6**	**721.0**	**Sum =**	**1027**
WORK DONE IN ROTATING HINGES							
Element for evaluating Mprd(phi)	AB	Ba	aD	bD	Eb		
MprA	0.0	0.0	402.5	452.1	0.0		
MprB	0.0	402.5	0.0	0.0	452.1		
MprA + MprB	0.0	402.5	402.5	452.1	452.1		
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.579	0.579	3.032		
Mpr*d(phi)	**0.0**	**402.5**	**232.9**	**261.6**	**1370.6**	**Sum =**	**2268**

Factor on lambda_p	0.547
lambda_p from first-order analysis	1.503
lambda_M	**0.822**

2.6 Load factor at failure, λ_M

$\Sigma P_2 \phi s d\phi = 1027 \phi$

$\Sigma M_{pr} = 2268 \phi$

$$\lambda_M = \lambda_p \left[1 - \left(\frac{P_2 \phi s d\phi}{\Sigma M_{pr} d\phi} \right) \right]$$

$$\lambda_M = 1.503 \left[1 - \left(\frac{1027 \phi}{2268 \phi} \right) \right] = 1.503 \times 0.547 = 0.822$$

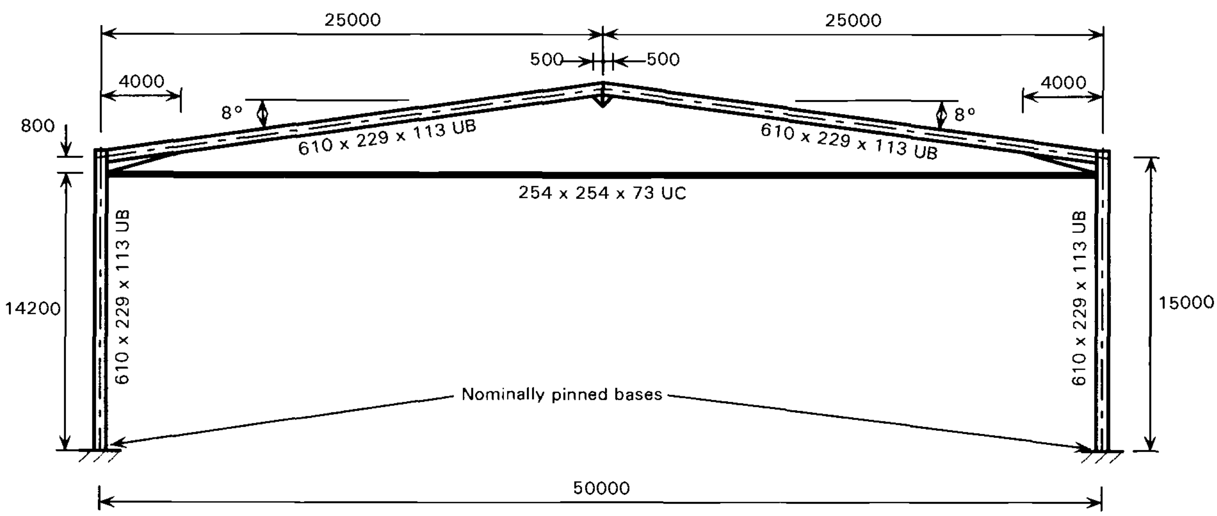
$\lambda_M < 1.0$, so the frame has failed the check for in-plane stability.

The above shows how the second-order effects have caused a major reduction in capacity of the frame due to in-plane instability effects. The "hand" method tends to be conservative, so analysis by another method might demonstrate that the reduction in capacity is not so great.

The analysis above would be less conservative if the stiffness of the haunches had been included in all the stiffness calculations. It would also be more economical if the frame were proportioned so that λ_1 were closer to λ_p.

1 INPUT FROM FIRST-ORDER ANALYSIS

1.1 General Arrangement



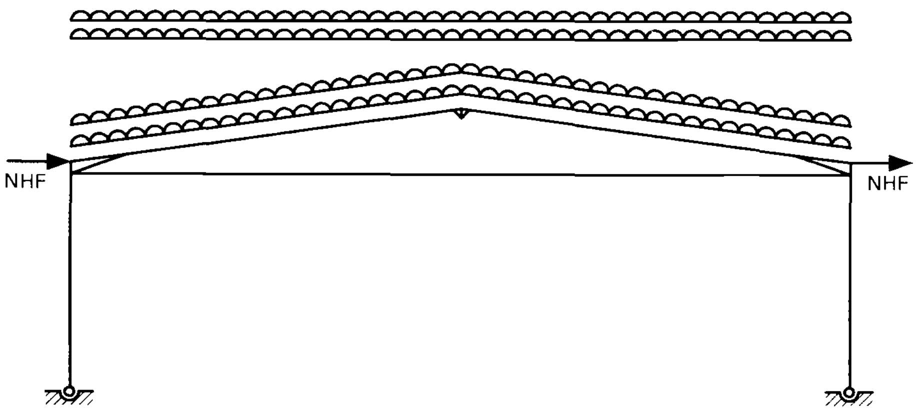
Angle of rafters: $\alpha_1 = \alpha_2 = 8°$

Span = 50 m

Developed length of rafter, apex to eaves $= \dfrac{25}{\cos 8°}$ = 25.246 m

Height of column from base to Neutral Axis of rafter = 15.0 m

1.2 Loading



Frame spacing = 8 m

Dead	=	$0.100 \times 8.000 \times 1.4$	=	1.12 kN/m along slope
Service	=	$0.150 \times 8.000 \times 1.4$	=	1.68 on plan
Imposed	=	$0.600 \times 8.000 \times 1.6$	=	7.68 on plan
Self Weight	=	$113 \times 10^{-2} \times 1.000 \times 1.4$	=	1.58 along slope

Taking Notional Horizontal Forces as 0.5% of the column base reactions — Clause 2.4.2.4

$\therefore$ Required NHF $= 0.005 \times 650$ kN $= 3.25$ kN

ULS loads on plan $= 1.68 + 7.68 = 9.36$ kN/m

ULS loads on slope $= 1.58 + 1.12 = 2.70$ kN/m

ULS load transverse to slope $= 9.36 \cos^2 8° + 2.70 \cos 8° = 11.85$ kN/m

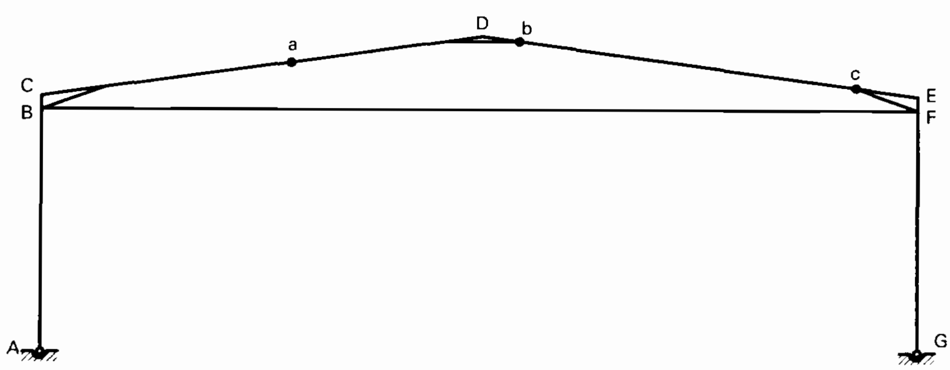
1.3 Hinge Incremental Rotations

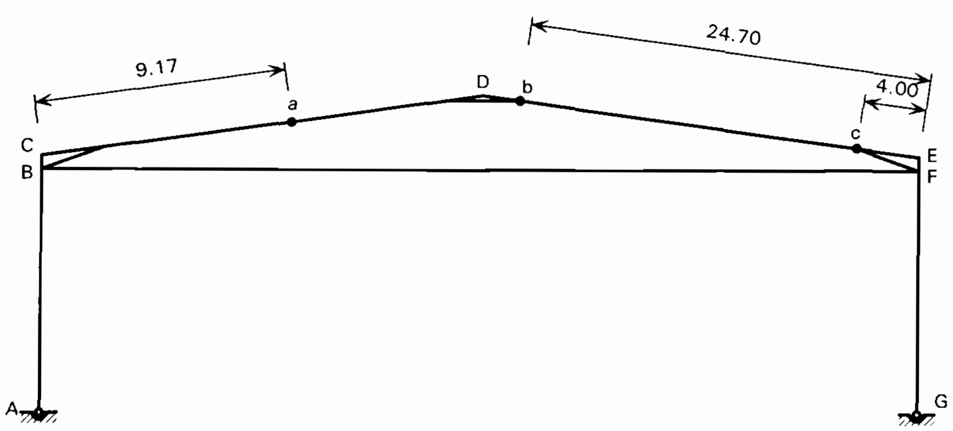
The values of incremental rotation of the hinges are taken from the first-order collapse mechanism. These are the incremental rotations as used to calculate the collapse factor of the frame using the classic Rigid-Plastic (Virtual Work) method.

The second-order analysis uses the relative magnitude of the instantaneous rotations, so the absolute magnitude of each rotation does not affect the calculations.

Where the analysis has been performed by methods other than the Rigid-Plastic method (e.g. by the Semi-Graphical method), the incremental rotations can be deduced from the geometry of the frame and the position of the hinges. It is not necessary to repeat the calculation of the collapse factor by the Rigid-Plastic method.

Failure Mechanism





Silwood Park, Ascot, Berks SL5 7QN
Telephone: (01344) 623345
Fax: (01344) 622944

Find Node Locations

Pt A: (0.0, 0.0) = (0.0, 0.0)

Pt B: (0.0, 14.2) = (0.0, 14.2)

Pt C: (0.0, 15.0) = (0.0, 15.0)

Pt D: (25.0, {15.0 + 25.0 Tan 8}) = (25.0, 18.514)

Pt E: (50.0, 15.0) = (50.0, 15.0)

Pt F: (50.0, 14.2) = (50.0, 14.2)

Pt G: (50.0, 0.0) = (50.0, 0.0)

Pt a: ({9.170 Cos 8}, {15.0 + 9.170 Sin 8}) = (9.081, 16.276)

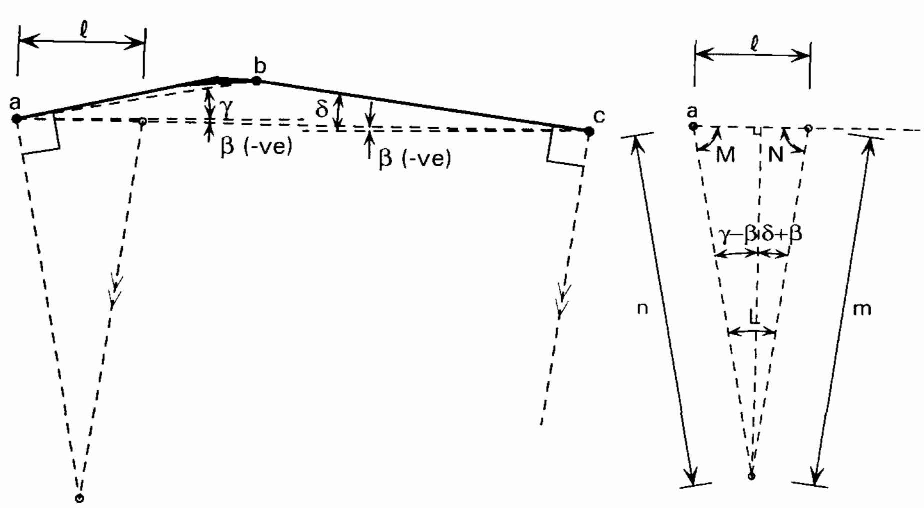
Pt b: ({50 – 24.700 Cos 8}, {15 – 24.700 Sin8}) = (25.540, 18.438)

Pt c: ({50 – 4.000 Cos 8}, {15 + 4.000 Sin 8}) = (46.039, 15.557)

Member Rotations

$\text{Rotation}_{ABC} = \theta$

$\text{Rotation}_{EFG} = \theta$



Find slope beta, β, of ac from horizontal

$\beta \quad = \text{Tan}^{-1}(Y_c - Y_a)/(X_c - X_a) = \text{Tan}^{-1}(15.557 - 16.276)/(46.039 - 9.081)$

$\quad = -1.12°$

Shortening of ac $= dX_{ac} \text{ Cos}\,\beta + dY_{ac} \text{ Sin}\,\beta$

$= [(Y_a - Y_A)\theta - (Y_c - Y_G)\theta]\text{Cos}\beta + [(X_a - X_A)\theta - (X_c - X_G)\theta] \text{ Sin}\,\beta$

$= (16.276\theta - 15.557\theta) \text{ Cos}\,\beta + (9.081\theta - (-3.961)\theta) \text{ Sin}\,\beta$

$= (0.720\theta)\ 0.9998 + (13.042\theta)0.0195 = 0.973\theta$

Find slope gamma, γ, of ab from horizontal

$\gamma \quad = \text{Tan}^{-1}(Y_b - Y_a)/(X_b - X_a) = \text{Tan}^{-1}(18.438 - 16.276)/(25.540 - 9.081)$

$\quad = 7.48°$

gamma - beta = 7.48 - (- 1.12) = 8.60°

Slope delta, δ, of cb

$\delta \quad = \text{Tan}^{-1}[(Y_b - Y_c)/(X_c - X_b)]$

$\quad = \text{Tan}^{-1}(18.438 - 15.557)/(46.039 - 25.540) = 8.00°$

Angle L = gamma + delta = 7.48 + 8.00 = 15.48°

Angle M = 90 - (gamma - beta) = 90 - 8.60 = 81.40°

Angle N = 90 - (delta + beta) = 90 - (8.0 - 1.12) = 83.12°

$\text{Sin}L/l \quad = \text{Sin}M/m \quad = \text{Sin}N/n$

$l \quad$ = shortening of ac $= 0.973\theta$

$m \quad = l(\text{Sin}M/\text{Sin}L) \quad = 0.973\theta(\text{Sin}81.40°/\text{Sin}15.48°) \quad = 3.61\theta$

$n \quad = l(\text{Sin}N/\text{Sin}L) \quad = 0.973\,\theta(\text{Sin}83.12°/\text{Sin}15.48°) \quad = 3.62\,\theta$

length ab $= [(Y_b - Y_a)^2 + (X_b - X_a)^2]^{1/2} = 16.601$

Rotation of ba $= n/(\text{length ab}) = 3.62\theta/16.601 = 0.218\theta$

length bc $= [(Y_c - Y_b)^2 + (X_c - X_b)^2]^{1/2} = 20.700$

Rotation of bc $= m/(\text{length bc}) = 3.61\theta/20.700 = 0.174\theta$

1.4 Axial forces at ULS from first-order analysis

The analysis is not sensitive to the accuracy of the axial forces. Therefore the axial forces may be calculated by elastic analysis, plastic analysis (factored down from collapse to ULS) or elastic-plastic analysis.

LH column: at base = 324 kN, at haunch = 302 kN

LH rafter: at column = 960 kN, at apex = 919 kN

RH rafter: at column = 960 kN, at apex = 919 kN

LH column: at base = 326 kN, at haunch = 302 kN

1.5 Reduced Plastic Moments at ULS from first-order analysis

Use the reduced moment capacity for the sections to account for the co-existent axial force. These may be taken from section tables.

610×229×113 UB $M_{rx} = p_y S_{rx} = 265 \times 3280 \times 10^{-3} = 869$ kNm — Annex I.2.

1.6 Load factor at formation of the first hinge, λ_1

From the frame analysis output, the load factor at the formation of the first hinge is:

$$\lambda_1 = 2.12$$

1.7 Plastic collapse factor, λ_p

From the frame analysis, the plastic collapse factor calculated by first-order analysis is:

$$\lambda_p = 2.28$$

1.8 Member inertias, I_x

LH column: $610 \times 229 \times 113$ UB: $I_x = 87300\ cm^4$

LH rafter: $610 \times 229 \times 113$ UB: $I_x = 87300\ cm^4$

RH rafter: as LH rafter

RH column: as LH column

1.9 Deflections of frame at λ_1 (formation of the first hinge)

The deflections are found from first order elastic or elastic-plastic analysis.

δ_{xB} = 22.1 mm $\quad\quad \delta_{yB}$ = 3.2 mm

δ_{xC} = 30.3 mm $\quad\quad \delta_{yC}$ = 3.3 mm

δ_{xD} = 49.1 mm $\quad\quad \delta_{yD}$ = 252.9 mm

δ_{xE} = 67.9 mm $\quad\quad \delta_{yE}$ = 3.3 mm

δ_{xF} = 73.0 mm $\quad\quad \delta_{yF}$ = 3.2 mm

2 SECOND-ORDER ANALYSIS

2.1 Axial force in members

The axial force in the members is calculated from the first-order analysis output. The values are taken as the mean of the axial force at the ends of the member where there is no major step in the axial force. Where there is a major step in the axial force, eg at the end of a tie in a tied rafter or at the underside of a crane bracket on a column, the value should be taken as the mean of the ends of the most highly loaded segment, e.g. between the ends of a tie and the apex in a tied rafter.

LH col: take mid-height $P_{ULS} = (324 + 302)/2$ = 313 kN

LH rafter: take mid-length $P_{ULS} = (960 + 919)/2$ = 940 kN

RH rafter: take mid-length P_{ULS} = (960 + 919)/2 = 940 kN

RH col: take mid-height P_{ULS} = (326 + 302)/2 = 314 kN

2.2 Bending deflections of the "elastic" frame — B.3

2.2.1 Stiffness reduction factors allowing for P.δ effects — B.3.2

LH column:

I_x = 87300 cm^4, h = 15000mm, α = 1.7 for nominally pinned bases

$P_{cr} = \pi^2 EI/(\alpha h)^2 = \pi^2 \times 205000 \times 87300 \times 10^4/(1.7 \times 15000)^2 = 2716$ kN

P_{ULS} = 313 kN

Stiffness reduction factor $(1 - P_{ULS}/P_{cr})$ = 1 - 313/2716 = 0.885

RH column:

P_{cr} = as LH column

P_{ULS} = 314 kN

Stiffness reduction factor $(1 - P_{ULS}/P_{cr})$ = 1 - 314/2716 = 0.884

LH rafter:

I_x = 87300 cm^4, L = 25246mm, α = 1.0

$P_{cr} = \pi^2 EI/(\alpha L)^2 = \pi^2 \times 205000 \times 87300 \times 10^4/(1.0 \times 25246)^2 = 2771$ kN

P_{ULS} = 940 kN

Stiffness reduction factor $(1 - P_{ULS}/P_{cr})$ = 1 - 940/2771 = 0.661

RH rafter:

P_{cr} = as LH rafter

P_{ULS} = 940 kN

Stiffness reduction factor $(1 - P_{ULS}/P_{cr})$ = 1 - 940/2771 = 0.661

2.2.2 Second-order magnification factors

B.3.3

Sway mode magnification factor $\lambda_{cr1}/(\lambda_{cr1} - 1)$

The notional sway deflection is calculated from the sum of the stiffnesses K for each of the column and rafter pairs:

Sway stiffness from column and rafter stiffness:

D.2.5

$$K_2 = \frac{1}{\left(\frac{S\,h^2}{3EI_{eff.R}} + \frac{h^3}{3EI_{eff.c}}\right)}$$

Sway stiffness from base stiffness:

D.4

$$K_b = \frac{1}{\left(\frac{5h^3}{2EI_c} + \frac{h^3}{3EI_{eff.c}}\right)}$$

LH rafter and column:

Rafter : $I_{eff.R} = I_x(1 - P_{ULS}/P_{cr}) = 87300 \times 0.661 = 57705$ cm^4

Column : $I_{eff.c} = I_x(1 - P_{ULS}/P_{cr}) = 87300 \times 0.885 = 77241$ cm^4

$$K_2 = \frac{1}{\left(\frac{25246\ (15000)^2}{3 \times 205000 \times 57705 \times 10^4} + \frac{(15000)^3}{3 \times 205000 \times 77241 \times 10^4}\right)}$$

$= 1/(0.01601 + 0.00711) \quad = 1/0.02311 \quad = 43.3$ N/mm

$$K_b = \frac{1}{\left(\frac{5\ (15000)^3}{2 \times 205000 \times 87300 \times 10^4} + \frac{(15000)^3}{3 \times 205000 \times 77241 \times 10^4}\right)}$$

$= 1/(0.04715 + 0.00711) \quad = 1/0.054 \quad = 18.4$ N/mm

RH rafter and column:

Rafter : $I_{eff.R} = I_x(1 - P_{ULS}/P_{cr}) = 87300 \times 0.661 = 57705$ cm^4

Column : $I_{eff.c} = I_x(1 - P_{ULS}/P_{cr}) = 87300 \times 0.884 = 77208$ cm^4

$$K_2 = \frac{1}{\left(\dfrac{25246\ (15000)^2}{3 \times 205000 \times 57705 \times 10^4} + \dfrac{(15000)^3}{3 \times 205000 \times 77208 \times 10^4}\right)}$$

$= 1/(0.01601 + 0.00711) \quad = 1/0.02311 \quad = 43.3$ N/mm

$$K_b = \frac{1}{\left(\dfrac{5\ (15000)^3}{2 \times 205000 \times 87300 \times 10^4} + \dfrac{(15000)^3}{3 \times 205000 \times 77241 \times 10^4}\right)}$$

$= 1/(0.04715 + 0.00711) \quad = 1/0.054 \quad = 18.4$ N/mm

Total frame:

$\Sigma K = 43.3 + 18.4 + 43.3 + 18.4 \quad = 123.4$ N/mm

$H = 0.005 \times$ [Sum of the column reactions]

$= 0.005 \times (650) \quad = 3.25$ kN

$\delta_{n2} = \Sigma H / \Sigma K \quad = 3250/123.4 \quad = 26.3$ mm

$\lambda_{cr1} = h/200\delta_{n2} \quad = 15000/(200 \times 26.3) \quad = 2.85$

Sway mode magnification factor $\lambda_{cr1}/(\lambda_{cr1} - 1) = 2.85/(2.85 - 1) = 1.54$

Symmetrical mode magnification

The symmetrical mode magnification is taken as the magnification arising from using the effective inertia $I_{eff} = I(1 - P_{ULS}/P_{cr})$ of the members.

2.2.3 Deflection calculations

B.3.4

Sway deflections

D.2.3

The first-order sway deflection δX_{1s} is calculated from the sum of the stiffnesses K for each of the column and rafter pairs. (The base stiffness of nominally pinned bases is not included because this is not a stability calculation like the calculation of λ_{cr})

$$K = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)}$$

LH rafter and column:

Rafter : I_R = 87300 cm^4

Column : I_c = 87300 cm^4

$$K = \frac{1}{\left(\dfrac{25246\ (15000)^2}{3\times205000\times87300\times10^4} + \dfrac{(15000)^3}{3\times205000\times87300\times10^4}\right)}$$

$= 1/(0.01058 + 0.00629) \quad = 1/0.01687 \quad = 59.3$ N/mm

RH rafter and column:

As LH rafter and column

Total frame:

First-order sway deflections:

$\Sigma K = 59.3 + 59.3 \quad = 118.6$ N/mm

$H = \lambda_1 \times H_{ULS} \quad = 2.12 \times 3.25 \quad = 6.89$ kN

$\Delta = \Sigma H/\Sigma K \quad = 6890/118.6 \quad = 58.1$ mm

$\delta X_{1s} = \Delta \quad = 58.1$ mm

Second-order deflections

$\delta X_2 = (\delta X_1 - \delta X_{1s}) + \delta X_{1s}\{\lambda_{cr1}/(\lambda_{cr1} - 1)\}$ B.3.4

$\delta X_B = (22.1 - 58.1) + 58.1\{1.54\} \quad = 35.1$ mm

$\delta X_C = (30.3 - 58.1) + 58.1\{1.54\} \quad = 47.5$ mm

$\delta X_D = (49.1 - 58.1) + 58.1\{1.54\} \quad = 75.9$ mm

δX_E = (67.9 - 58.1) + 58.1{1.54} = 104.4 mm

δX_F = (73.0 - 58.1) + 58.1{1.54} = 112.1 mm

The hinges at "b" and "c" are so close to points D and E that the deflections at the hinges can be assumed to be at the same points. The effect on the Energy summation is insignificant.

Deflection at point where hinge "a" occurs

In the absence of more detailed analysis, the deflection at hinge "a" may be taken as the mid-span deflection of the section of the rafter between the haunch to the apex. The bending moment at the ends of this segment of rafter may be assumed to be $wL^2/16$ hogging, which gives the midspan bending moment equal to $wL^2/16$ sagging.

At λ_1 , load transverse to slope = 2.12 × 11.85 = 25.13 kN/m

$$\text{Deflection} = \frac{2}{384}\frac{wL^4}{EI_{\text{eff}}} = \frac{2}{384}\frac{25.13(21610)^4}{205000 \times 57705 \times 10^4} = 223.8 \text{ mm}$$

Vertical deflection = 223.8 Cos 8° = 221.6 mm

Horizontal deflection = 223.8 Sin 8° = 31.1 mm

The total deflection of "a" is taken as the mean deflection of C and D plus the deflection calculated above:

δX_a = $(\delta X_C + \delta X_D)/2$ + 31.1 = (47.5 + 75.9)/2 + 31.1 = 92.9 mm

2.3 Bending deflections of the "plastic" frame B.4

2.3.1 Stiffness reduction factors to allow for P.δ effects B.4.2

LH column:

As "elastic" frame, stiffness reduction factor $(1 - P_{ULS}/P_{cr})$ = 0.885

RH column:

As "elastic" frame, stiffness reduction factor $(1 - P_{ULS}/P_{cr})$ = 0.884

LH rafter:

As "elastic" frame, stiffness reduction factor $(1 - P_{ULS}/P_{cr})$ $= 0.661$

RH rafter:

The RH rafter has a plastic hinge at each, so the stiffness value is zero.

2.3.2 Second-order magnification factors

B.4.3

Sway mode magnification factor $\lambda_{crp}/(\lambda_{crp} - 1)$

The notional sway deflection is calculated from the sum of the stiffnesses K of the LH column and rafter pair (the RH rafter is hinged at each end reducing the stiffness to zero) and the LH and RH base stiffness:

Sway stiffness from column and rafter stiffness:

D.3.5

$$K_2 = \frac{1}{\left(\frac{S\,h^2}{3EI_{\text{eff.R}}} + \frac{h^3}{3EI_{\text{eff.c}}}\right)}$$

Sway stiffness from base stiffness:

D.4

$$K_b = \frac{1}{\left(\frac{5h^3}{2EI_c} + \frac{h^3}{3EI_{\text{eff.c}}}\right)}$$

LH rafter and column:

K_2 and K_b are the same as for the "elastic" frame.

RH rafter and column:

K_b is the same as for the "elastic" frame. K_2 is zero because the rafter is hinged at both ends.

Total frame:

ΣK $= 43.3 + 18.4 + 0 + 18.4$ $= 80.1$ N/mm

H $= 3.25$ kN as calculated above

$\Delta \quad = \Sigma H / \Sigma K \quad = 3250/80.1 \quad = 40.6$ mm

$\lambda_{crp} \quad = h/200\delta_{np} \quad = 15000/(200 \times 40.6) \quad = 1.85$

Sway mode magnification factor $\lambda_{crp}/(\lambda_{crp} - 1) \quad = 1.85/(1.85 - 1) \quad = 2.18$

2.3.3 Deflections from vertical loads

B.4.5

The loads applied to the "plastic" frame $= (\lambda_p - \lambda_l)$(loads at ULS)

$(\lambda_p - \lambda_l) \quad = (2.28 - 2.12) \quad = 0.16$

$w_P \quad = 0.16(9.36\text{Cos}^2 8° + 2.70\text{Cos}8°) \quad = 1.90$ kN/m

Sway

This arises due to the rotation of the column without an adjacent hinge caused by the end rotation of the rafter spanning from eaves to apex.

End slope of the rafter, $\theta_{R2} = \dfrac{w_P\, S^3}{24\, EI_R}\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}$

$$\theta_{R2} = \frac{3.03 \times 25246^3}{24 \times 205000 \times 87300 \times 10^4}\, 2.18 = 0.0155 \text{ radians}$$

Horizontal deflection of eaves $= h\theta_R \quad = 15000 \times 0.0155 \quad = 232.1$ mm

Horizontal deflection of tie $\quad = 14200 \times 0.0155 \quad = 219.7$ mm

Drop of mid-rafter hinge "a"

Midspan deflection of the rafter, $\delta_{R2} = \dfrac{5}{384}\dfrac{w_P\, S^4}{E\, I_R}\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}$

$$\delta_{R2} = \frac{5}{384}\frac{1.7 \times 25246^4}{205000 \times 87300 \times 10^4}\, 2.18 = 122.1 \text{ mm}$$

X deflection $= \delta_{R2} \text{Sin}\, \alpha = 122.1 \times 0.139 \quad = 17.0$ mm

Y deflection $= \delta_{R2} \text{Cos}\, \alpha = 122.1 \times 0.990 \quad = 120.9$ mm

2.3.4 Deflections from horizontal loads

B.4.6

The loads applied to the "plastic" frame = $(\lambda_p - \lambda_l)$(loads at ULS)

$(\lambda_p - \lambda_l)$ = (2.28 - 2.12) = 0.16

The ULS horizontal load in this load case is the notional horizontal force

$H = (\lambda_p - \lambda_l) \times \text{NHF} = 0.160 \times 3.25 = 0.52$ kN

The sway deflection is calculated from the sum of the stiffnesses K of the LH column and rafter pair (the RH rafter is hinged at each end reducing the stiffness to zero). The base stiffness of nominally pinned bases is not included because this is not a stability calculation like the calculation of λ_{cr}.

$$K_s = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} \frac{1}{\left(\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}\right)}$$

The first-order stiffness of the LH rafter and column pair is the same as for the "elastic" frame:

$$\frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} = 59.3$$

$K_s = 59.3 \times (1/2.18) = 27.2$ N/mm

Sway deflection = 520/27.2 = 19.1 mm

Drop of mid-rafter hinge at "a"

The sway deflection induces an additional deflection at the hinge point "a"

$$\delta_{sm2} = \frac{M_i\, S^2}{16\, EI_R} \frac{\lambda_{crp}}{\lambda_{crp} - 1}$$ where $M = \Sigma H_i h_i = 520 \times 15000 = 7.8$ kNm

$$\delta_{sm2} = \frac{7.8\times10^6\times25246^2}{16\times205000\times87300\times10^4} \times 2.18 = 3.8 \text{ mm}$$

X deflection $= \delta_{sm2} \operatorname{Sin}\alpha = 3.8 \times 0.139 = 0.5$ mm

Y deflection $= \delta_{sm2} \cos\alpha \quad = 3.8 \times 0.990 \quad = 3.7$ mm

2.4 Apex drop

B.5

Apex drop from the first-order elastic deflections

B.5.2

Y deflection at λ_1, $\delta_1 \quad = 252.9$ mm

Apex drop at ULS, $\delta_{ULS} \quad = \delta_1 / \lambda_1 \quad = 252.9/2.12 \quad = 119.3$ mm

Apex drop from curvature shortening δ_c

B.5.3

Rafter transverse deflections at ULS:

ULS transverse load on rafters $\quad = 11.85$ kN/m

Take total transverse deflection $\quad = \delta_{cs2} = \dfrac{2}{384} \dfrac{wS_r^4}{EI_{eff,R}}$

$$= \frac{2}{384} \frac{11.85 \times (21610)^4}{205000 \times 57705 \times 10^4} = 105.6 \text{ mm}$$

Taking shortening $= \dfrac{\pi^2 (\delta_{cs2})^2}{4 S_r} = \dfrac{\pi^2 (105.6)^2}{4 \times 21610} = 1.3$ mm

Apex drop $= 1.3/\text{Sin } 8° = 9.3$ mm

Calculate the apex drop from increased rafter axial force

B.5.4

(i) Vertical component of rafter axial, V_1

$= P_{R1} \sin\alpha \quad = 919 \text{ Sin } 8° \quad = 127.9$ kN

(ii) Reduced slope

Drop from first-order + curvature, $d_1 = \delta_1 + \Delta/\sin\alpha_1$

$= 119.3 + 1.3/\text{Sin } 8° \quad = 119.3 + 9.3 \quad = 128.6$ mm

$S_r \quad = (25000 - 4000)/\text{Cos } 8° \quad = 21400/\text{Cos } 8° \quad = 21610$

Un-stressed rafter rise above haunch end, $h_1 = 21206 \text{ Sin } 8°$ $= 2951$ mm

Reduced rafter rise above haunch end, h_2 $= 2951 - 128.6$ $= 2823$ mm

Reduced rafter slope, $\alpha_2 = \text{Tan}^{-1}\,(2823/21000)$ $= 7.66°$

(iii) Reduced vertical component of the rafter, V_2

$= P_{R1} \text{ Sin } \alpha_2 = 919 \times \text{Sin } 7.66° = 122.4 \text{ kN}$

(iv) Required increase in rafter axial force, δP_R

$= (V_1 - V_2)/\text{Sin } \alpha_2$

$= (127.9 - 122.4)/\text{Sin } \alpha_2$ $= 5.5/\text{Sin } 7.66°$ $= 41.1$ kN

(v) Resultant increase in horizontal reaction at column top, δH_c

$= \delta P_R \text{ Cos } \alpha_2$ $= 41.1 \text{ Cos } \alpha_2$ $= 40.7$ kN

(vi) Resulting increase in tie force, δT

Haunch length $= 4000/\text{Cos } \alpha$ $= 4039$ mm

c $= 800 + 4039(\text{Sin } \alpha - \text{Sin } \alpha_2)$ $= 824$ mm

δT $= \delta H_c\,[(c + h_T)/h_T]$ $= 41.1\,(824 + 14200)/14200$ $= 43.1$ kN

(vii) Horizontal movement of Z due to tie stretching, δX_T

Lateral displacement of the end of the tie:

$$\delta X_T = \frac{\delta T \times \text{halfspan}}{A_T E} = \frac{43.1 \times 10^3 \times 25000}{93.1 \times 100 \times 205000} = 0.6 \text{ mm}$$

e $= 800 + 4000\text{Tan } 8°$ $= 1362$ mm

$$\delta X_{ZT} = \delta X_T \left(\frac{e + h_T}{h_T}\right) = 0.6 \left(\frac{1362 + 14200}{14200}\right) = 0.6 \text{ mm}$$

(viii) Horizontal movement of Z due to column bending, δ_{ZC} :

$$= \frac{\delta H_c c^2 (h_T + c)}{3EI_{eff.c}} = \frac{40.7 \times 10^3 \times 824^2 \times (14200 + 824)}{3 \times 205000 \times 77241} = 0.9 \text{ mm}$$

(ix) Rafter shortening from increased axial strain:

$$\delta S_R = \frac{PL}{AE} = \frac{\delta P_R \times S_R}{A_R E} = \frac{41.1 \times 10^3 \times 25246}{144.0 \times 100 \times 205000} = 0.4 \text{ mm}$$

(x) Increased rafter drop d_A

$\delta X_Z = \delta X_{ZT} + \delta X_{ZC} = 0.6 + 0.9 = 1.5$ mm

$d_A = \delta X_Z / \text{Tan}\alpha_2 + \delta S_r / \text{Sin}\alpha_2 = 13.7$ mm

(xi) Revised rafter slope:

$d_2 = \delta_1 + \Delta / \text{Sin}\,\alpha_1 + d_A = 119.3 + 1.3/\text{Sin}\,7.66° + 13.7$

$= 119.3 + 9.7 + 13.7 = 142.8$ mm

Revised rafter slope $\alpha_2 = \text{Sin}^{-1}\,[(2951 - 142.8)/21206] = 7.61°$

(xii) Check if d_2 / d_1 is less than 1.03

$d_2 / d_1 = 142.8/128.6 = 1.11 > 1.03$ so repeat steps (iii) to (xii)

2nd (iii) Reduced vertical component of the rafter, V_2

$= P_{R1}\,\text{Sin}\alpha_2 = 919 \times \text{Sin}7.61° = 121.7$ kN

2nd (iv) Required increase in rafter axial force, δP_R

$= (V_1 - V_2)/\text{Sin}\,\alpha_2$

$= (127.9 - 121.7)/\text{Sin}\,\alpha_2 = 6.2/\text{Sin}\,7.61° = 46.7$ kN

2nd (v) Resultant increase in horizontal reaction at column top, δH_c

$= \delta P_R\,\text{Cos}\,\alpha_2 = 46.3\,\text{Cos}\,\alpha_2 = 46.3$ kN

2nd (vi) Resulting increase in tie force, δT

Haunch length $= 4000/\text{Cos}\,\alpha \quad = 4039$ mm

c $= 800 + 4039(\text{Sin}\alpha - \text{Sin}\alpha_2) \quad = 827$ mm

$\delta T \quad = \delta H_c\,[(c + h_T)/h_T] \quad = 46.3 \times (827 + 14200)/14200 = 49.0$ kN

2nd (vii) Horizontal movement of Z due to tie stretching, δX_T

Lateral displacement of the end of the tie:

$$\delta X_T = \frac{\delta T \times \text{halfspan}}{A_T E} = \frac{49.0 \times 10^3 \times 25000}{144.0 \times 100 \times 205000} = 0.6 \text{ mm}$$

$$e = 800 + 4000\,\text{Tan}8^\circ = 1362 \text{ mm}$$

$$\delta X_{ZT} = \delta X_T \left(\frac{e + h_T}{h_T} \right) = 0.7 \text{ mm}$$

2nd (viii) Horizontal movement of Z due to column bending, δ_{ZC} :

$$= \frac{\delta H_c c^2 (h_T + c)}{3EI_{\text{eff,c}}} = \frac{46.3 \times 10^3 \times 827^2 \times (14200 + 827)}{3 \times 205000 \times 77241} = 1.0 \text{ mm}$$

2nd (ix) Rafter shortening from increased axial strain:

$$\delta S_r = \frac{\delta P_R \times S_r}{A_R E} = \frac{46.7 \times 10^3 \times 25246}{144.0 \times 100 \times 205000} = 0.4 \text{ mm}$$

2nd (x) Increased rafter drop d_A

$$\delta X_Z = \delta X_{ZT} + \delta X_{ZC} = 0.7 + 1.0 = 1.7 \text{ mm}$$

$$d_A = \delta X_Z/\text{Tan}\,\alpha_2 + \delta S_r/\text{Sin}\,\alpha_2 = 15.8 \text{ mm}$$

2nd (xi) Revised rafter slope:

revised $d_2 = \delta_1 + \Delta/\text{Sin}\,\alpha_1 + d_A \quad = 119.3 + 1.5/\text{Sin}\,7.43° + 15.8$

$= 119.3 + 9.8 + 15.8 \quad = 144.9$ mm

Revised rafter slope $\alpha_2 = \text{Sin}^{-1}\,[(2951 - 144.9)/21206] = 7.605°$

2nd (xii) Check if revised $d_2\,/d_2$ is less than 1.03

revised $d_2\,/d_2 \quad = 144.9/142.8 \quad = 1.01$

The additional apex drop was 1% of previous - accept this value

Second-order axial force in rafters $= P_1 + \delta P_R \quad = P_1 + 46.7$ kN

2.5 Axial forces for the energy calculation

The total of the axial loads in the columns is not affected, so $P_2 = P_1$ which is taken as the mid-height value calculated in 2.1 above.

The axial force in the rafters is the first-order force calculated in 2.1 above plus the second-order increase in force, δP_R , from 2.4 above

LH col: $P_2 = 313$ kN

LH rafter: $P_2 = 940 + 46.7 = 987$ kN

RH rafter: $P_2 = 940 + 46.7 = 987$ kN

RH col: $P_2 = 314$ kN

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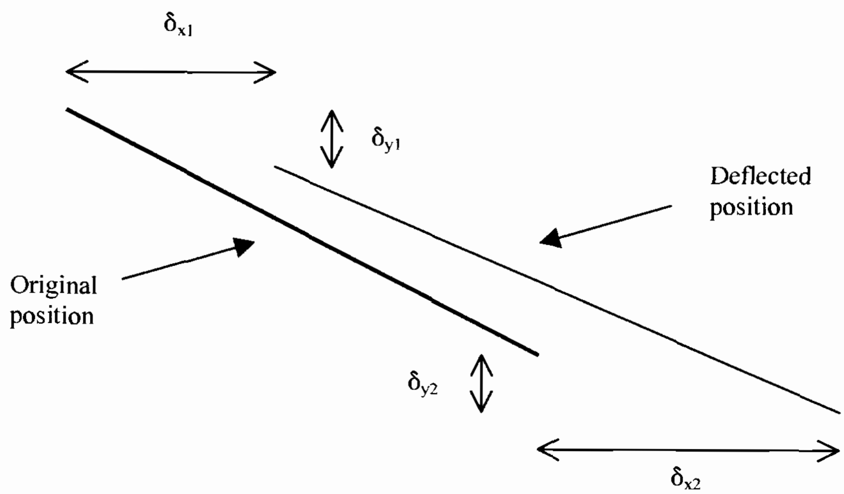
2.6 Second-order Energy Summation

B.2.2



The energy summation is required to calculate λ_M following the methods in B.2.2

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CALCULATION SHEET

Element for evaluation of Pc*phi*s*d(phi)	AB	BC	Ca	aD	Db	bc	cE	GF	FE		
X-AXIS DEFLECTIONS											
Deflections from the "elastic" frame											
dxa	0.0	35.1	47.5	92.9	75.9	75.9	104.4	0.0	112.1		
dxb	35.1	47.5	92.9	75.9	75.9	104.4	104.4	112.1	104.4		
(dxb - dxa)	**35.1**	**12.4**	**45.4**	**-16.9**	**0.0**	**28.4**	**0.0**	**112.1**	**-7.7**		
Deflections from the "plastic" frame											
From tranverse loads on rafter											
Sway of top of elastic column											
dxa	0.0	219.7	232.1	232.1	232.1	232.1	232.1	0.0	219.7		
dxb	219.7	232.1	232.1	232.1	232.1	232.1	232.1	219.7	232.1		
(dxb - dxa)	**219.7**	**12.4**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**219.7**	**12.4**		
Drop of mid-rafter hinge											
dxa	0.0	0.0	0.0	17.0	0.0	0.0	0.0	0.0	0.0		
dxb	0.0	0.0	17.0	0.0	0.0	0.0	0.0	0.0	0.0		
(dxb - dxa)	**0.0**	**0.0**	**17.0**	**-17.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**		
From horizontal loads											
Sway											
dxa	0.0	19.1	19.1	19.1	19.1	19.1	19.1	0.0	19.1		
dxb	19.1	19.1	19.1	19.1	19.1	19.1	19.1	19.1	19.1		
(dxb - dxa)	**19.1**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**19.1**	**0.0**		
Drop of mid-rafter hinge											
dxa	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0		
dxb	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0		
(dxb - dxa)	**0.0**	**0.0**	**0.5**	**-0.5**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**		
Total of (dxb - dxa) at collapse	**312.9**	**63.8**	**101.4**	**5.1**	**39.0**	**67.4**	**39.0**	**389.9**	**43.7**		
Y-AXIS DEFLECTIONS											
Relative deflections from apex drop											
dya	0.0	0.0	0.0	80.3	144.9	142.0	20.7	0.0	0.0		
dyb	0.0	0.0	80.3	144.9	142.0	20.7	0.0	0.0	0.0		
(dyb - dya)	**0.0**	**0.0**	**80.3**	**64.5**	**-2.9**	**-121.3**	**-20.7**	**0.0**	**0.0**		
Deflections from the "elastic" frame											
Drop of mid-rafter hinge											
dxa	0.0	0.0	0.0	221.6	0.0	0.0	0.0	0.0	0.0		
dxb	0.0	0.0	221.6	0.0	0.0	0.0	0.0	0.0	0.0		
(dxb - dxa)	**0.0**	**0.0**	**221.6**	**-221.6**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**		
Deflections from the "plastic" frame											
Drop of mid-rafter hinge from transverse loads on rafter											
dxa	0.0	0.0	0.0	120.9	0.0	0.0	0.0	0.0	0.0		
dxb	0.0	0.0	120.9	0.0	0.0	0.0	0.0	0.0	0.0		
(dxb - dxa)	**0.0**	**0.0**	**120.9**	**-120.9**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**		
Drop of mid-rafter hinge from sway loads											
dxa	0.0	0.0	0.0	3.7	0.0	0.0	0.0	0.0	0.0		
dxb	0.0	0.0	3.7	0.0	0.0	0.0	0.0	0.0	0.0		
(dxb - dxa)	**0.0**	**0.0**	**3.7**	**-3.7**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**		
Totals of (dyb - dya) at collapse	**0.0**	**0.0**	**426.6**	**-281.7**	**-2.9**	**-121.3**	**-20.7**	**0.0**	**0.0**		
SHORTENING											
psi (angle from X axis)	90.0	90.0	8.0	8.0	-8.0	-8.0	-8.0	90.0	90.0		
[(dxb - dxa) at collapse]*Sin(psi)	312.9	63.8	14.1	0.7	-5.4	-9.4	-5.4	389.9	43.7		
[(dyb - dya) at collapse]*Cos(psi)	0.0	0.0	422.4	-278.9	-2.8	-120.2	-20.5	0.0	0.0		
phi * s at collapse	312.9	63.8	436.5	-278.2	-8.3	-129.5	-25.9	389.9	43.7		
Incremental rotn = d(phi) from mechanism	1.000	1.000	1.000	0.218	0.218	0.174	1.000	1.000	1.000		
Shortening = phi*s*d(phi) (modulus)	**312.9**	**63.8**	**436.5**	**60.7**	**1.8**	**22.6**	**25.9**	**389.9**	**43.7**		
AXIAL FORCES											
Pc for columns and rafters at ULS	313.0	313.0	939.5	939.5	939.5	939.5	939.5	314.0	314.0		
Second-order increase in rafter axial			46.7	46.7	46.7	46.7	46.7				
Design Axial	313.0	313.0	986.2	986.2	986.2	986.2	986.2	314.0	314.0		
Incremental energy = Pc*phi*s*d(phi)	**97.9**	**20.0**	**430.5**	**59.8**	**1.8**	**22.3**	**25.5**	**122.4**	**13.7**	**Sum =**	**794**
WORK DONE IN ROTATING HINGES											
Element for evaluating Mpr d(phi)	AB	BC	Ca	aD	Db	bc	cE	GF	FE		
MprA	0.0	0.0	0.0	869.2	0.0	869.2	869.2	0.0	0.0		
MprB	0.0	0.0	869.2	0.0	869.2	869.2	0.0	0.0	0.0		
MprA + MprB	0.0	0.0	869.2	869.2	869.2	1738.4	869.2	0.0	0.0		
Incremental rotn = d(phi) from mechanism	1.000	1.000	1.000	0.218	0.218	0.174	1.000	1.000	1.000		
Mpr*d(phi)	**0.0**	**0.0**	**869.2**	**189.5**	**189.5**	**302.8**	**869.2**	**0.0**	**0.0**	**Sum =**	**2420**

Factor on lambda_p	0.672
lambda_p from first-order analysis	2.280
lambda_M	**1.532**

2.6 Load factor at failure, λ_M

B.7

$\Sigma P_2 \phi s d\phi = 794 \phi$

$\Sigma M_{pr} d\phi = 2420 \phi$

$$\lambda_M = \lambda_p \left[\frac{1}{1 + \left(\dfrac{P_2 \phi s d\phi}{\Sigma M_{pr} d\phi} \right)} \right]$$

$$\lambda_M = 2.280 \left[\frac{1}{1 + \left(\dfrac{794 \phi}{2420 \phi} \right)} \right] = 2.280 \times 0.672 = 1.532$$

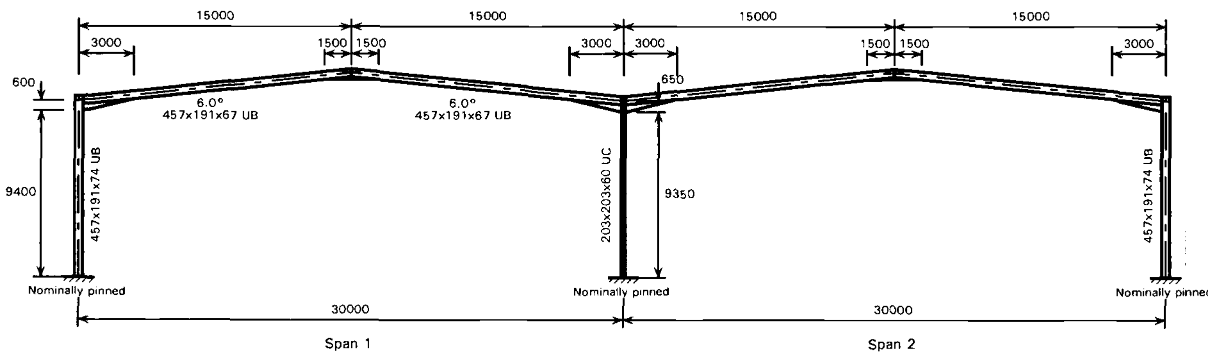
$\lambda_M > 1.0$, so the frame has passed the check for in-plane stability.

The Steel Construction Institute Silwood Park, Ascot, Berks SL5 7QN Telephone: (01344) 623345 Fax: (01344) 622944 **CALCULATION SHEET**	Job No: CDS 139	Page 1 of 22	Rev
	Job Title: BS 5950 Portals		
	Subject: Second-order Worked Example: Two-span Portal Frame		
	Client: DETR	Made by: CMK	Date: May 2001
		Checked by: WIS	Date: July 2001

1. INPUT FROM FIRST-ORDER ANALYSIS

The calculations have been carried out using spreadsheet software. The numerical values presented below are the values from the spreadsheet rounded to a suitable number of significant figures.

1.1 General Arrangement



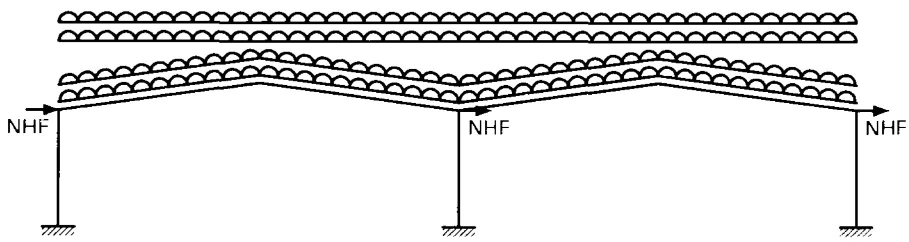
Angle of rafters: $\alpha_1 = \alpha_2 = 6°$

Span = 30 m

$$\text{Developed length of rafter} = \frac{30}{\cos 6°} = 30.165 \text{ m}$$

Height of column from base to Neutral Axis of rafter = 10.0 m

1.2 Loading



Frame spacing = 6 m

Dead = 0.100 × 6.000 × 1.4 = 0.840 kN/m along slope
Service = 0.150 × 6.000 × 1.4 = 1.260 on plan
Imposed = 0.600 × 6.000 × 1.6 = 5.760 on plan
Self Weight = 80 × 10^{-2} × 1.000 × 1.4 = 1.120 along slope

Developed Length of Rafters $= \dfrac{30}{\cos 6^\circ} = 30.165$

Notional Horizontal Force — Cl 2.4.2.4.

0.5% factored load on span

= 0.5% × 2 × [30.165 × (0.84 + 1.12) + 30 × (1.26 + 5.76)]

= 0.005 × 2 × [59.1 + 210.6] =2.7 kN

1.3 Plastic Hinge Incremental Rotations

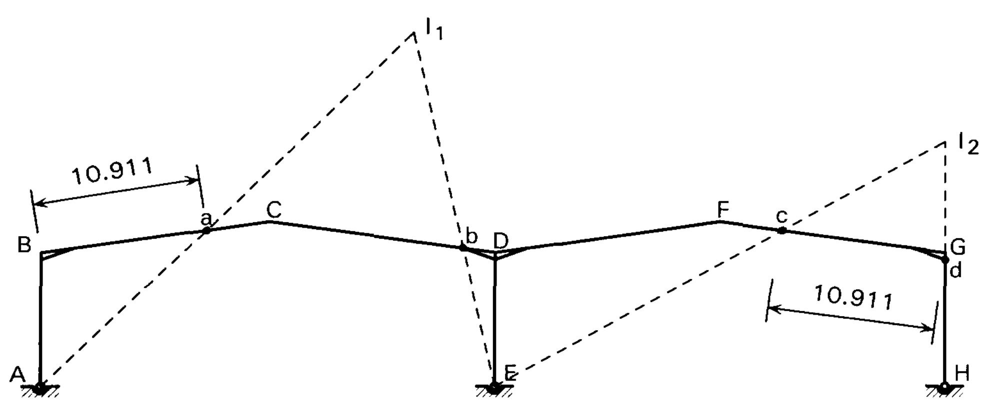
The values of incremental rotation of the hinges are taken from the first-order collapse mechanism (determined elsewhere). These are the incremental rotations as used to calculate the collapse factor of the frame using the classic Rigid-Plastic (Virtual Work) method.

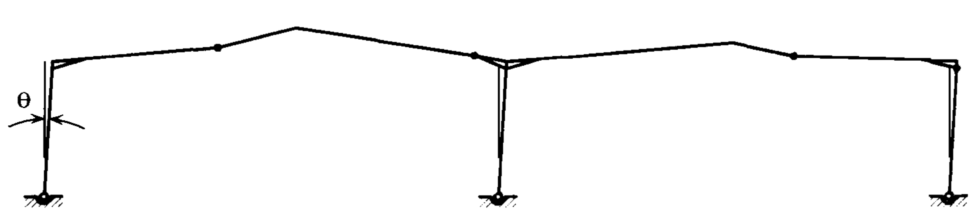
The second-order analysis uses the relative magnitude of the instantaneous rotations, so the absolute magnitude of each rotation does not affect the calculations.

Where the analysis has been performed by methods other than the Rigid-Plastic method (e.g. by the Semi-Graphical method), the incremental rotations can be deduced from the geometry of the frame and the position of the hinges. It is not necessary to repeat the calculation of the collapse factor by the Rigid-Plastic method.

Failure Mechanism

The mechanism will have two instantaneous centres of rotation, as shown below (locations of hinges determined from analysis).





Node Locations (coordinates)

Pt A → (0.0, 0.0) = (0.0, 0.0)

Pt B → (0.0, 10.0) = (0.0, 10.0)

Pt C → (15.0, {10.0 + 15.0Tan6}) = (15.0, 11.577)

Pt D → (30.0, 10.0) = (30.0, 10.0)

Pt E → (30.0, 0.0) = (30.0, 0.0)

Pt F → ({30.0+15.0}, {10.0 + 15.0Tan6}) = (45.0, 11.577)

Pt G → (60.0, 10.0) = (60.0, 10.0)

Pt H → (60.0, 0.0) = (60.0, 0.0)

Pt a → (10.911Cos6, {10.0 + 10.911Sin6}) = (10.851, 11.141)

Pt b → ({30.0-2.993Cos6}, {10.0 + 2.993Sin6}) = (27.023, 10.313)

Pt c → ({60.0-10.911Cos6}, {10.0 + 10.911Sin6}) = (49.149, 11.141)

Pt d → (60.0, 9.400) = (60.0, 9.400)

Find Centre of Rotation I_1

$$Y_{I1} = \frac{X_E - X_A}{\left(\frac{dX}{dY}\right)_{Aa} - \left(\frac{dX}{dY}\right)_{Eb}} = \frac{30.0 - 0.0}{\left(\frac{10.851}{11.141}\right) - \left(\frac{30-27.023}{10.313}\right)} = 23.759 \text{ m}$$

$$X_{I1} = X_A + \left[\left(\frac{dX}{dY}\right)_{Aa} \times Y_{I1}\right] = 0.0+(0.97397\times 23.760) = 23.142 \text{ m}$$

Pt I_1 → (23.142, 23.759)

Find Centre of Rotation I_2

$$Y_{I2} = \frac{X_H - X_E}{\left(\frac{dX}{dY}\right)_{Ec} - \left(\frac{dX}{dY}\right)_{Hd}} = \frac{60.0 - 30.0}{(1.719) - (0.0)} = 17.454 \text{ m}$$

X_{I2} = 60.0 (obvious by inspection)

Pt I_2 → (17.454, 60.0)

Hinge Rotations

Taking the instantaneous rotation about A as θ

$$\theta_{I1} = \theta \times \frac{Y_a}{Y_{I1} - Y_a} = \theta \times \frac{11.141}{23.759 - 11.141} = 0.883\ \theta$$

$$\theta_E = \theta_{I1} \times \frac{Y_{I1} - Y_b}{Y_b} = 0.883\theta \times \frac{23.759 - 10.313}{10.313} = 1.151\ \theta$$

$$\theta_{I2} = \theta_E \times \frac{Y_c}{Y_{I2} - Y_c} = 1.151\theta \times \frac{11.141}{17.454 - 11.141}$$

$$= 1.151\ \theta \times 1.765 = 2.031\ \theta$$

$$\theta_H = \theta_{I2} \times \frac{Y_{I2} - Y_d}{Y_d} = 2.031\theta \times \frac{17.454 - 9.400}{9.400}$$

$$= 2.031\ \theta \times 0.857 = 1.740\ \theta$$

1.4 Axial forces at ULS from first-order plastic analysis

Span 1

LH column:	at base	= 131.9 kN,	at haunch	= 122.3 kN
LH rafter:	at column	= 59.8 kN,	at apex	= 46.0 kN
RH rafter:	at column	= 62.0 kN,	at apex	= 48.2 kN
RH column:	at base	= 291.5 kN,	at haunch	= 283.3 kN

Span 2

LH rafter:	at column	= 62.0 kN,	at apex	= 48.3 kN
RH rafter:	at column	= 60.1 kN,	at apex	= 46.3 kN
RH column:	at base	= 132.8 kN,	at haunch	= 122.6 kN

1.5 Reduced Plastic Moments at ULS from first-order analysis

Use the reduced moment capacity for the sections to account for the co-existent axial force, calculated in accordance with BS 5950-1:2000, Annex I.2. These may be taken from section tables.

$M_{rx} = p_y S_{rx} \leq 1.2 p_y Z_x$ Cl 4.2.5

For the axial forces in this frame under this load case

M_{pr} rafters(457 × 191 × 67 UB) = 404 kNm

M_{pr} external columns (457 × 191 × 74 UB) = 454 kNm

1.6 Load factor at formation of the first hinge, λ_1

From the frame analysis output, the load factor at the formation of the first hinge is:

$\lambda_1 = 0.957$

1.7 Plastic collapse factor, λ_p

From the frame analysis, the plastic collapse factor calculated by first-order analysis is:

$\lambda_p = 1.122$

1.8 Member inertias, I_x

External columns: 457×191 × 74 UB: I_x = 33320 cm^4

Rafters: 457×191 × 67 UB: I_x = 29380 cm^4

Internal column: 203×203 × 60 UC: I_x = 6125 cm^4

1.9 Deflections of frame at λ_1 (formation of the first hinge)

The following deflections are from the first order frame analysis output.

δ_{xB} = −33.0 mm δ_{yB} = 0.6 mm

δ_{xC} = − 8.8 mm δ_{yC} = 234.7 mm

δ_{xD} = 15.3 mm δ_{yD} = 1.7 mm

δ_{xF} = 38.3 mm δ_{yF} = 224.6 mm

δ_{xG} = 61.4 mm δ_{yG} = 0.6 mm

2. SECOND-ORDER ANALYSIS

2.1 Axial forces in members

Use the average axial forces in the members, from first order analysis

Span 1

LH col: take mid-height P_{ULS} = (131.9 + 122.3)/2 = 127.1 kN

LH rafter: take mid-length P_{ULS} = (59.8 + 46.0)/2 = 52.9 kN

RH rafter: take mid-length P_{ULS} = (62.0 + 48.2)/2 = 55.1 kN

RH col: take mid-height P_{ULS} = (291.5 +283.3)/2 = 287.4 kN

Span 2

LH rafter: take mid-length P_{ULS} = (62.0 + 48.3)/2 = 55.2 kN

RH rafter: take mid-length P_{ULS} = (60.1 + 46.3)/2 = 53.2 kN

RH col: take mid-height P_{ULS} = (132.8 +122.6)/2 = 127.7 kN

2.2 Bending Deflections of the "elastic" frame

2.2.1 Stiffness reduction factors allowing for P.δ effects (A.3.2)

Columns

The stiffness of each external column differs from the stiffness of the internal column. Therefore the reduction in frame stiffness is calculated from the sum of the ULS loads in the columns and the sum of the critical loads of the columns.

Sum of load in columns, ΣP_{ULS} = 127.1 + 287.4 + 127.7 = 542.2 kN

External columns: I_x = 33320 cm^4 (A.3.2)
h = 10000 mm
Nominal pinned bases α = 1.7 (A.2.4)

$P_{cr} = \pi^2 EI/(\alpha h)^2 = \pi^2 \times 205000 \times 33320 \times 10^4/(1.7 \times 10000)^2 = 2333$ kN

Internal column: I_x = 6125 cm^4
h = 10000 mm
Nominally pinned bases α = 1.7

$P_{cr} = \pi^2 EI/(\alpha h)^2 = \pi^2 \times 205000 \times 6125 \times 10^4/(1.7 \times 10000)^2 = 429$ kN

Sum of load in columns, ΣP_{cr} = 2333 + 429 +2333 = 5094 kN

$(1 - \Sigma P_{ULS}/\Sigma p_{cr})$ = (1 - 542.2/5094) = (1 - 0.147) = 0.894

$1/(1 - \Sigma P_{ULS}/\Sigma p_{cr})$ = 1.119

Rafters (A3.2)

Span 1

Average P_{ULS} = (52.9 + 55.1)/2 = 54.0 kN

I_x = 29380 cm^4, L = 30165mm, α = 1.0

$P_{cr} = \pi^2 EI/(\alpha L)^2 = \pi^2 \times 205000 \times 29380 \times 10^4/(1.0 \times 30165)^2 = 653$ kN

Reduction factor is given by:

$(1 - P_{ULS}/P_{cr})$ = (1 - 54.0/653) = (1 - 0.083) = 0.917

Span 2

Average P_{ULS} = (55.2 + 53.2)/2 = 54.2 kN

I_x = 29380 cm^4, L = 30165mm, α = 1.0

$P_{cr} = \pi^2 EI/(\alpha L)^2 = \pi^2 \times 205000 \times 29380 \times 10^4/(1.0 \times 30165)^2 = 653$ kN

Reduction factor is given by:

$(1 - P_{ULS}/P_{cr}) = (1 - 54.2/653) = (1 - 0.083) = 0.917$

2.2.2 Second order magnification factors

The notional sway deflection is calculated from the sum of the stiffnesses K for each of the column and rafter pairs:

Sway stiffness from column and rafter stiffness

$$K = \frac{H}{\delta} = \frac{1}{\left(\dfrac{Sh^2}{3EI_{eff,R}} + \dfrac{h^3}{3EI_{eff,c}}\right)}$$

D.2.5

LH Rafter and Column

$I_{eff,R} = I_R\,(1 - P/P_{cr})$ = 29380 (1 − 54/653) = 26951

$I_{eff,c} = I_c\,(1 - P/P_{cr})$ = 33320 (1 − 127.1/2333) = 31505

$$\frac{Sh^2}{3EI_{eff,R}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26951 \times 10^4} = 0.01820$$

$$\frac{h^3}{3EI_{eff,c}} = \frac{10000^3}{3 \times 205000 \times 31505 \times 10^4} = 0.00516$$

$$K_{Sub1} = \frac{1}{0.01820 + 0.00516} = 42.8 \text{ N/mm}$$

RH Rafter and Column

$I_{eff,R} = I_R\,(1 - P/P_{cr})$ = 29380 (1 − 54.2/653) = 26944

$I_{eff,c} = I_c\,(1 - P/P_{cr})$ = 33320 (1 − 122.6/2333) = 31496

$$\frac{Sh^2}{3EI_{\text{eff,R}}} = \frac{30165\times(10000)^2}{3\times205000\times26944\times10^4} = 0.01820$$

$$\frac{h^3}{3EI_{\text{eff,c}}} = \frac{10000^3}{3\times205000\times31496\times10^4} = 0.00516$$

$$K_{\text{Sub2}} = \frac{1}{0.01820 + 0.00516} = 42.8 \text{ N/mm}$$

Sway stiffness from nominal base fixity

$$K_{\text{b}} = \frac{H}{\delta} = \frac{1}{\left(\frac{5h^3}{2EI} + \frac{h^3}{3EI_{\text{eff}}}\right)}$$ D.4

$$K_{\text{b ext}} = \frac{1}{(0.0366 + 0.00516)} = 23.9 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K = K_{\text{Sub 1}} + K_{\text{b ext}} + K_{\text{Sub 2}} + K_{\text{b ext}}$$

$$\Sigma K = 42.8 + 23.9 + 42.8 + 23.9 = 133.5 \text{ N/mm}$$

H = 0.005 × [sum of column reactions] = 0.005 × [542.2] = 2.711 kN

$$\delta_{n2} = \frac{H}{\Sigma K_2} = \frac{2.711\times1000}{133.5} = 20.31 \text{ mm}$$

$$\lambda_{\text{cr1}} = \frac{h}{200\,\delta_{\text{n2}}} = \frac{10000}{200\times20.31} = 2.5$$

Sway mode magnification

$$\left[\frac{\lambda_{\text{cr1}}}{\lambda_{\text{cr1}} - 1}\right] = 1.7$$

2.2.3 Deflection calculations A.3.4

Sway deflections D.2.3

The first-order sway deflection δX_{1s} is calculated from the sum of the stiffnesses K for each of the column and rafter pairs. (The base stiffness of nominally pinned bases is not included because this is not a stability calculation like the calculation of λ_{cr}).

$$K = \frac{H}{\delta} = \frac{1}{\left(\frac{Sh^2}{3EI_R} + \frac{h^3}{3EI_c}\right)}$$

Sub Frame 1 (Elastic)

$$\frac{Sh^2}{3EI_R} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 29380 \times 10^4} = 0.01669$$

$$\frac{h^3}{3EI_c} = \frac{10000^3}{3 \times 205000 \times 33320 \times 10^4} = 0.00488$$

$$K_{Sub1} = \frac{1}{0.01669 + 0.00488} = 46.4 \text{ N/mm}$$

Sub Frame 2 (Elastic)

As Sub frame 1

$$K_{Sub2} = \frac{1}{0.01669 + 0.00488} = 46.4 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K = K_{Sub\,1} + K_{Sub\,2}$$

$$\Sigma K = 46.4 + 46.4 = 92.7 \text{ N/mm}$$

First-order sway deflection

$$\delta X_{1s} = \frac{\lambda_l H}{\Sigma K_2} = \frac{0.957 \times 2.711 \times 1000}{92.7} = 27.99 \text{ mm}$$

A.3.4

$$\delta X_2 = (\delta X_1 - \delta X_{1s}) \times \left[\frac{\lambda_{cr2}}{\lambda_{cr2} - 1}\right] + \delta X_{1s} \times \left[\frac{\lambda_{cr1}}{\lambda_{cr1} - 1}\right]$$

$$\delta Y_2 = \delta Y_1 \times \left[\frac{\lambda_{cr2}}{\lambda_{cr2} - 1}\right]$$

Values of δX_{I} and δY_{I} are taken from first order analysis (See Sheet 7).

δ_{xB} = (-33.0 - 27.99) × 1.12 + 27.99 × 1.7 = -21.1 mm
δ_{yB} = 0.6 × 1.12 = 0.7 mm

δ_{xC} = (-8.8 - 27.99) × 1.12 + 27.99 × 1.7 = 6.0 mm
δ_{yC} = 234.7 × 1.12 = 262.7 mm

δ_{xD} = (15.3 - 27.99) × 1.12 + 27.99 × 1.7 = 32.9 mm
δ_{yD} = 1.7 × 1.12 = 1.9 mm

δ_{xF} = (38.3 - 27.99) × 1.12 + 27.99 × 1.7 = 58.7 mm
δ_{yF} = 224.6 × 1.12 = 251.4 mm

δ_{xG} = (61.4 - 27.99) × 1.12 + 27.99 × 1.7 = 84.5 mm
δ_{yG} = 0.6 × 1.12 = 0.7 mm

2.3 Bending deflections of the "plastic" frame

A.4

2.3.1 Stiffness reduction factors to allow for P.δ effects

Columns: as the "elastic" frame

External Column RHS $(1 - \Sigma P_{ULS} / \Sigma P_{cr}) = 0.946$

Internal Column $(1 - \Sigma P_{ULS} / \Sigma P_{cr}) = 0.330$

External Column LHS $(1 - \Sigma P_{ULS} / \Sigma P_{cr}) = 0.945$

Rafters: as the "elastic" frame, because that used $\alpha = 1.0$

Span 1: $(1 - P_{ULS} / P_{cr}) = 0.917$

Span 2: $(1 - P_{ULS} / P_{cr}) = 0.917$

2.3.2 Second Order Magnification Factor

Sway mode magnification factor

The notional sway deflection is calculated from the sum of the stiffnesses K of the rafter and column pairs between plastic hinges and the base stiffness of each column.

Sway stiffness from column and rafter stiffness.

$$K_2 = \frac{1}{\left(\frac{Sh^2}{3EI_{\text{eff,R}}} + \frac{h^3}{3EI_{\text{eff,c}}}\right)}$$ D.3.5

Sub Frame 1: LH Column and Rafter Span 1

$$\frac{Sh^2}{3EI_{\text{eff,R}}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26951 \times 10^4} = 0.01820$$

$$\frac{h^3}{3EI_{\text{eff,c}}} = \frac{10000^3}{3 \times 205000 \times 31505 \times 10^4} = 0.00516$$

$$K_{\text{sub 1}} = \frac{1}{0.01820 + 0.00516} = 42.8 \text{ N/mm}$$

Sub Frame 2: Internal Column and Rafter Span 2

$$\frac{Sh^2}{3EI_{\text{eff,R}}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26944 \times 10^4} = 0.01820$$

$$\frac{h^3}{3EI_{\text{eff,c}}} = \frac{10000^3}{3 \times 205000 \times 2020 \times 10^4} = 0.0805$$

$$K_{\text{sub 2}} = \frac{1}{0.01820 + 0.0805} = 10.1 \text{ N/mm}$$

Nominal Base fixity

$$K_b = \frac{H}{\delta} = \frac{1}{\left(\frac{5h^3}{2EI} + \frac{h^3}{3EI_{\text{eff}}}\right)}$$ D.4

External Column

$$K_{\text{b ext}} = \frac{1}{(0.0387 + 0.00516)} = 22.8 \text{ N/mm}$$

Internal Column

$$K_{\text{b int}} = \frac{1}{(0.2106 + 0.0805)} = 1.5 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K = K_{\text{sub 1}} + K_{\text{b ext}} + K_{\text{sub 2}} + K_{\text{b int}} + K_{\text{b ext}}$$

$$\Sigma K = 42.8 + 22.8 + 10.1 + 1.5 + 22.8 = 100.0 \text{ N/mm}$$

$$\delta_{\text{np}} = \frac{\Sigma H}{\Sigma K_2} = \frac{2.711 \times 1000}{110.0} = 27.1 \text{ mm}$$ D.3.5

$$\lambda_{\text{crp}} = \frac{h}{200\,\delta_{\text{np}}} = \frac{10000}{200 \times 27.1} = 1.84$$

Sway mode magnification factor

$$\left[\frac{\lambda_{\text{crp}}}{\lambda_{\text{crp}} - 1}\right] = 2.185$$

2.3.3 Deflections of the "plastic" frame from gravity loads

Loads

The loads applied to the "plastic" frame = $(\lambda_p - \lambda_1)$(loads at ULS)

λ_p and λ_1 are taken from the first order plastic analysis

$(\lambda_p - \lambda_1) = (1.122 - 0.957) = 0.165$

Slope of rafters $\alpha_1 = \alpha_2 = \alpha = 6°$, giving $\text{Cos}\alpha = 0.9945$

Service load and imposed load are specified "on plan",

at ULS, $w_{\text{v.plan}} = 1.26 + 5.76 = 7.02$ kN/m

giving a transverse load on the "plastic frame"

$$= (\lambda_p - \lambda_1)(w_{\text{v.plan}} \text{ at ULS})\text{Cos}^2\alpha$$

$$= 0.165 \times 7.02(0.9945)^2 = 1.15 \text{ kN/m}$$

Dead load and self-weight are values "along the slope",

at ULS, $w_{v.slope} = 0.84 + 1.12 = 1.96$ kN/m

giving a transverse load on the "plastic frame"

$= (\lambda_p - \lambda_1)(w_{v.slope}$ at ULS$)\text{Cos}\alpha$

$= 0.165 \times 1.96 \times 0.866 = 0.32$ kN/m

Summing loads from components "on plan" and "along the slope",

$w_P = 1.15 + 0.32 = 1.47$ kN/m

Sway — A.4.5

This arises due to the rotation of the column without an adjacent hinge.

First-order end slope of the rafter as a simply supported beam, $\theta_R = \dfrac{w_P\, S^3}{24\, EI_R}$

Second-order end slope of the rafter, $\theta_{R2} = \dfrac{w_P\, S^3}{24\, EI_R} \left(\dfrac{\lambda_{crp}}{\lambda_{crp} - 1} \right)$

$E = 205000$ N/mm^2

$I_x = 29380$ cm^4

$$\theta_{R2} = \frac{1.47 \times 30165^3}{24 \times 205000 \times 29380 \times 10^4} \times 2.185 = 0.06087 \text{ radians}$$

Horizontal deflection of Point B, C, D $= h\theta_R$

$= 10000 \times 0.06087 = 608.7$ mm

Mid-span drop

Deflection given by value for simply supported beam of span equal to the developed length of the rafters.

Span 1:

$$\delta_{b2} = \frac{5}{384} \frac{w_P\, S^4}{E\, I_R} \left[\frac{\lambda_{crp}}{\lambda_{crp} - 1} \right]$$

$$\delta_{b2} = \frac{5}{384} \times \frac{1.47 \times 30165^4}{205000 \times 29380 \times 10^4} \times 2.185 = 573.8 \text{ mm}$$

$$\delta_{apex} = \frac{\delta_{B2}}{\cos\alpha} = \frac{573.8}{0.9945} = 577.0 \text{ mm}$$

Span 2: as Span 1

Spread A.4.5

This is caused by the drop of the angle in the rafter which is at the apex in this frame. Because this frame is a symmetrical pitched roof portal, the drop of the angle is the mid-span drop calculated above.

Span 1:

Full span $\delta_{spread} = \delta_{B2}(\text{Sin}\alpha_1 + \text{Sin}\alpha_2) = \delta_B(2\text{Sin } 6°)$

$= 573.8\ (2 \times 0.1045) = 120.0$ mm

Half span $\delta_{spread} = \delta_{B2}\ \text{Sin}\ \alpha_1 = 573.8 \times 0.1045 = 60.0$ mm

Span 2: as Span 1

Column hinge horizontal displacement

The hinge occurs at the underside of the haunch, which is at a distance from the neutral axis of the rafter, causing an additional horizontal displacement.

Second-order end slope of the rafter, $\theta_{R2} = 0.06087$ radians (Sheet 15)

Off-set of the hinge below the rafter, $e = 10000 - 9400 = 600$ mm

Horizontal deflection of Point M $= e\theta_R$

$= 600 \times 0.06087 = 36.5$ mm

2.3.4 Deflections of the "plastic" frame due to horizontal loads A.4.6

Loads

The unfactored loads applied to the "elastic frame" included the horizontal loads and were in proportion to the ULS loads, so the additional horizontal load on the plastic frame $= (\lambda_p - \lambda_1)$(loads at ULS)

$(\lambda_p - \lambda_1) = (1.122 - 0.957) = 0.165$

Slope of rafters $\alpha_1 = \alpha_2 = \alpha = 6°$, giving $\text{Cos}\alpha = 0.9945$

Sway — A.4.6

Total of column forces $= 542.2$ kN — (Sheet 8)

0.5% × of total of column forces (elastic frame) $= 0.005 \times 542.2 = 2.7$ kN — (Sheet 2)

Additional horizontal load on plastic frame

$H = 0.165 \times 2.7 \times 1000 = 447$ N

For a multi-span frame, the sway deflection is calculated from the sum of the stiffnesses K for each of the spans:

$$K_s = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} \times \frac{1}{\left[\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}\right]}$$

$$K_{sub\ 1} = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} = \frac{1}{0.0167 + 0.0055} = 45.1$$

$$K_{sub\ 2} = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} = \frac{1}{0.0167 + 0.0297} = 21.5$$

Total Frame Stiffness

$\Sigma K_s = K_{sub\ 1} + K_{sub\ 2}$

$\Sigma K_s = 45.1 + 21.5 = 66.6$ N/mm

Second-order least sway deflection of the column top

$$\Delta_2 = \frac{H}{\Sigma K_s} = \frac{0.447 \times 1000}{66.6} \times 2.185 = 14.7 \text{ mm}$$

Mid-span drop

Second order sagging deflection of a straight rafter:

$$\delta_{sm2} = \frac{ML_r^2}{16EI_R}\left[\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right] \quad \text{where } M = \Sigma\, H_i\, h_i$$

H_i is the proportion of the horizontal force carried by each sub frame calculated as a proportion of the stiffness K_2.

Sub frame 1:

Drop in the apex of rafter span 1 due to horizontal load.

$$\therefore \quad \delta_{sm2} = \frac{0.293 \times 1000 \times 10000\ (30165)^2}{16 \times 205000 \times 29380 \times 10^4} \times 2.185 = 6.1 \text{ mm}$$

Sub frame 2:

Drop in the apex of rafter span 2 due to horizontal load.

$$\therefore \quad \delta_{sm2} = \frac{0.154 \times 1000 \times 10000\ (30165)^2}{16 \times 205000 \times 29380 \times 10^4} \times 2.185 = 3.2 \text{ mm}$$

Spread

Span 1:

Full span spread $= \delta_{sm2}\ (\text{Sin}\alpha_1 + \text{Sin}\alpha_2) = 6.1\ (\text{Sin } 6° + \text{Sin } 6°)$

$= 6.1\ (0.1045 + 0.1045) = 1.3$ mm

Half span spread $= \delta_{sm2}\ (\text{Sin}\alpha_1) = 6.1\ (\text{Sin } 6°)$

$= 6.1\ (0.1045) = 0.6$ mm

Span 2:

Full span spread $= \delta_{sm2}\ (\text{Sin}\alpha_1 + \text{Sin}\alpha_2) = 3.2\ (\text{Sin } 6° + \text{Sin } 6°)$

$= 3.2\ (0.1045 + 0.1045) = 0.7$ mm

Half-span spread $= \delta_{sm2}\ (\text{Sin}\alpha_1) = 3.2\ (\text{Sin } 6°)$

$= 3.2\ (0.1045) = 0.3$ mm

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2.4 Axial forces for the energy calculation

The total of the axial forces in the columns is not affected by second-order effects because of vertical equilibrium, so $P_2 = P_1$ which is taken as the mid-height value calculated in 2.2 above.

Column Forces

Span 1:

LH col: take mid-height P_2 = 127.1 kN

RH col: take mid-height P_2 = 287.4 kN (Sheet 7)

Span 2:

RH col: take mid-height P_2 = 127.7 kN

Rafter Forces

The rafter axial forces are affected by the drop of the rafters at mid-span.

Span 1:

Total mid-span drop = 845.7 mm

Increase in P_2 = $\{1/[1-(\delta_a/h_a)]-1\}$ = $\{1/[1-(845.7/11577)] - 1\}$

= 0.079

LH rafter: take mid-length P_1 = 52.9 kN

Mid-span axial = 46.0 (sheet 5), giving $P\Delta$ increase = 0.079×46.0 = 3.6 kN

P_2 = 52.9 + 3.6 = 56.5 kN

RH rafter: take mid-length P_1 = (62.0 + 48.2)/2 = 55.1 kN

Mid-span axial = 48.2(Sheet 5), giving $P\Delta$ increase = 0.079×48.2 = 3.8 kN

P_2 = 55.1 + 3.8 = 58.9 kN

Span 2:

Total mid-span drop = 831.5 mm

Increase in P_2 $= \{1/[1-(\delta_a/h_a)]-1\} = \{1/[1-(831.5/11577)] - 1\}$

$= 0.077$

LH rafter: take mid-length $P_1 = (62.0 + 48.3)/2 = 55.2$ kN

Mid-span axial = 48.3, giving $P\Delta$ increase = $0.077 \times 48.3 = 3.7$ kN

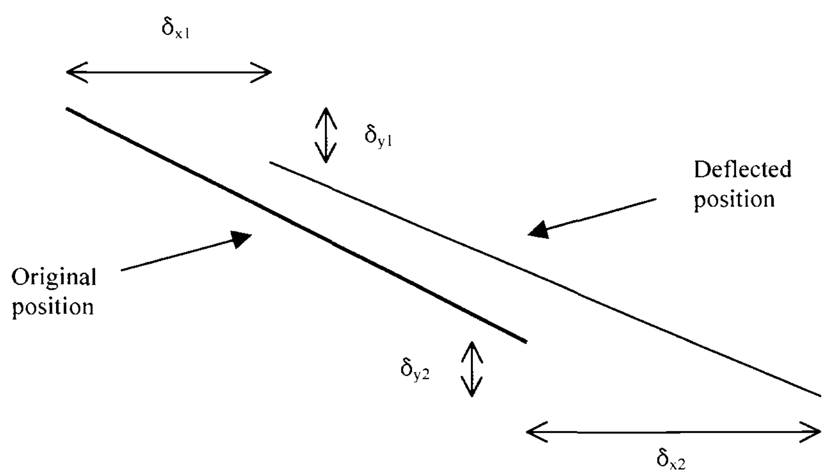
$P_2 = 55.2 + 3.7 = 58.9$ kN

RH rafter: take mid-length $P_1 = (60.1 + 46.3)/2 = 53.2$ kN

Mid-span axial = 46.3, giving $P\Delta$ increase = $0.077 \times 46.3 = 3.6$ kN

$P_2 = 53.2 + 3.6 = 56.8$ kN

2.5 Second-order Energy Summation



The following spreadsheet shows the second order energy summation.

Element for evaluation of Pc*phi*s*d(phi)	AB	BC	CD	ED	DF	FG	dG	Hd
X-AXIS DEFLECTIONS								
Deflections from the "elastic" frame								
dxa	0.0	-21.1	6.0	0.0	32.9	58.7	84.5	0.0
dxb	-21.1	6.0	32.9	32.9	58.7	84.5	84.5	84.5
(dxb - dxa)	**-21.1**	**27.1**	**27.0**	**32.9**	**25.7**	**25.9**	**0.0**	**84.5**
Deflections from the "plastic" frame								
From gravity loads								
Sway of top of elastic column								
dxa	0.0	608.7	608.7	0.0	608.7	608.7	608.7	0.0
dxb	608.7	608.7	608.7	608.7	608.7	608.7	608.7	608.7
(dxb - dxa)	**608.7**	**0.0**	**0.0**	**608.7**	**0.0**	**0.0**	**0.0**	**608.7**
Spread								
dxa	0.0	0.0	60.0	0.0	120.0	179.9	239.9	239.9
dxb	0.0	60.0	120.0	120.0	179.9	239.9	239.9	0.0
(dxb - dxa)	**0.0**	**60.0**	**60.0**	**120.0**	**60.0**	**60.0**	**0.0**	**-239.9**
Column hinge horizontal displacement								
dxa	0.0	0.0	0.0	0.0	0.0	0.0	36.5	0.0
dxb	0.0	0.0	0.0	0.0	0.0	0.0	0.0	36.5
(dxb - dxa)	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**-36.5**	**36.5**
From horizontal loads								
Sway								
dxa	0.0	14.7	14.7	0.0	14.7	14.7	14.7	0.0
dxb	14.7	14.7	14.7	14.7	14.7	14.7	14.7	14.7
(dxb - dxa)	**14.7**	**0.0**	**0.0**	**14.7**	**0.0**	**0.0**	**0.0**	**14.7**
Spread								
dxa	0.0	0.0	0.6	0.0	1.3	1.6	1.9	0.0
dxb	0.0	0.6	1.3	1.3	1.6	1.9	1.9	1.9
(dxb - dxa)	**0.0**	**0.6**	**0.6**	**1.3**	**0.3**	**0.3**	**0.0**	**1.9**
Total of (dxb-dxa) at collapse	**602.2**	**87.7**	**87.6**	**777.5**	**86.0**	**86.2**	**-36.5**	**506.4**
Y-AXIS DEFLECTIONS								
Deflections from the "elastic" frame								
dya	0.0	0.7	262.7	0.0	1.9	251.4	0.7	0.0
dyb	0.7	262.7	1.9	1.9	251.4	0.7	0.7	0.7
(dyb - dya)	**0.7**	**262.0**	**-260.8**	**1.9**	**249.5**	**-250.7**	**0.0**	**0.7**
Deflections from the "plastic" frame								
Mid-span drop from gravity loads								
dya	0.0	0.0	577.0	0.0	0.0	577.0	0.0	0.0
dyb	0.0	577.0	0.0	0.0	577.0	0.0	0.0	0.0
(dyb - dya)	**0.0**	**577.0**	**-577.0**	**0.0**	**577.0**	**-577.0**	**0.0**	**0.0**
Mid-span drop from horizontal loads								
dya	0.0	0.0	6.1	0.0	0.0	3.2	0.0	0.0
dyb	0.0	6.1	0.0	0.0	3.2	0.0	0.0	0.0
(dyb - dya)	**0.0**	**6.1**	**-6.1**	**0.0**	**3.2**	**-3.2**	**0.0**	**0.0**
Total of (dyb-dya) at collapse	**0.7**	**845.0**	**-843.8**	**1.9**	**829.6**	**-830.8**	**0.0**	**0.7**
SHORTENING								
psi (angle from X axis)	90.0	6.0	-6.0	90.0	6.0	-6.0	90.0	90.0
[(dxb - dxa) at collapse]*Sin(psi)	602.2	9.2	-9.2	777.5	9.0	-9.0	-36.5	506.4
[(dyb - dya) at collapse]*Cos(psi)	0.0	840.4	-839.1	0.0	825.0	-826.3	0.0	0.0
phi * s at collapse	602.2	849.5	-848.3	777.5	834.0	-835.3	-36.5	506.4
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.883	1.151	1.151	2.031	2.031	1.740
Shortening = phi*s*d(phi) (modulus)	**602.2**	**849.5**	**748.9**	**895.0**	**960.1**	**1696.7**	**74.2**	**881.3**
AXIAL FORCES								
Pc for columns and rafters at ULS	127.1	52.9	55.1	287.4	55.2	53.2	127.7	127.7
Total midspan drop		845.7	845.7		831.5	831.5		
Midspan height		11577	11577		11577	11577		
Increase rafter mispan axial by {1/(1-drop/height) - 1}		0.079	0.079		0.077	0.077		
Midspan axial		46.0	48.2		48.3	46.3		
Increase in rafter axial		3.6	3.8		3.7	3.6		
Design axial	127.1	56.5	58.9	287.4	58.9	56.8	127.7	127.7
Incremental energy = Pc*phi*s*d(phi)	**76.5**	**48.0**	**44.1**	**257.2**	**56.5**	**96.3**	**9.5**	**112.5**

Sum = 701

WORK DONE IN ROTATING HINGES								
Element for evaluating Mprd(phi)	AB	Ba	ab	bD	Dc	cG	dG	Hd
MprA	0.0	0.0	404.0	0.0	0.0	404.0	454.0	0.0
MprB	0.0	404.0	404.0	404.0	404.0	0.0	0.0	454.0
MprA + MprB	0.0	404.0	808.0	404.0	404.0	404.0	454.0	454.0
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.883	1.151	1.151	2.031	2.031	1.740
Mpr*d(phi)	**0.0**	**404.0**	**713.3**	**465.0**	**465.0**	**820.7**	**922.2**	**790.1**

Sum = 4580

Factor on lambda_p	0.847
lambda_p from first-order analysis	1.122
lambda_M	**0.950**

2.6 Load factor at failure, λ_m

$\Sigma P_2 \phi s d\phi \quad = 701 \phi$

$\Sigma M_{pr} d\phi \quad = 4580 \phi$

$$\lambda_M / \lambda_p = \left[1 - \left(\frac{\Sigma (P_2 \phi s d\phi)}{\Sigma (M_{pr} d\phi)}\right)\right]$$

A.2.2

$$\lambda_M / \lambda_p = \left[1 - \left(\frac{701 \phi}{4580 \phi}\right)\right] = 0.847$$

$\lambda_M = 0.847 \times \lambda_p = 0.847 \times 1.122 = 0.950$

$\lambda_M < 1.0$, so the frame has failed the check for in-plane stability.

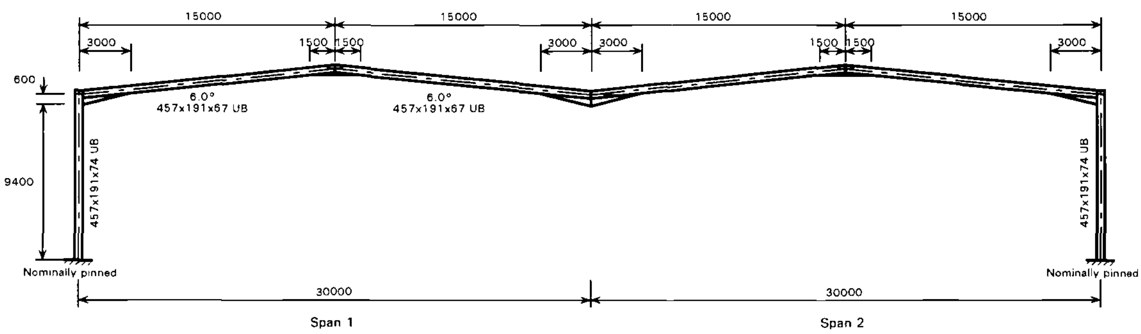
The above shows how the second-order effects have caused a major reduction in capacity of the frame due to in-plane instability effects. The "hand" method tends to be conservative, so analysis by another method might demonstrate that the reduction in capacity is not so great.

The analysis above would be less conservative if the stiffness of the haunches had been included in all the stiffness calculations. It would also be more economical if the frame were proportioned so that λ_l were closer to λ_p.

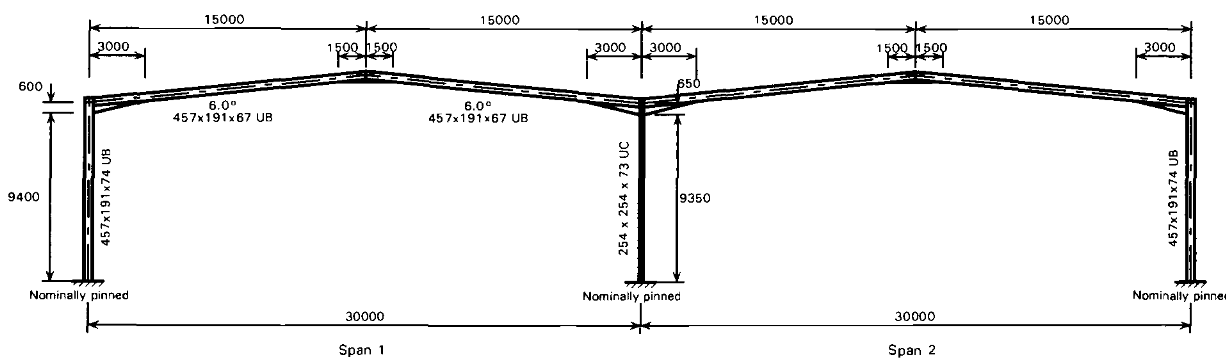
1. INPUT FROM FIRST-ORDER ANALYSIS

1.1 General Arrangement

Miss Frame



Hit Frame



Angle of rafters: $\alpha_1 = \alpha_2 = 6°$

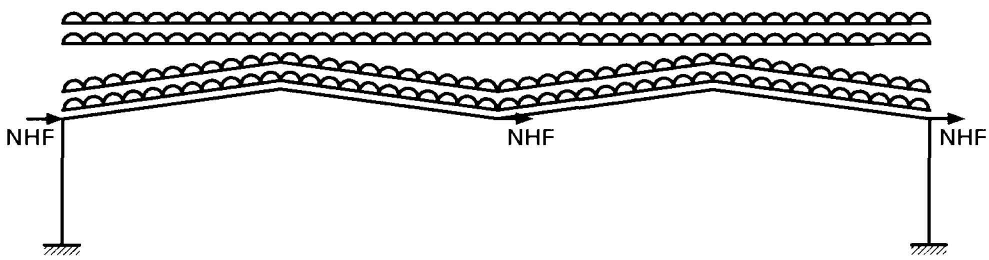
Spans = 30 m

Developed length of rafter = $\dfrac{30}{\cos 6°}$ = 30.165 m

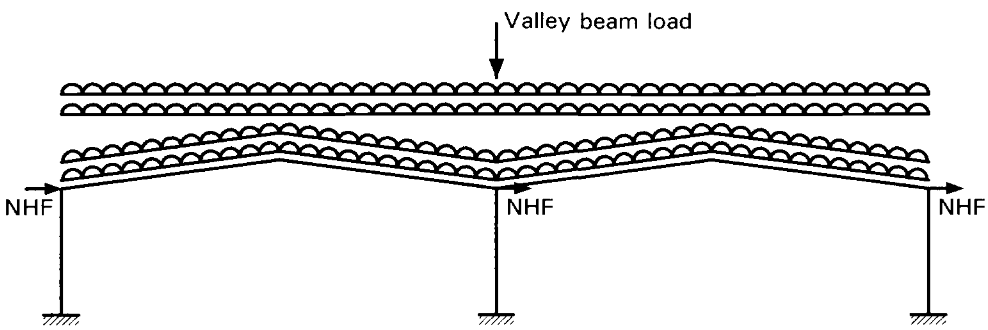
Height of column from base to Neutral Axis of rafter = 10.0 m

1.2 Loading

Miss Frame



Hit Frame



Loading

Dead	= 0.100 × 6.000 × 1.4	=	0.840 kN/m	along slope
Service	= 0.150 × 6.000 × 1.4	=	1.260 kN/m	on plan
Imposed	= 0.600 × 6.000 × 1.6	=	5.760 kN/m	on plan
Self Weight	= 80 × 10^{-2} × 1.000 × 1.4	=	1.120 kN/m	along slope

Valley beam factored load = 300 kN

MISS FRAME

1.3 INPUT FROM FIRST-ORDER ANALYSIS

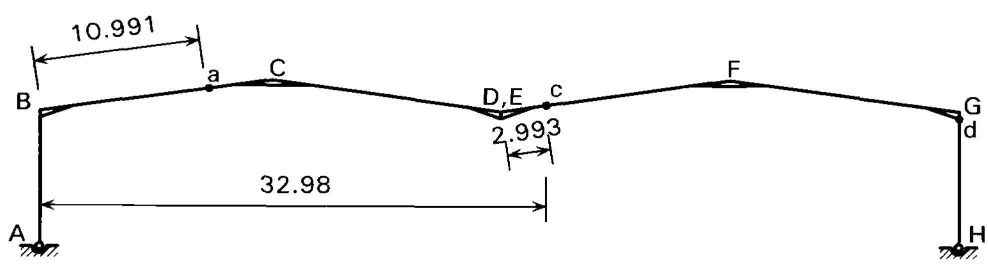
1.3.1 General Arrangement

The values of incremental rotation of the hinges are taken from the first-order collapse mechanism. These are the incremental rotations as used to calculate the collapse factor of the frame using the classic Rigid-Plastic (Virtual Work) method.

The second-order analysis uses the relative magnitude of the instantaneous rotations, so the absolute magnitude of each rotation does not affect the calculations.

Where the analysis has been performed by methods other than the Rigid-Plastic method (e.g. by the Semi-Graphical method), the incremental rotations can be deduced from the geometry of the frame and the position of the hinges. It is not necessary to repeat the calculation of the collapse factor by the Rigid-Plastic method.

Failure Mechanism for the Miss Frame



Find Node Locations

Pt A → (0.0, 0.0) = (0.0, 0.0)

Pt B → (0.0, 10.0) = (0.0, 10.0)

Pt C → (15.0, {10.0 + 15.0 Tan 6}) = (15.0, 11.577)

Pt D → (30.0, 10.0) = (30.0, 10.0)

Pt E → (30.0, 10.0) = (30.0, 10.0)

Pt F → ({30.0+15.0}, {10.0 + 15.0 Tan 6}) = (45.0, 11.577)

Pt G → (60.0, 10.0) = (60.0, 10.0)

Pt H → (60.0, 0.0) = (60.0, 0.0)

Pt a → (10.911 Cos 6, {10.0 + 10.911 Sin 6}) = (10.851, 11.141)

Pt c → ({30.0+2.993 Cos 6}, {10.0 + 2.993 Sin 6}) = (32.977, 10.313)

Pt d → (60.0, 9.400) = (60.0, 9.400)

Hinge Rotations

Taking the instantaneous rotation about A as θ

$$\theta_E / \theta_A = \frac{(X_a - X_A)}{(X_E - X_a)} = 0.567$$

$$\gamma = \tan^{-1}\left(\frac{Y_d - Y_c}{X_d - X_c}\right) = 1.935$$

$$\theta_{dc} / \theta_A = \frac{dY_c \, \text{Cos}(\gamma) + dX_c \, \text{Sin}(\gamma)}{cd \; \theta_A} = 0.077$$

$$\theta_{dH} / \theta_A = \frac{dX_d}{Y_{(d-H)} \; \theta_A} = 1.229$$

1.4 Axial forces at ULS from first-order analysis

Span 1

LH column: at base = 132.0 kN, at haunch = 122.4 kN

LH rafter: at column = 60.0 kN, at apex = 46.2 kN

RH rafter: at column = 62.1 kN, at apex = 48.4 kN

Span 2

LH rafter: at column = 62.1 kN, at apex = 48.3 kN

RH rafter: at column = 60.1 kN, at apex = 46.3 kN

RH column: at base = 132.5 kN, at haunch = 122.9 kN

Notional Horizontal Forces

Cl 2.4.2.4

Span 1:

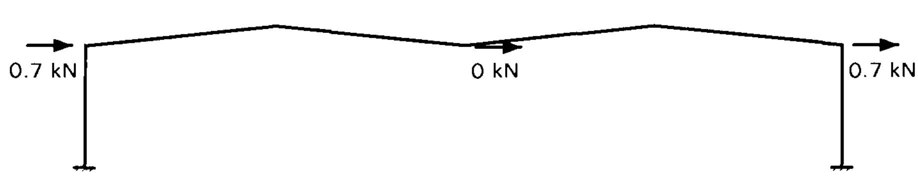
External Column

0.5% × 127.2 = 0.636 kN

Span 2:

External Column

0.5% × 127.7 = 0.638 kN



1.5 Reduced Plastic Moments at ULS from first-order analysis

Use the reduced moment capacity for the sections to account for the co-existent axial force, calculated in accordance with BS5950-1:2000 Annex I.2. These may be taken from section tables.

$M_{rx} = p_y S_{rx}$

For the axial forces in this frame under this load case

M_{pr} rafters = 404 kNm

M_{pc} external columns = 454 kNm

1.6 Load factor at formation of the first hinge, λ_1

$\lambda_1 = 0.960$ (from the frame analysis output)

1.7 Plastic collapse factor, λ_p

$\lambda_p = 1.12$ (from the frame analysis output)

1.8 Member inertias, I_x

External columns: 457×191 × 74 UB: $I_x = 33320\ cm^4$

Rafters: 457×191 × 67 UB: $I_x = 29380\ cm^4$

1.9 Deflections of frame at λ_1 (formation of the first hinge)

$\delta_{xB} = -35.7$ mm $\delta_{yB} = 0.6$ mm

$\delta_{xC} = -11.3$ mm $\delta_{yC} = 236.7$ mm

$\delta_{xD} = 13.1$ mm $\delta_{yD} = 3.4$ mm

$\delta_{xF} = 36.4$ mm $\delta_{yF} = 225.4$ mm

$\delta_{xG} = 59.6$ mm $\delta_{yG} = 0.6$ mm

2. SECOND-ORDER ANALYSIS

2.1 Axial forces in members

Span 1

LH col: take mid-height P_{ULS} = (132.0 + 122.4)/2 = 127.2 kN

LH rafter: take mid-length P_{ULS} = (60.0 + 46.2)/2 = 53.1 kN

RH rafter: take mid-length P_{ULS} = (62.1 + 48.4)/2 = 55.3 kN

Span 2

LH rafter: take mid-length P_{ULS} = (62.1 + 48.3)/2 = 55.2 kN

RH rafter: take mid-length P_{ULS} = (60.1 + 46.3)/2 = 53.2 kN

RH col: take mid-height P_{ULS} = (132.5 + 122.9)/2 = 127.7 kN

2.2 Bending Deflections of the "elastic" frame

A.3.2

2.2.1 Stiffness reduction factors allowing fore P.δ effects

Columns

The stiffness of each external column differs from the stiffness of the internal column. Therefore the reduction in frame stiffness is calculated from the sum of the ULS loads in the columns and the sum of the critical loads of the columns.

Sum of columns P_{ULS}, ΣP_{ULS} = 127.2 + 127.7 = 254.9 kN

External columns: I_x = 33320 cm^4, h = 10000 mm, α = 1.7

$P_{cr} = \pi^2 EI/(\alpha h)^2 = \pi^2 \times 205000 \times 33320 \times 10^4/(1.7 \times 10000)^2 = 2333$ kN

Sum of columns P_{cr}, ΣP_{cr} = 2333 + 2333 = 4666 kN

$(1 - \Sigma P_{ULS}/\Sigma P_{cr}) = (1 - 254.9/4666) = 0.945$

$1/(1 - \Sigma P_{ULS}/\Sigma P_{cr}) = 1.058$

Rafters

Span 1:

Average P_{ULS} = (53.1 + 55.3)/2 = 54.2 kN

I_x = 29380 cm^4, L = 30165mm, α = 1.0

$P_{cr} = \pi^2 EI/(\alpha L)^2 = \pi^2 \times 205000 \times 29380 \times 10^4/(1.0 \times 30165)^2 = 653$ kN

$(1 - P_{ULS}/P_{cr}) = (1 - 54.2/653) = (1 - 0.083) = 0.917$

Span 2:

Average P_{ULS} = (55.2 + 53.2)/2 = 54.2 kN

I_x = 29380 cm^4, L = 30165mm, α = 1.0

$P_{cr} = \pi^2 EI/(\alpha L)^2 = \pi^2 \times 205000 \times 29380 \times 10^4/(1.0 \times 30165)^2 = 653$ kN

$(1 - P_{ULS}/P_{cr}) = (1 - 54.2/653) = (1 - 0.083) = 0.917$

2.2.2 Second order magnification factors

The notional sway deflection is calculated from the sum of the stiffnesses *K* for each of the column and rafter pairs:

Sway stiffness from column and rafter stiffness

$$K = \frac{H}{\delta} = \frac{1}{\left(\dfrac{Sh^2}{3EI_{eff,R}} + \dfrac{h^3}{3EI_{eff,c}}\right)}$$ D.2.5

Sub Frame 1 (Elastic)

$$\frac{Sh^2}{3EI_{eff,R}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26944 \times 10^4} = 0.01820$$

$$\frac{h^3}{3EI_{eff,c}} = \frac{10000^3}{3 \times 205000 \times 31503 \times 10^4} = 0.00516$$

$$K_{Sub1} = \frac{1}{0.01820 + 0.00516} = 42.8 \text{ N/mm}$$

Sub Frame 2 (Elastic)

$$\frac{Sh^2}{3EI_{eff,R}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26942 \times 10^4} = 0.01821$$

$$\frac{h^3}{3EI_{eff,c}} = \frac{10000^3}{3 \times 205000 \times 31505 \times 10^4} = 0.00516$$

$$K_{Sub2} = \frac{1}{0.01821 + 0.00516} = 42.8 \text{ N/mm}$$

Nominal Base fixity D.4

$$K_b = \frac{H}{\delta} = \frac{1}{\left(\frac{5h^3}{2EI} + \frac{h^3}{3EI_{eff}}\right)}$$

External Column

$$K_{b\ ext} = \frac{1}{(0.0366 + 0.00516)} = 23.95 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K = K_{sub\ 1} + K_{b\ ext} + K_{sub\ 2} + K_{b\ int} + K_{b\ ext}$$

$$\Sigma K = 42.8 + 23.95 + 42.8 + 23.95 = 133.5 \text{ N/mm}$$

Second -order notional sway deflection A.2.5

$$\delta_{n2} = \frac{H}{\Sigma K_2} = \frac{1.274 \times 1000}{133.5} = 9.55 \text{ mm}$$

Critical Buckling Ratio

$$\lambda_{cr1} = \frac{h}{200\,\delta_{n2}} = \frac{10000}{200 \times 9.55} = 5.2$$

Sway mode magnification

$$\left[\frac{\lambda_{cr1}}{\lambda_{cr1} - 1}\right] = 1.24$$

2.2.3 Deflection calculations

Sway Deflections A.3.4

The first-order sway deflection δX_{1s} is calculated from the sum of the stiffnesses K for each of the column and rafter pairs. (The base stiffness of nominally pinned bases is not included because this is not a stability calculation like the calculation of λ_{cr}).

$$K = \frac{H}{\delta} = \frac{1}{\left(\dfrac{Sh^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)}$$

D.2.3

Sub Frame 1 (Elastic)

$$\frac{Sh^2}{3EI_R} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 29380 \times 10^4} = 0.01669$$

$$\frac{h^3}{3EI_c} = \frac{10000^3}{3 \times 205000 \times 33320 \times 10^4} = 0.00488$$

$$K_{Sub1} = \frac{1}{0.01669 + 0.00488} = 46.4 \text{ N/mm}$$

Sub Frame 2 (Elastic)

As Sub frame 1

$$K_{Sub2} = \frac{1}{0.01669 + 0.00488} = 46.4 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K = K_{sub\,1} + K_{sub\,2}$$

$$\Sigma K = 46.4 + 46.4 = 92.7 \text{ N/mm}$$

First-order sway deflection

$$\delta X_{1s} = \frac{\lambda_l H}{\Sigma K_2} = \frac{0.960 \times 1.275 \times 1000}{92.7} = 13.20 \text{ mm}$$

$$\left[\frac{\lambda_{cr2}}{\lambda_{cr2} - 1}\right] = 1.09$$

A.3.4

$$\delta X_2 = (\delta X_1 - \delta X_{1s}) \times \left[\frac{\lambda_{cr2}}{\lambda_{cr2} - 1}\right] + \delta X_{1s} \times \left[\frac{\lambda_{cr1}}{\lambda_{cr1} - 1}\right]$$

$$\delta Y_2 = \delta Y_1 \times \left[\frac{\lambda_{cr2}}{\lambda_{cr2} - 1}\right]$$

Values of δX_1 and δY_1 are taken from first order analysis (See Sheet 7).

δ_{xB} = (−35.7 − 13.19) × 1.09 + 13.19 × 1.24 = −37.0 mm
δ_{yB} = 0.6 × 1.09 = 0.7 mm

δ_{xC} = (−11.3 − 13.19) × 1.09 + 13.19 × 1.24 = −10.4 mm
δ_{yC} = 236.7 × 1.09 = 258.1 mm

δ_{xD} = (13.1 − 13.19) × 1.09 + 13.19 × 1.24 = 16.2 mm
δ_{yD} = 3.4 × 1.09 = 3.7 mm

δ_{xF} = (38.3 − 13.19) × 1.09 + 13.19 × 1.24 = 41.6 mm
δ_{yF} = 225.4 × 1.09 = 245.8 mm

δ_{xG} = (59.6 − 13.19) × 1.09 + 13.19 × 1.24 = 66.9 mm
δ_{yG} = 0.6 × 1.09 = 0.7 mm

Interpolation of deflections at hinge 'c'

Across span ratio $\alpha = \dfrac{x}{S} = \dfrac{2977}{30165} = 0.099$

Ratio of hinge deflection to maximum deflection

$$\frac{y\alpha}{y\beta} = 3.2\left(\alpha^4 - 2\alpha^3 + \alpha\right) = 0.312$$ E.1

$\delta_{XF} = 41.6$ mm $\qquad \delta_{YF} = 245.8$ mm

$\delta_{XD} = 16.2$ mm $\qquad \delta_{YD} = 3.7$ mm

$$\delta_{Xc} = \delta_{XD} + \frac{\alpha L}{\beta L}\left(\delta_{XF} - \delta_{XD}\right) = 19.5 \text{ mm}$$ E.2

$$\delta_{Yc} = \delta_{YD} + \frac{y\alpha}{y\beta}\left(\delta_{YF} - \delta_{YD}\right) = 79.1 \text{ mm}$$ E.1

2.3 Bending deflections of the plastic frame

2.3.1 Stiffness reduction factors to allow for P.δ effects

A.4.2

Columns: as the "elastic" frame

LH Column: $(1 - P/P_{cr}) = 0.945$

Rafters: as the "elastic" frame because that used $\alpha = 1.0$

Span 1: $(1 - P/P_{cr}) = 0.917$

Span 2: $(1 - P/P_{cr}) = 0.917$

2.3.2 Second Order Magnification Factor

Sway mode magnification factor

The notional sway deflection is calculated from the sum of the stiffnesses K of the rafter and column pairs between plastic hinges and the base stiffness of each column.

Sway stiffness from column and rafter stiffness.

$$K_2 = \frac{1}{\left(\dfrac{Sh^2}{3EI_{eff,R}} + \dfrac{h^3}{3EI_{eff,c}}\right)}$$

D.3.5

$$I_{eff.R} = I_R\,(1 - P/P_{cr}) = 26944 \text{ cm}^4$$

A.4.2

$$I_{eff.c} = I_c\,(1 - P/P_{cr}) = 31503 \text{ cm}^4$$

Sub Frame 1 (Plastic)

$$\frac{Sh^2}{3EI_{eff,R}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26944 \times 10^4} = 0.01820$$

$$\frac{h^3}{3EI_{eff,c}} = \frac{10000^3}{3 \times 205000 \times 31503 \times 10^4} = 0.00516$$

$$K_{sub\ 1} = \frac{1}{0.01820 + 0.00516} = 42.8 \text{ N/mm}$$

Nominal Base fixity D.4

$$K_b = \frac{H}{\delta} = \frac{1}{\left(\frac{5h^3}{2EI} + \frac{h^3}{3EI_{eff}}\right)}$$

External Column

$$K_{b\ ext} = \frac{1}{(0.0366 + 0.00516)} = 23.95 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K_2 = K_{sub\ 1} + K_{b\ ext} + K_{b\ ext}$$

$$\Sigma K_2 = 42.8 + 23.95 + 23.95 = 90.7 \text{ N/mm}$$

Second-order notional sway deflection for the plastic frame

$$\delta_{np} = \frac{H}{\Sigma K_2} = \frac{1.274 \times 1000}{90.7} = 14.05 \text{ mm}$$

Critical Buckling Ratio D.3.5

$$\lambda_{crp} = \frac{h}{200\delta_{np}} = \frac{10000}{200 \times 14.05} = 3.56$$

Magnification Factor

$$\left[\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right] = 1.391$$

2.3.3 Deflections of the "plastic" frame from gravity loads

Loads

The loads applied to the "plastic" frame $= (\lambda_p - \lambda_l)$(loads at ULS)

$(\lambda_p - \lambda_l) = (1.12 - 0.96) = 0.16$

Slope of rafters $\alpha_1 = \alpha_2 = \alpha = 6°$, giving $\cos\alpha = 0.9945$

Assuming both service load and imposed load are specified "on plan", at ULS, $w_{v.plan} = 1.26 + 5.76 \quad = 7.02$ kN/m

giving a transverse load on the "plastic frame"

$= (\lambda_p - \lambda_1)(w_{v.plan} \text{ at ULS})\text{Cos}^2\alpha$

$= 0.16 \times 7.02(0.9945)^2 \; = 1.11$ kN/m

Assuming both dead load and self-weight are values "along the slope",

at ULS, $w_{v.slope} \quad = 0.84 + 1.12 \quad = 1.96$ kN/m

giving a transverse load on the "plastic frame"

$= (\lambda_p - \lambda_1)(w_{v.slope} \text{ at ULS})\text{Cos}\alpha$

$= 0.16 \times 1.96 \times 0.9945 \quad = 0.31$ kN/m

Summing loads from components "on plan" and "along the slope",

$w_P \quad = 1.11 + 0.31 \quad = 1.42$ kN/m

Sway A.4.5

This arises due to the rotation of the column without an adjacent hinge.

First-order end slope of the rafter as a simply supported beam, $\theta_R = \dfrac{w_P\, S^3}{24\, EI_R}$

Second-order end slope of the rafter, $\theta_{R2} = \dfrac{w_P\, S^3}{24\, EI_R}\left(\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}\right)$

$E = 205000$ N/mm^2

$I_x = 29380$ cm^4

$$\theta_{R2} = \frac{1.42 \times 30165^3}{24 \times 205000 \times 29380 \times 10^4} \times 1.391 = 0.03472 \text{ radians}$$

Horizontal deflection of Point B, C, D $= h\theta_R$

$= 10000 \times 0.03472$

$= 347.2$ mm

Mid-span drop

Deflection given by value for simply supported beam of span equal to the developed length of the rafters.

Span 1:

$$\delta_{b2} = \frac{5}{384}\frac{w_P\, S^4}{E\, I_R}\left[\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right]$$

$$\delta_{b2} = \frac{5}{384} \times \frac{1.42 \times 30165^4}{205000 \times 29380 \times 10^4} \times 1.391 = 354.3 \text{ mm}$$

$$\delta_{apex} = \delta_{apex} = \frac{\delta_{B2}}{\cos\,\alpha} = \frac{354.3}{0.9945} = 356.2 \text{ mm}$$

Span 2: as Span 1

Hinge Drop From interpolation

$$\delta Y = \frac{Y\alpha}{Y\beta}\,\delta_{Apex} = 0.312 \times 356.2 = 111.0 \text{ mm}$$

Spread

This is caused by the drop of the angle in the rafter which is at the apex in this frame. Because this frame is a symmetrical pitched roof portal, the drop of the angle is the mid-span drop calculated above.

Span 1:

Full span $\delta_{spread} = \delta_{B2}\,(\text{Sin}\alpha_1 + \text{Sin}\alpha_2) = \delta_{B2}\,(2\text{Sin } 6°)$

$= 354.3\ (2 \times 0.1045) = 74.1$ mm

Half span $\delta_{spread} = \delta_{B2}\ \text{Sin}\ \alpha_1 = 354.3 \times 0.1045 = 37.0$ mm

Span 2: as Span 1

Spread at hinge from Interpolation

$$\delta X = \delta_{\text{SpreadSpan}1} + \frac{y\alpha}{y\beta}\delta_{\text{SpreadSpan}2} = 74.1 + (0.312 \times 74.1) = 97.1 \text{ mm}$$

Column hinge horizontal displacement

The hinge occurs at the underside of the haunch, which is at a distance from the neutral axis of the rafter, causing an additional horizontal displacement.

Second-order end slope of the rafter, $\theta_{R2} = 0.03472$ radians (Sheet 14)

Off-set of the hinge below the rafter, $e = 10000 - 9400 = 600$ mm

Horizontal deflection of Point M $= e\theta_R$

$= 600 \times 0.03472$

$= 20.8$ mm

2.3.4 Deflections of the "plastic" frame from horizontal loads

Loads

The unfactored loads applied to the "elastic frame" included the horizontal loads and were in proportion to the ULS loads, so the loads applied to the "plastic" frame $= (\lambda_p - \lambda_l)$(loads at ULS)

$(\lambda_p - \lambda_l) = (1.12 - 0.96) = 0.16$

Slope of rafters $\alpha_1 = \alpha_2 = \alpha = 6°$, giving $\cos\alpha = 0.9945$

Sway

Total of column forces = 254.9 kN (Sheet 7)

0.5% × of total of column forces $= 0.005 \times 254.9 = 1.27$ kN (Sheet 5)

Additional horizontal load on plastic frame

$H = 0.16 \times 1.27 = 0.204$ kN $= 204$ N

For a multi-span frame, the sway deflection is calculated from the sum of the stiffnesses K for each of the spans:

$$K_s = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} \times \frac{1}{\left[\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}\right]}$$

A.4.6

$$K_{sub\ 1} = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} = \frac{1}{0.0167 + 0.0052} = 45.8$$

Total Frame Stiffness

$\Sigma K_s = K_{sub\ 1}$

$\Sigma K_s = 45.8$ N/mm

Second-order least sway deflection of the column top

$$\Delta_2 = \frac{H}{\Sigma K_s} = \frac{0.204 \times 1000}{45.8} \times 1.391 = 6.2 \text{ mm}$$

Mid-span drop

Second order sagging deflection of a straight rafter:

$$\delta_{sm2} = \frac{ML_r^2}{16EI_R}\left[\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right] \quad \text{where } M = \Sigma\, H_i\, h_i$$

A.4.6

H_i is the proportion of the horizontal force carried by each sub frame calculated as a proportion of the stiffness K_2.

Sub frame 1:

Drop in the apex of rafter span 1 due to horizontal load.

$$\therefore \quad \delta_{sm2} = \frac{0.204 \times 1000 \times 10000\ (30165)^2}{16 \times 205000 \times 29380 \times 10^4} \times 1.391 = 2.7$$

Spread

Span 1:

Full span spread $= \delta_{sm2}\,(\text{Sin}\alpha_1 + \text{Sin}\alpha_2) \quad = 2.7\,(\text{Sin } 6° + \text{Sin } 6°)$

$= 2.7\,(0.1045 + 0.1045) \quad = 0.6$ mm

Half span spread $= \delta_{sm2}\,(\text{Sin}\alpha_1) \quad = 2.7\,(\text{Sin } 6°)$

$= 2.7\,(0.1045) \quad = 0.3$ mm

2.4 Axial forces for the energy calculation

A.5

The total of the axial loads in the columns is not affected by second-order effects because of vertical equilibrium, so $P_2 = P_1$ which is taken as the mid-height value calculated in 2.2 above.

Span 1:

LH col: take mid-height $P_1 \quad = (132.0 + 122.4)/2 \quad = 127.2$ kN

Span 2:

RH col: take mid-height $P_1 \quad = (132.5 + 122.9)/2 \quad = 127.7$ kN

The rafter axial forces are affected by the drop of the rafters at mid-span.

Span 1:

Total Mid-span drop = 617.0 mm

Increase in $P_2 = \{1/[1-(\delta_a/h_a)]-1\} \quad = \{1/[1-(617.0/11577)] - 1\}$

$= 0.056$

LH rafter: take mid-length $P_1 \quad = (60.0 + 46.2)/2 \quad = 53.1$ kN

Mid-span axial = 46.2, giving $P\Delta$ increase $= 0.056 \times 46.2 \quad = 2.6$ kN

$P_2 = 53.1 + 2.6 = 55.7$ kN

RH rafter: take mid-length P_1 = (62.1 + 48.4)/2 = 55.3 kN

Mid-span axial = 48.4, giving $P\Delta$ increase = 0.056×48.4 = 2.7 kN

P_2 = 55.3 + 2.7 = 58.0 kN

Span 2:

Total Mid-span drop = 602.0 mm

Increase in P_2 = $\{1/[1-(\delta_a/h_a)]-1\}$ = {1/[1-(602.0 /11577)] - 1} = 0.055

LH rafter: take mid-length P_1 = (62.1 + 48.3)/2 = 55.2 kN

Mid-span axial = 48.3, giving $P\Delta$ increase = 0.055×48.3 = 2.6 kN

P_2 = 55.2 + 2.6 = 57.8 kN

RH rafter: take mid-length P_1 = (60.1 + 46.3)/2 = 53.2 kN

Mid-span axial = 46.3, giving $P\Delta$ increase = 0.055×46.3 = 2.5 kN

P_2 = 53.2 + 2.5 = 55.7 kN

2.5 Second-order Energy Summation

A.6

The energy summation is required to calculate λ_M following the methods in A.2.2

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CALCULATION SHEET

Element for evaluation of Pc*phi*s*d(phi)	AB	BC	CD	ED	Dc	cG	dG	Hd		
X-AXIS DEFLECTIONS				E=D						
Deflections from the "elastic" frame										
dxa	0.0	-37.0	-10.4		16.2	19.5	66.9	0.0		
dxb	-37.0	-10.4	16.2		19.5	66.9	66.9	66.9		
(dxb - dxa)	**-37.0**	**26.6**	**26.6**		**3.3**	**47.4**	**0.0**	**66.9**		
Deflections from the "plastic" frame										
From gravity loads										
Sway of top of elastic column										
dxa	0.0	347.2	347.2		347.2	347.2	347.2	0.0		
dxb	347.2	347.2	347.2		347.2	347.2	347.2	347.2		
(dxb - dxa)	**347.2**	**0.0**	**0.0**		**0.0**	**0.0**	**0.0**	**347.2**		
Spread										
dxa	0.0	0.0	37.0		74.1	97.1	148.1	0.0		
dxb	0.0	37.0	74.1		97.1	148.1	148.1	148.1		
(dxb - dxa)	**0.0**	**37.0**	**37.0**		**23.1**	**51.0**	**0.0**	**148.1**		
Column hinge horizontal displacement										
dxa	0.0	0.0	0.0		0.0	0.0	20.8	0.0		
dxb	0.0	0.0	0.0		0.0	0.0	0.0	20.8		
(dxb - dxa)	**0.0**	**0.0**	**0.0**		**0.0**	**0.0**	**-20.8**	**20.8**		
From horizontal loads										
Sway										
dxa	0.0	6.2	6.2		6.2	6.2	6.2	0.0		
dxb	6.2	6.2	6.2		6.2	6.2	6.2	6.2		
(dxb - dxa)	**6.2**	**0.0**	**0.0**		**0.0**	**0.0**	**0.0**	**6.2**		
Spread										
dxa	0.0	0.0	0.3		0.6	0.6	0.6	0.0		
dxb	0.0	0.3	0.6		0.6	0.6	0.6	0.6		
(dxb - dxa)	**0.0**	**0.3**	**0.3**		**0.0**	**0.0**	**0.0**	**0.6**		
Total of (dxb - dxa) at collapse	**316.4**	**63.9**	**63.9**		**26.4**	**98.4**	**-20.8**	**589.8**		
Y-AXIS DEFLECTIONS										
Deflections from the "elastic" frame										
dya	0.0	0.7	258.1		3.7	79.1	0.7	0.0		
dyb	0.7	258.1	3.7		79.1	0.7	0.7	0.7		
(dyb - dya)	**0.7**	**257.5**	**-254.4**		**75.4**	**-78.5**	**0.0**	**0.7**		
Deflections from the "plastic" frame										
Mid-span drop from gravity loads										
dya	0.0	0.0	356.2		0.0	111.0	0.0	0.0		
dyb	0.0	356.2	0.0		111.0	0.0	0.0	0.0		
(dyb - dya)	**0.0**	**356.2**	**-356.2**		**111.0**	**-111.0**	**0.0**	**0.0**		
Deflections from the "plastic" frame										
Mid-span drop from horizontal loads										
dya	0.0	0.0	2.7		0.0	0.0	0.0	0.0		
dyb	0.0	2.7	0.0		0.0	0.0	0.0	0.0		
(dyb - dya)	**0.0**	**2.7**	**-2.7**		**0.0**	**0.0**	**0.0**	**0.0**		
Total of (dyb - dya) at collapse	**0.7**	**616.4**	**-613.3**		**186.4**	**-189.5**	**0.0**	**0.7**		
psi (angle from X axis)	90.0	6.0	-6.0		6.0	-6.0	90.0	90.0		
[(dxb - dxa) at collapse]*Sin(psi)	316.4	6.7	-6.7		2.8	-10.3	-20.8	589.8		
[(dyb - dya) at collapse]*Cos(psi)	0.0	613.0	-609.9		185.4	-188.4	0.0	0.0		
phi * s at collapse	316.4	619.7	-616.6		188.1	-198.7	-20.8	589.8		
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.567		0.567	0.077	0.077	1.229		
Shortening = phi*s*d(phi) (modulus)	**316.4**	**619.7**	**349.4**		**106.6**	**15.3**	**1.6**	**724.9**		
AXIAL FORCES										
Pc for columns and rafters at ULS	127.2	53.1	55.3		55.2	53.2	127.7	127.7		
Total midspan drop		617.0	617.0		602.0	602.0				
Midspan height		11577	11577		11577	11577				
Increase rafter mispan axial by {1/(1-drop/height) - 1}		0.056	0.056		0.055	0.055				
Midspan axial		46.2	48.4		48.3	46.3				
Increase in rafter axial		2.6	2.7		2.6	2.5				
Design axial	127.2	55.7	58.0		57.8	55.7	127.7	127.7		
Incremental energy = Pc*phi*s*d(phi)	**40.2**	**34.5**	**20.3**		**6.2**	**0.9**	**0.2**	**92.6**	**Sum =**	**195**
WORK DONE ON ROTATING HINGES										
Element for evaluating Mprd(phi)	AB	Ba	ab	bD	Dc	cG	dG	Hd		
MprA	0.0	0.0	404.0		0.0	404.0	454.0	0.0		
MprB	0.0	404.0	404.0		404.0	0.0	0.0	454.0		
MprA + MprB	0.0	404.0	808.0		404.0	404.0	454.0	454.0		
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.567		0.567	0.077	0.077	1.229		
Mpr*d(phi)	**0.0**	**404.0**	**457.9**		**228.9**	**31.0**	**34.9**	**558.0**	**Sum =**	**1715**

Factor on lambda_p	0.886
lambda_p from first-order analysis	1.120
lambda_M	**0.993**

2.6 Load factor at failure, λ_M

A.6

$\Sigma\ P_2\ \phi s d\phi\ = 195\ \phi$

$\Sigma\ M_{pr}\ d\phi\ = 1715\ \phi$

$$\lambda_M\ /\ \lambda_p\ = \left[1 - \left(\frac{\Sigma\ (P_2\ \phi\ s\ d\phi)}{\Sigma\ (M_{pr}\ d\phi)}\right)\right]$$

$$\lambda_M\ /\ \lambda_p\ = \left[1 - \left(\frac{195\ \phi}{1715\ \phi}\right)\right] = 0.886$$

$\lambda_M = \quad 0.886 \times \lambda_p \quad = 0.886 \times 1.120 \quad = 0.993$

The above shows how the second-order effects have caused a major reduction in capacity of the frame due to in-plane instability effects. The "hand" method tends to be conservative, so analysis by another method might demonstrate that the reduction in capacity is not so great.

The analysis above would be less conservative if the stiffness of the haunches had been included in all the stiffness calculations. It would also be more economical if the frame were proportioned so that λ_1 were closer to λ_p.

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CALCULATION SHEET

HIT FRAME

As geometry of the hit frame is similar to the two-span portal in the previous worked example some of the checks are not explicitly shown in this example.

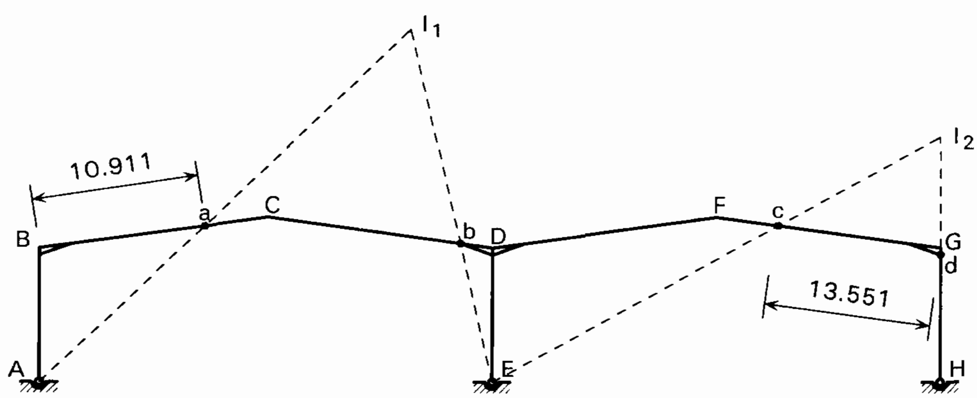
1.3 Hinge Incremental Rotations

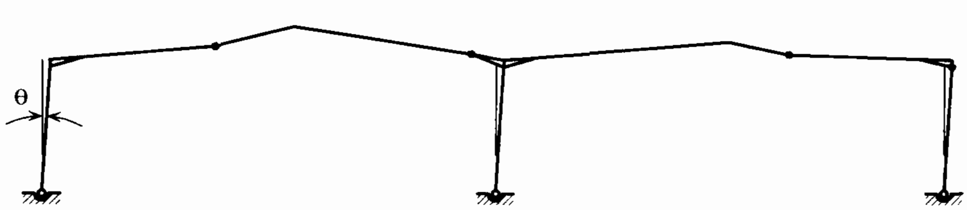
The values of incremental rotation of the hinges are taken from the first-order collapse mechanism. These are the incremental rotations as used to calculate the collapse factor of the frame using the classic Rigid-Plastic (Virtual Work) method.

The second-order analysis uses the relative magnitude of the instantaneous rotations, so the absolute magnitude of each rotation does not affect the calculations.

Where the analysis has been performed by methods other than the Rigid-Plastic method (e.g. by the Semi-Graphical method), the incremental rotations can be deduced from the geometry of the frame and the position of the hinges. It is not necessary to repeat the calculation of the collapse factor by the Rigid-Plastic method.

Failure Mechanism for Hit Frame





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CALCULATION SHEET

Job No:	CDS139	Page	23 of 41	Rev	A
Job Title	BS 5950 Portal				
Subject	Second-order Worked Example: Two-span Portal with Hit/Miss Internal Columns.				
Client	DETR	Made by	CMK	Date	May 2001
		Checked by	WIS	Date	July 2001

Find Node Locations

Pt A → (0.0, 0.0) = (0.0, 0.0)

Pt B → (0.0, 10.0) = (0.0, 10.0)

Pt C → (15.0, {10.0 + 15.0Tan6}) = (15.0, 11.577)

Pt D → (30.0, 10.0) = (30.0, 10.0)

Pt E → (30.0, 0.0) = (30.0, 0.0)

Pt F → ({30.0+15.0}, {10.0 + 15.0Tan6}) = (45.0, 11.577)

Pt G → (60.0, 10.0) = (60.0, 10.0)

Pt H → (60.0, 0.0) = (60.0, 0.0)

Pt a → (10.911Cos6, {10.0 + 10.911Sin6}) = (10.851, 11.141)

Pt b → ({30.0-2.993Cos6}, {10.0 + 2.993Sin6}) = (27.023, 10.313)

Pt c → ({60.0-13.551Cos6}, {10.0 + 13.551Sin6})= (46.523, 11.416)

Pt d → (60.0, 9.400) = (60.0, 9.400)

Find Centre of Rotation I_1

$$Y_{I1} = \frac{X_E - X_A}{\left(\frac{dX}{dY}\right)_{Aa} - \left(\frac{dX}{dY}\right)_{Eb}} = \frac{30.0 - 0.0}{(0.974) - (-0.289)} = 23.759 \text{ m}$$

$$X_{I1} = X_A + \left[\left(\frac{dX}{dY}\right)_{Aa} \times Y_{I1}\right] = 0.0 + (0.97397 \times 23.760) = 23.142\text{m}$$

Pt I → (23.142, 23.759)

Find Centre of Rotation I_2

$$Y_{I2} = \frac{X_H - X_E}{\left(\frac{dX}{dY}\right)_{Ec} - \left(\frac{dX}{dY}\right)_{Hd}} = \frac{60.0 - 30.0}{(1.447) - (0.0)} = 20.728 \text{ m}$$

$$X_{I2} = X_E + \left[\left(\frac{dX}{dY}\right)_{Ec} \times Y_{I2}\right] = 30.0 + (1.447 \times 20.728) = 60.0 \text{ m}$$

Note that $X_{I2} = 60.0$ is obvious without calculation!

Pt I → (20.728, 60.0)

Hinge Rotations

Taking the instantaneous rotation about A as θ

$$\theta_{I1} = \theta \times \frac{Y_a}{Y_{I1} - Y_a} = \theta \times \frac{11.141}{23.759 - 11.141} = 0.883\,\theta$$

$$\theta_E = \theta_{I1} \times \frac{Y_{I1} - Y_b}{Y_b} = 0.883\,\theta \times \frac{23.759 - 10.313}{10.313} = 1.151\,\theta$$

$$\theta_{I2} = \theta_E \times \frac{Y_c}{Y_{I2} - Y_c} = 1.151\,\theta \times \frac{11.141}{20.728 - 11.141}$$

$$= 1.151\,\theta \times 1.162 = 1.411\,\theta$$

$$\theta_H = \theta_{I2} \times \frac{Y_{I2} - Y_d}{Y_d} = 1.411\,\theta \times \frac{20.728 - 9.400}{9.400}$$

$$= 1.411\,\theta \times 1.205 = 1.701\,\theta$$

1.4 Axial forces at ULS from first-order analysis

Span 1:

LH column: at base = 131.7 kN, at haunch = 122.0 kN

LH rafter: at column = 59.1 kN, at apex = 45.4 kN

RH rafter: at column = 61.3 kN, at apex = 47.6 kN

RH column: at base = 593.3 kN, at haunch = 583.9 kN

Span 2:

LH rafter: at column = 62.0 kN, at apex = 48.2 kN

RH rafter: at column = 60.1 kN, at apex = 46.4 kN

RH column: at base = 133.1 kN, at haunch = 123.5 kN

Notional Horizontal Forces

Span 1:

External Column

0.5% × Axial Force in Column = 0.005 × 126.9 = 0.635

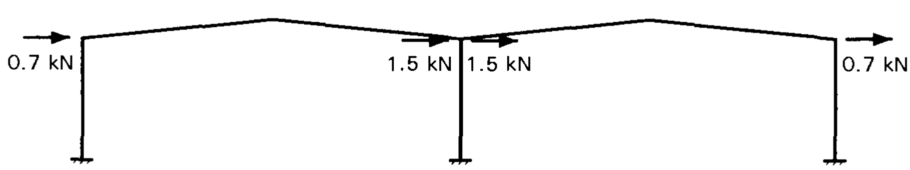
Internal Column

0.5% × Axial Force in Column = 0.005 × 588.6 = 2.943

Span 2

External Column

0.5% × Axial Force in Column = 0.005 × 128.3 = 0.642



1.5 Reduced Plastic Moments at ULS from first-order analysis

Use the reduced moment capacity for the sections to account for the co-existent axial force, calculated in accordance with BS5950-1:2000, Annex I.2. These may be taken from section tables.

$M_{rx} = p_y S_{rx} \leq 1.2\, p_y Z_X$ — Cl 4.2.5

For the axial forces in this frame under this load case

M_{pr} rafters = 404 kNm

M_{pc} external columns = 454 kNm

1.6 Load factor at formation of the first hinge, λ_1

$\lambda_1 = 0.945$ (From the frame analysis output)

1.7 Plastic collapse factor, λ_p

$\lambda_p = 1.117$ (From the frame analysis)

1.8 Member inertias, I_x

External columns: 457×191 × 74 UB: I_x = 33320 cm^4

Rafters: 457×191 × 67 UB: I_x = 29380 cm^4

Internal column: 254×254 × 73 UC: I_x = 11410 cm^4

1.9 Deflections of frame at λ_1 (formation of the first hinge)

δ_{xB} = −28.9 mm δ_{yB} = 0.6 mm

δ_{xC} = 4.8 mm δ_{yC} = 233.5 mm

δ_{xD} = 19.0 mm δ_{yD} 2.9 mm

δ_{xF} = 41.7 mm δ_{yF} = 223.1 mm

δ_{xG} = 64.7 mm δ_{yG} = 0.6 mm

2. SECOND-ORDER ANALYSIS

2.1 Axial forces in members

Use the average axial forces in the members, from first order analysis

Span 1

LH col: take mid-height P_{ULS} = (131.7 + 122.0)/2 = 126.9 kN

LH rafter: take mid-length P_{ULS} = (59.1 + 45.4)/2 = 52.3 kN

RH rafter: take mid-length P_{ULS} = (61.3 + 47.6)/2 =54.5 kN

RH col: take mid-height P_{ULS} = (583.9 +593.3)/2 = 588.6 kN

Span 2

LH rafter: take mid-length P_{ULS} = (62.0 + 48.2)/2 = 55.1 kN

RH rafter: take mid-length P_{ULS} = (60.1 + 46.4)/2 = 53.3 kN

RH col: take mid-height P_{ULS} = (133.1 +123.5)/2 = 128.3 kN

2.2 Bending deflections of the "elastic" frame

2.2.1 Stiffness reduction factors allowing for P.δ effects — A.3.2

Columns

Sum of columns P_{ULS}, ΣP_{ULS} = 126.9 + 588.6 + 128.3 = 843.8 kN

External columns: I_x = 33320 cm^4, h = 10000mm, α = 1.7

P_{cr} = 2333 kN

Internal column: I_x = 11410 cm^4, h = 10000mm, α = 1.7

P_{cr} = 799 kN

Sum of columns P_{cr}, ΣP_{cr} = 2333 + 799 + 2333 = 5464 kN

$(1 - \Sigma P_{ULS} / \Sigma P_{cr})$ = 0.846

$1/(1 - \Sigma P_{ULS} / \Sigma P_{cr})$ = 1.18

Rafters

Span 1:

Average P_{ULS} = (52.3 + 54.5)/2 = 53.4 kN

I_x = 29380 cm^4, L = 30165mm, α = 1.0

$P_{cr} = \pi^2 EI/(\alpha L)^2$ = 653 kN

$(1 - P_{ULS}/P_{cr})$ = 0.918

Span 2:

Average P_{ULS} = (55.1 + 53.3)/2 = 54.2 kN

I_x = 29380 cm^4, L = 30165mm, α = 1.0

$P_{cr} = \pi^2 EI/(\alpha L)^2$ = 653 kN

$(1 - P_{ULS}/P_{cr})$ = 0.917

2.2.2 Second order magnification factors

A.3.3

The notional sway deflection is calculated from the sum of the stiffnesses K for each of the column and rafter pairs:

Sway stiffness from column and rafter stiffness

$$K = \frac{H}{\delta} = \frac{1}{\left(\dfrac{Sh^2}{3EI_{\text{eff.R}}} + \dfrac{h^3}{3EI_{\text{eff.c}}}\right)}$$

D.2.5

Sub Frame 1 (Elastic)

$$\frac{Sh^2}{3EI_{\text{eff.R}}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26981 \times 10^4} = 0.01818$$

$$\frac{h^3}{3EI_{\text{eff.c}}} = \frac{10000^3}{3 \times 205000 \times 31508 \times 10^4} = 0.00516$$

$$K_{\text{Sub1}} = \frac{1}{0.01818 + 0.00516} = 42.8 \text{ N/mm}$$

Sub Frame 2 (Elastic)

$$\frac{Sh^2}{3EI_{\text{eff.R}}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26981 \times 10^4} = 0.01818$$

$$\frac{h^3}{3EI_{\text{eff.c}}} = \frac{10000^3}{3 \times 205000 \times 31508 \times 10^4} = 0.00516$$

$$K_{\text{Sub2}} = \frac{1}{0.01818 + 0.00516} = 42.8 \text{ N/mm}$$

Nominal Base fixity D.4

$$K_{\text{b}} = \frac{H}{\delta} = \frac{1}{\left(\frac{5h^3}{2EI} + \frac{h^3}{3EI_{\text{eff}}}\right)}$$

External Column

$$K_{\text{b ext}} = \frac{1}{(0.0366 + 0.00516)} = 24.0 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K = K_{\text{sub 1}} + K_{\text{b ext}} + K_{\text{sub 2}} + K_{\text{b int}} + K_{\text{b ext}}$$

$$\Sigma K = 42.8 + 24.0 + 42.8 + 24.0 = 133.6 \text{ N/mm}$$

$$\delta_{\text{n2}} = \frac{H}{\Sigma K_2} = \frac{4.219 \times 1000}{133.6} = 31.6 \text{ mm}$$

$$\lambda_{\text{cr1}} = \frac{h}{200\,\delta_{\text{n2}}} = \frac{10000}{200 \times 31.6} = 1.6$$

Sway mode magnification

$$\left[\frac{\lambda_{\text{cr1}}}{\lambda_{\text{cr1}} - 1}\right] = 2.7$$

2.2.3 Deflection calculations

Sway deflections A.3.4

The first order sway deflection δX_{1s} is calculated from the sum of the stiffnesses K for each of the column and rafter pairs. (The base stiffness of nominally pinned bases is not included because this is not a stability calculation like the calculation of λ_{cr}).

$$K = \frac{H}{\delta} = \frac{1}{\left(\dfrac{Sh^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)}$$ D.3.3

Sub Frame 1 (Elastic)

$$\frac{Sh^2}{3EI_R} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 29380 \times 10^4} = 0.01669$$

$$\frac{h^3}{3EI_c} = \frac{10000^3}{3 \times 205000 \times 33320 \times 10^4} = 0.00488$$

$$K_{Sub1} = \frac{1}{0.01669 + 0.00488} = 46.4 \text{ N/mm}$$

Sub Frame 2 (Elastic)

As Sub frame 1

$$K_{Sub2} = \frac{1}{0.01669 + 0.00488} = 46.4 \text{ N/mm}$$

Total Frame Stiffness

$\Sigma K \;\; = K_{sub\,1} + K_{sub\,2}$

$\Sigma K \;\; = 46.4 + 46.4 \;\; = 92.7$ N/mm

First-order sway deflection

$$\delta X_{1s} = \frac{\lambda_1 H}{\Sigma K_2} = \frac{0.950 \times 4.219 \times 1000}{92.7} = 43.23 \text{ mm}$$ D.3.3

A.3.4

$$\delta X_2 = (\delta X_1 - \delta X_{1s}) \times \left[\frac{\lambda_{cr2}}{\lambda_{cr2} - 1}\right] + \delta X_{1s} \times \left[\frac{\lambda_{cr1}}{\lambda_{cr1} - 1}\right]$$

$$\delta Y_2 = \delta Y_1 \times \left[\frac{\lambda_{cr2}}{\lambda_{cr2} - 1}\right]$$

Values of δX_1 and δY_1 are taken from first order analysis (See Sheet 7).

δ_{xB} = (-28.9 - 43.23) × 1.183 + 43.23 × 2.7 = 32.1 mm
δ_{yB} = 0.6 × 1.183 = 0.7 mm

δ_{xC} = (4.8 - 43.23) × 1.183 + 43.23 × 2.7 = 71.9 mm
δ_{yC} = 233.5 × 1.183 = 276.1 mm

δ_{xD} = (19.0 - 43.23) × 1.183 + 43.23 × 2.7 = 88.7 mm
δ_{yD} = 2.9 × 1.183 = 3.4 mm

δ_{xF} = (41.7 - 43.23) × 1.183 + 43.23 × 2.7 = 115.6 mm
δ_{yF} = 223.1 × 1.183 = 263.8 mm

δ_{xG} = (64.7 - 43.23) × 1.183 + 43.23 × 2.7 = 142.8 mm
δ_{yG} = 0.6 × 1.183 = 0.7 mm

2.3 Bending deflections of the "plastic" frame

A.4

2.3.1 Stiffness reduction factors to allow for P.δ effects

A.4.2

Columns: as the "elastic" frame

LH Column: $(1 - P_{ULS} / P_{cr})$ = 0.946

Central Column: $(1 - P_{ULS} / P_{cr})$ = 0.263

Rafters: as the "elastic" frame because that used $\alpha = 1.0$

Span 1: $(1 - P_{ULS} / P_{cr})$ = 0.917

Span 2: $(1 - P_{ULS} / P_{cr})$ = 0.917

2.3.2 Second Order Magnification Factor

A.4.3

Sway mode magnification factor

The notional sway deflection is calculated from the sum of the stiffnesses K of the rafter and column pairs between plastic hinges and the base stiffness of each column.

Sway stiffness from column and rafter stiffness.

$$K_2 = \frac{1}{\left(\dfrac{Sh^2}{3EI_{\text{eff.R}}} + \dfrac{h^3}{3EI_{\text{eff.c}}}\right)}$$

D.3.5

Sub Frame 1 (Plastic)

$$\frac{Sh^2}{3EI_{\text{eff.R}}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26981 \times 10^4} = 0.01818$$

$$\frac{h^3}{3EI_{\text{eff.c}}} = \frac{10000^3}{3 \times 205000 \times 31508 \times 10^4} = 0.00516$$

$$K_{\text{sub 1}} = \frac{1}{0.01818 + 0.00516} = 42.84 \text{ N/mm}$$

Sub Frame 2 (Plastic)

$$\frac{Sh^2}{3EI_{\text{eff.R}}} = \frac{30165 \times (10000)^2}{3 \times 205000 \times 26981 \times 10^4} = 0.01818$$

$$\frac{h^3}{3EI_{\text{eff.c}}} = \frac{10000^3}{3 \times 205000 \times 3003 \times 10^4} = 0.05416$$

$$K_{\text{sub 2}} = \frac{1}{0.01818 + 0.05416} = 13.82 \text{ N/mm}$$

Nominal Base fixity

D.4

$$K_b = \frac{H}{\delta} = \frac{1}{\left(\dfrac{5h^3}{2EI} + \dfrac{h^3}{3EI_{\text{eff}}}\right)}$$

External Column

$$K_{b\ ext} = \frac{1}{(0.0366 + 0.00516)} = 23.95 \text{ N/mm}$$

Internal Column

$$K_{b\ int} = \frac{1}{(0.1069 + 0.0542)} = 6.2 \text{ N/mm}$$

Total Frame Stiffness

$$\Sigma K = K_{sub\ 1} + K_{b\ int} + K_{sub\ 2} + K_{b\ int} + K_{b\ ext}$$

$$\Sigma K = 42.84 + 23.95 + 13.82 + 6.2 + 23.95 \quad = 110.8 \text{ N/mm}$$

Second-order notional sway deflection for the plastic frame

$$\delta_{np} = \frac{H}{\Sigma K_2} = \frac{4.219 \times 1000}{110.8} = 39.8 \text{ mm}$$

Critical Buckling Ratio D.3.5

$$\lambda_{crp} = \frac{h}{200\,\delta_{np}} = \frac{10000}{200 \times 39.8} = 1.31$$

Magnification Factor A.4.3

$$\left[\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right] = 4.20$$

2.3.3 Deflections of the "plastic" frame from gravity loads A.4.5

Loads

The loads applied to the "plastic" frame = $(\lambda_p - \lambda_1)$(loads at ULS)

$(\lambda_p - \lambda_1) = (1.117 - 0.95) = 0.167$

Slope of rafters $\alpha_1 = \alpha_2 = \alpha = 6°$, giving Cos α = 0.9945

Assuming both service load and imposed load are specified "on plan",

at ULS, $w_{v.plan} = 1.26 + 5.76 \quad = 7.02$ kN/m giving a transverse load on the "plastic frame"

$= (\lambda_p - \lambda_1)(w_{v.plan} \text{ at ULS})\text{Cos}^2\alpha$

$= 0.171 \times 7.02(0.9945)^2 \quad = 1.16 \text{ kN/m}$

Assuming both dead load and self-weight are values "along the slope",

at ULS, $w_{v.slope} = 0.84 + 1.12 \quad = 1.96 \text{ kN/m}$

giving a transverse load on the "plastic frame"

$= (\lambda_p - \lambda_1)(w_{v.slope} \text{ at ULS})\text{Cos}\,\alpha$

$= 0.171 \times 1.96 \times 0.9945 \quad = 0.33 \text{ kN/m}$

Summing loads from components "on plan" and "along the slope",

$w_P = 1.16 + 0.33 \quad = 1.49 \text{ kN/m}$

Sway A.4.5

This arises due to the rotation of the column without an adjacent hinge.

First-order end slope of the rafter as a simply supported beam, $\theta_R = \dfrac{w_P S^3}{24\, EI_R}$

Second-order end slope of the rafter, $\theta_{R2} = \dfrac{w_P S^3}{24\, EI_R}\left(\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}\right)$

$E = 205000 \text{ N/mm}^2$

$I_x = 29380 \text{ cm}^4$

$$\theta_{R2} = \frac{1.49 \times 30165^3}{24 \times 205000 \times 29380 \times 10^4} \times 4.20 = 0.11834 \text{ radians}$$

Horizontal deflection of Point B, C, D $= h\theta_R$

$= 10000 \times 0.11834$

$= 1183.4 \text{ mm}$

Mid-span drop

Deflection given by value for simply supported beam of span equal to the developed length of the rafters.

Span 1:

$$\delta_{b2} = \frac{5}{384}\frac{w_P\, S^4}{E\, I_R}\left[\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right]$$

$$\delta_{b2} = \frac{5}{384} \times \frac{1.47 \times 30165^4}{205000 \times 29380 \times 10^4} \times 4.20 = 1115.5 \text{ mm}$$

$$\delta_{apex} = \frac{\delta_{B2}}{\cos\,\alpha} = \frac{1115.5}{0.9945} = 1121.7 \text{ mm}$$

Span 2: as Span 1

Spread — A.4.5

This is caused by the drop of the angle in the rafter which is at the apex in this frame. Because this frame is a symmetrical pitched roof portal, the drop of the angle is the mid-span drop calculated above.

Span 1:

Full span δ_{spread} $= \delta_{B2}\,(\text{Sin}\,\alpha_1 + \text{Sin}\,\alpha_2) = \delta_{B2}\,(2\text{Sin}\,6°)$

$= 1115.5\,(2 \times 0.1045) = 233.2$ mm

Half span δ_{spread} $= \delta_{B2}\,\text{Sin}\,\alpha_1 = 1115.5 \times 0.1045 = 116.6$ mm

Span 2: as Span 1

Column hinge horizontal displacement

The hinge occurs at the underside of the haunch, which is at a distance from the neutral axis of the rafter, causing an additional horizontal displacement.

Second-order end slope of the rafter, $\theta_{R2} = 0.11834$ radians — (Sheet 34)

Off-set of the hinge below the rafter, e $= 10000 - 9400 = 600$ mm

Horizontal deflection of Point $M = e\theta_R$ $= 600 \times 0.11834 = 71.0$ mm

2.3.4 Deflections of the "plastic" frame from horizontal loads

Loads

The unfactored loads applied to the "elastic frame" included the horizontal loads and were in proportion to the ULS loads, so the loads applied to the "plastic" frame = $(\lambda_p - \lambda_1)$(loads at ULS)

$(\lambda_p - \lambda_1) = (1.117 - 0.950) = 0.167$

Slope of rafters $\alpha_1 = \alpha_2 = \alpha = 6°$, giving $\cos\alpha = 0.9945$

Sway

Total of column forces = 843.8 kN (Sheet 27)

0.5% × of total of column forces (elastic frame) = 0.005×843.75 = 4.22 kN

Additional horizontal load on plastic frame

$H = 0.167 \times 4.22 \times 1000 = 705\text{N}$

For a multi-span frame, the sway deflection is calculated from the sum of the stiffnesses K for each of the spans:

$$K_s = \frac{1}{\left(\dfrac{S\,h^2}{3EI_R} + \dfrac{h^3}{3EI_c}\right)} \times \frac{1}{\left[\dfrac{\lambda_{crp}}{\lambda_{crp} - 1}\right]}$$ A.4.6

$K_{sub\,1} = 44.5$

$K_{sub\,2} = 29.8$

Total Frame Stiffness (First Order)

$\Sigma K_s = K_{sub\,1} + K_{sub\,2}$

$\Sigma K_s = 44.5 + 29.8 = 74.3$ N/mm

Second-order least sway deflection of the column top

$$\Delta_2 = \frac{H}{\Sigma K_s} = \frac{0.705 \times 1000}{74.3} \times 4.20 = 39.8 \text{ mm}$$

Mid-span drop

Second order sagging deflection of a straight rafter:

$$\delta_{sm2} = \frac{ML_r^2}{16EI_R}\left[\frac{\lambda_{crp}}{\lambda_{crp} - 1}\right] \quad \text{where } M = \Sigma\, H_i\, h_i$$

H_i is the proportion of the horizontal force carried by each sub frame calculated as a proportion of the stiffness K_2.

Sub frame 1:

Drop in the apex of rafter span 1 due to horizontal load.

$$\therefore \quad \delta_{sm2} = \frac{0.425 \times 1000 \times 10000\ (30165)^2}{16 \times 205000 \times 26951 \times 10^4} \times 4.20 = 16.8 \text{ mm}$$

Sub frame 2:

Drop in the apex of rafter span 2 due to horizontal load.

$$\therefore \quad \delta_{sm2} = \frac{0.280 \times 1000 \times 10000\ (30165)^2}{16 \times 205000 \times 26951 \times 10^4} \times 4.20 = 11.1\text{mm}$$

Spread

Span 1:

Full span spread $= \delta_{sm2}\,(\text{Sin}\,\alpha_1 + \text{Sin}\,\alpha_2) = 16.8\ (\text{Sin } 6° + \text{Sin } 6°)$

$= 16.8\ (0.1045 + 0.1045) = 3.5$ mm

Half span spread $= \delta_{sm2}\,(\text{Sin}\alpha_1) = 16.8\ (\text{Sin } 6°)$

$= 16.8\ (0.1045) = 1.8$ mm

Span 2:

Full span spread $= \delta_{sm2}\,(\text{Sin}\,\alpha_1 + \text{Sin}\,\alpha_2) = 11.1\ (\text{Sin } 6° + \text{Sin } 6°)$

$= 11.1\ (0.1045 + 0.1045) = 1.2$ mm

Half-span spread $= \delta_{sm2}\,(\text{Sin}\,\alpha_1) = 11.1\ (\text{Sin } 6°)$

$= 11.1\ (0.1045) = 2.3$ mm

2.4 Axial forces for the energy calculation

The total of the axial loads in the columns is not affected by second-order effects because of vertical equilibrium, so $P_2 = P_1$ which is taken as the mid-height value calculated in 2.2 above.

Span 1:

LH col: take mid-height $P_1 = 126.9$ kN

RH col: take mid-height $P_1 = 588.6$ kN

Span 2:

RH col: take mid-height $P_1 = 128.3$ kN

The rafter axial forces are affected by the drop of the rafters at mid-span.

Span 1:

Mid-span drop = 1414.7 mm

Increase in P_2 $= \{1/[1-(\delta_a/h_a)]-1\} = \{1/[1-(1414.7/11577)] - 1\}$

$= 0.139$

LH rafter: take mid-length $P_1 = 52.3$ kN

Mid-span axial = 45.4, giving $P\Delta$ increase $= 0.139 \times 45.4 = 6.3$ kN

$P_2 = 52.3 + 6.3 = 58.6$ kN

RH rafter: take mid-length $P_1 = 54.5$ kN

Mid-span axial = 47.6, giving $P\Delta$ increase $= 0.139 \times 47.6 = 6.6$ kN

$P_2 = 54.5 + 6.6 = 61.1$ kN

Span 2:

Total Mid-span drop = 1396.6 mm

Increase in P_2 $= \{1/[1-(\delta_a/h_a)]-1\} = \{1/[1-(1396.6/11577)] - 1\}$

$= 0.137$

LH rafter: take mid-length $P_1 = 55.1$ kN

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CALCULATION SHEET

Mid-span axial = 48.2, giving $P\Delta$ increase = 0.137×48.2 = 6.6 kN

P_2 = 55.1 + 6.6 = 61.7 kN

RH rafter: take mid-length P_1 = 53.3 kN

Mid-span axial = 46.4, giving $P\Delta$ increase = 0.137×46.4 = 6.4 kN

P_2 = 53.3 + 6.4 = 59.6 kN

2.5 Second-order Energy Summation

A.6

The energy summation is required to calculate λ_M following the methods in A.2.2

Element for evaluation of Pc*phi*s*d(phi)	AB	BC	CD	ED	DF	FG	dG	Hd		
X-AXIS DEFLECTIONS										
Deflections from the "elastic" frame										
dxa	0.0	32.1	71.9	0.0	88.7	115.6	142.8	0.0		
dxb	32.1	71.9	88.7	88.7	115.6	142.8	142.8	142.8		
(dxb - dxa)	**32.1**	**39.9**	**16.8**	**88.7**	**26.8**	**27.2**	**0.0**	**142.8**		
Deflections from the "plastic" frame										
From gravity loads										
Sway of top of elastic column										
dxa	0.0	1183.4	1183.4	1183.4	1183.4	1183.4	1183.4	0.0		
dxb	1183.4	1183.4	1183.4	1183.4	1183.4	1183.4	1183.4	1183.4		
(dxb - dxa)	**1183.4**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**1183.4**		
Spread										
dxa	0.0	0.0	116.6	0.0	233.2	583.0	699.6	0.0		
dxb	0.0	116.6	233.2	233.2	583.0	699.6	699.6	699.6		
(dxb - dxa)	**0.0**	**116.6**	**116.6**	**233.2**	**349.8**	**116.6**	**0.0**	**699.6**		
Column hinge horizontal displacement										
dxa	0.0	0.0	0.0	0.0	0.0	0.0	71.0	0.0		
dxb	0.0	0.0	0.0	0.0	0.0	0.0	0.0	71.0		
(dxb - dxa)	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**-71.0**	**71.0**		
From horizontal loads	AB	BC	CD	ED	DF	FG	dG	Hd		
Sway										
dxa	0.0	39.8	39.8	0.0	39.8	39.8	39.8	0.0		
dxb	39.8	39.8	39.8	39.8	39.8	39.8	39.8	39.8		
(dxb - dxa)	**39.8**	**0.0**	**0.0**	**39.8**	**0.0**	**0.0**	**0.0**	**39.8**		
Spread										
dxa	0.0	0.0	1.8	0.0	3.5	4.7	5.8	0.0		
dxb	0.0	1.8	3.5	3.5	4.7	5.8	5.8	5.8		
(dxb - dxa)	**0.0**	**1.8**	**1.8**	**3.5**	**1.2**	**1.2**	**0.0**	**5.8**		
Total of (dxb - dxa) at collapse	**1255.2**	**158.2**	**135.2**	**365.2**	**377.8**	**145.0**	**-71.0**	**2142.4**		
Y-AXIS DEFLECTIONS										
Deflections from the "elastic" frame										
dya	0.0	0.7	276.1	0.0	3.4	263.8	0.7	0.0		
dyb	0.7	276.1	3.4	3.4	263.8	0.7	0.7	0.7		
(dyb - dya)	**0.7**	**275.4**	**-272.7**	**3.4**	**260.4**	**-263.1**	**0.0**	**0.7**		
Deflections from the "plastic" frame										
Mid-span drop from gravity loads										
dya	0.0	0.0	1121.7	0.0	0.0	1121.7	0.0	0.0		
dyb	0.0	1121.7	0.0	0.0	1121.7	0.0	0.0	0.0		
(dyb - dya)	**0.0**	**1121.7**	**-1121.7**	**0.0**	**1121.7**	**-1121.7**	**0.0**	**0.0**		
Deflections from the "plastic" frame										
Mid-span drop from horizontal loads										
dya	0.0	0.0	16.8	0.0	0.0	11.1	0.0	0.0		
dyb	0.0	16.8	0.0	0.0	11.1	0.0	0.0	0.0		
(dyb - dya)	**0.0**	**16.8**	**-16.8**	**0.0**	**11.1**	**-11.1**	**0.0**	**0.0**		
Total of (dyb - dya) at collapse	**0.7**	**1414.0**	**-1411.2**	**3.4**	**1393.2**	**-1395.9**	**0.0**	**0.7**		
psi (angle from X axis)	90.0	6.0	-6.0	90.0	6.0	-6.0	90.0	90.0		
[(dxb - dxa) at collapse]*Sin(psi)	1255.2	16.5	-14.1	365.2	39.5	-15.2	-71.0	2142.4		
[(dyb - dya) at collapse]*Cos(psi)	0.0	1406.2	-1403.5	0.0	1385.6	-1388.3	0.0	0.0		
phi * s at collapse	1255.2	1422.7	-1417.6	365.2	1425.0	-1403.4	-71.0	2142.4		
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.883	1.151	1.151	1.411	1.411	1.701		
Shortening = phi*s*d(phi) (modulus)	**1255.2**	**1422.7**	**1251.6**	**420.4**	**1640.4**	**1980.7**	**100.2**	**3643.8**		
AXIAL FORCES										
Pc for columns and rafters at ULS	126.9	52.3	54.5	588.6	55.1	53.3	128.3	128.3		
Total midspan drop		1414.7	1414.7		1396.6	1396.6				
Midspan height		11577	11577		11577	11577				
Increase rafter mispan axial by {1/(1-drop/height) - 1}		0.139	0.139		0.137	0.137				
Midspan axial		45.4	47.6		48.2	46.4				
Increase in rafter axial		6.3	6.6		6.6	6.4				
Design axial	126.9	58.6	61.1	588.6	61.7	59.6	128.3	128.3		
Incremental energy = Pc*phi*s*d(phi)	**159.2**	**83.3**	**76.4**	**247.5**	**101.2**	**118.1**	**12.9**	**467.5**	**Sum =**	**1266**
WORK DONE ROTATING HINGES										
Element for evaluating Mprd(phi)	**AB**	**Ba**	**ab**	**bD**	**Dc**	**cG**	**dG**	**Hd**		
MprA	0.0	0.0	404.0	0.0	0.0	404.0	454.0	0.0		
MprB	0.0	404.0	404.0	404.0	404.0	0.0	0.0	454.0		
MprA + MprB	0.0	404.0	808.0	404.0	404.0	404.0	454.0	454.0		
Incremental rotn = d(phi) from mechanism	1.000	1.000	0.883	1.151	1.151	1.411	1.411	1.701		
Mpr*d(phi)	**0.0**	**404.0**	**713.3**	**465.0**	**465.0**	**570.2**	**640.7**	**772.2**	**Sum =**	**4031**

Factor on lambda_p	0.686
lambda_p from first-order analysis	1.117
lambda_M	**0.766**

2.6 Load factor at failure, λ_M

A.6

$\Sigma\, P_2\, \phi s d\phi \;=\; 1266\ \phi$

$\Sigma\, M_{pr}\, d\phi \;=\; 4031\ \phi$

$$\lambda_M\ /\ \lambda_p = \left[1 - \left(\frac{\Sigma\ (P_2\ \phi\ s\ d\phi)}{\Sigma\ (M_{pr}\ d\phi)}\right)\right]$$

$$\lambda_M\ /\ \lambda_p = \left[1 - \left(\frac{1266\ \phi}{4031\ \phi}\right)\right] = 0.686$$

$\lambda_M \;=\; 0.686 \times \lambda_p \;=\; 0.686 \times 1.117 \;=\; 0.766$

This frame exceeds the $h/1000$ limit for the sway-check method by a factor of about 3. The above calculations demonstrate that such a flexible frame has a serious reduction in capacity from in-plane stability effects.